

# Nonlinear supersymmetry

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## ABSTRACT

We describe the Akulov-Volkov nonlinear realization of supersymmetry its superfield formulation and the corresponding standard form for nonlinear superfields are presented. We derive the Akulov-Volkov nonlinear effective Lagrangian and show its relation to the spontaneous breaking of supersymmetry.

## RESUMEN

Se describe la realización no-lineal de la supersimetría de Akulov-Volkov. Se presenta su formulación de supercampos y su correspondiente forma estándar para supercampos no-lineales. Se obtiene el lagrangiano efectivo de Akulov-Volkov y se muestra su relación con el rompimiento espontáneo de supersimetría.

## 1. INTRODUCCIÓN

Supersymmetry is one of the most important recent discoveries in Theoretical Physics [1]. It is a boson-fermion symmetry. Exact supersymmetry demands, for example, boson-fermion mass degeneracy. Since this degeneracy is not observed in the phenomenology of elementary particles at the presently measured energies, supersymmetry must be broken. It can be broken either explicitly or spontaneously. We will consider the spontaneous breaking of supersymmetry as described by nonlinear realizations and phenomenological Lagrangians, which has been extremely successful in

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the study of low-energy pion interactions [2]. In section 2 we will present the nonlinear Akulov-Volkov (AV) realization of simple supersymmetry and its extension to other fields [3]. In section 3 we will describe the corresponding nonlinear superfields [4] and its constraint eqs. In section 4, following Ivanov and Kapustnikov [5], we show that any superfields corresponding to a linear realization of supersymmetry can be transformed to a standart form with its components transforming nonlinearly under supersymmetry. Finally, in section 5 we construct the nonlinear Akulov-Volkov phenomenological Lagrangian.

## 2. THE AKULOV-VOLKOV NONLINEAR REALIZATION OF SUPERSYMMETRY.

The concepts and methods of nonlinear realizations and phenomenological Lagrangians as developed in ref. 2 can be applied to supersymmetric theories. Taking for G the supergroup of N = 1 supersymmetry transformations and the Poincaré group and H as the Poincaré group, we can treat supersymmetry and look for its low-energy manifestations. The lack of supersymmetry multiplets suggest that supersymmetry must be strongly and spontaneously broken. From our experience with the sigma model we imagine that the supersymmetric partners of the known elementary particles are very heavy and therefore there must be an energy range where the contributions to the effective Lagrangian of the supersymmetry partners can be neglected. The supergroup G is generated by the supersymmetry algebra [1].

$$\left\{ \Omega_\alpha, \bar{\Omega}_\alpha \right\} = 2\sigma_{\alpha\dot{\alpha}}^m P_m \quad \left\{ \Omega_\alpha, \Omega_\beta \right\} = \left\{ \bar{\Omega}_\alpha, \bar{\Omega}_\beta \right\} = [\Omega_\alpha, P_m] = [\bar{\Omega}_\alpha, P_m] = 0$$

Instead of Goldstone bosons, the spontaneous breaking of rigid supersymmetry gives rise to Goldstone fermions. The Goldstone fermions transform nonlinearly under the broken supersymmetry generators  $\Omega_\alpha, \bar{\Omega}_\alpha$  and linearly under the Poincaré generators. The first nonlinear realization of simple (N = 1) supersymmetry was found by Akulov and Volkov [3]. It involves as a basic entity the nonlinearly and inhomogeneously transforming Goldstone spinor  $\tilde{\lambda}(x)$ , according to the following transformation law:

$$\begin{aligned} \delta_\xi \tilde{\lambda}_\alpha(x) &= \frac{1}{k} \xi_\alpha - ik (\tilde{\lambda}(x) \sigma^m \bar{\xi} - \xi \sigma^m \tilde{\lambda}(x)) \partial_m \tilde{\lambda}_\alpha(x) \\ \delta_\xi \bar{\tilde{\lambda}}_{\dot{\alpha}}(x) &= \frac{1}{k} \bar{\xi}_{\dot{\alpha}} - ik (\tilde{\lambda}(x) \sigma^m \bar{\xi} - \xi \sigma^m \tilde{\lambda}(x)) \partial_m \bar{\tilde{\lambda}}_{\dot{\alpha}}(x) \end{aligned} \quad (1)$$

$\lambda_\alpha(x) = \begin{pmatrix} \lambda_1(x) \\ \lambda_2(x) \end{pmatrix}$  is a two-component spinor field, a linear representation of the unbroken Poincaré subgroup of the full supergroup. The parameter k is an arbitrary constant of dimension (mass)<sup>-2</sup> and k<sup>-1/2</sup> denotes the energy scale of supersymmetry breaking. It may be suppressed and then reinstated at any stage of our discussion, by dimensional analysis. It

is not hard to show that the transformation (1) closes into a translation as demanded by the supersymmetry algebra:

$$\begin{aligned} [\delta_{\eta'}, \delta_{\xi}] \tilde{\lambda}_{\alpha} &= -2i (n\sigma^m \bar{\xi} - \xi \sigma^m \bar{\eta}) \partial_m \tilde{\lambda}_{\alpha} \\ [\delta_{\eta'}, \delta_{\xi}] \tilde{\lambda}_{\dot{\alpha}} &= -2i (n\sigma^m \bar{\xi} - \xi \sigma^m \bar{\eta}) \partial_m \tilde{\lambda}_{\dot{\alpha}} \end{aligned} \quad (2)$$

With the help of the intrinsically nonlinearly transforming AV-field  $(\tilde{\lambda}(x), \tilde{\lambda}(x))$  it is possible to realize supersymmetry transformations nonlinearly on any Lorentz covariant field  $\tilde{C}_I(x)$ :

$$\delta_{\xi} \tilde{C}_I(x) = -i (\tilde{\lambda}(x) \sigma^m \bar{\xi} - \xi \sigma^m \tilde{\lambda}(x)) \partial_m \tilde{C}_I(x) \quad (3)$$

where I represents any set of Lorentz or other indexes. Again

$$[\delta_{\eta'}, \delta_{\xi}] \tilde{C}_I(x) = -2i (\eta \sigma^m \bar{\xi} - \xi \sigma^m \bar{\eta}) \partial_m \tilde{C}_I(x).$$

The transformation (3) is the analogue of the standard form of a nonlinear transformation according to ref. 2. A field transforming this way will be called a "standard matter" field (SM-field).

The AV nonlinear transformation law (1) can be derived in the following way: consider a covariantly transforming hypersurface in superspace defined as

$$k \tilde{\lambda}_{\alpha}(x) = \theta_{\alpha}, \quad k \tilde{\lambda}_{\dot{\alpha}}(x) = \bar{\theta}_{\dot{\alpha}}$$

If we realize a pure supersymmetric motion in superspace as:

$$\begin{aligned} x^m + x'^m &= x^m + i \theta \sigma^m \bar{\xi} - i \xi \sigma^m \bar{\theta} \\ \theta + \theta' &= \theta + \xi \\ \bar{\theta} + \bar{\theta}' &= \bar{\theta} + \bar{\xi} \end{aligned} \quad (4)$$

then the transformation of the surface under this motion is

$$\begin{aligned} \tilde{\lambda}'_{\alpha}(x') &= \tilde{\lambda}_{\alpha}(x) + \frac{1}{k} \xi_{\alpha} \\ \tilde{\lambda}'_{\dot{\alpha}}(x') &= \tilde{\lambda}_{\dot{\alpha}}(x) + \frac{1}{k} \bar{\xi}_{\dot{\alpha}} \end{aligned} \quad (5)$$

and therefore

$$\begin{aligned} \tilde{\lambda}'_{\alpha}(x + i \theta \sigma \bar{\xi} - i \xi \sigma \bar{\theta}) &= \tilde{\lambda}_{\alpha}(x) + \frac{1}{k} \xi_{\alpha} \\ \tilde{\lambda}'_{\dot{\alpha}}(x + i \theta \sigma \bar{\xi} - i \xi \sigma \bar{\theta}) &= \tilde{\lambda}_{\dot{\alpha}}(x) + \frac{1}{k} \bar{\xi}_{\dot{\alpha}} \end{aligned}$$

From this we may compute the changes of the fields at the same space-time point:

$$\delta_{\xi} \tilde{\lambda}_{\alpha} = \tilde{\lambda}'_{\alpha}(x) - \tilde{\lambda}_{\alpha}(x) = \frac{1}{k} \xi_{\alpha} - ik (\tilde{\lambda}(x) \sigma^m \bar{\xi} - \xi \sigma^m \tilde{\lambda}(x)) \partial_m \tilde{\lambda}_{\alpha}(x) + \dots$$

$$\delta \xi \tilde{\lambda}_\alpha = \tilde{\lambda}'_\alpha(x) - \tilde{\lambda}_\alpha(x) = \frac{1}{k} \tilde{\xi}_\alpha - ik (\tilde{\lambda}(x) \sigma^m \tilde{\xi} - \xi \sigma^m \tilde{\lambda}(x)) \partial_m \tilde{\lambda}_\alpha(x) + \dots$$

This nonlinear transformation law mixes  $\tilde{\lambda}(x)$  and  $\tilde{\lambda}'(x)$ . We can find a simpler nonlinear realization taking into account that supersymmetry may also be realized as another motion in superspace:

$$\begin{aligned} x'^m &= x^m + 2i \theta \sigma^m \tilde{\xi} \\ \theta'_\alpha &= \theta_\alpha + \xi_\alpha \\ \bar{\theta}'_\alpha &= \bar{\theta}_\alpha + \tilde{\xi}_\alpha \end{aligned} \quad (6)$$

corresponding to the following parametrization of left cosets of superspace  $\approx$  Super-Poincaré group/Poincaré group

$$G(x, \theta, \bar{\theta}) = e^{i(-x \cdot P + \theta Q)} e^{i \bar{\theta} \bar{Q}}$$

The corresponding nonlinear transformation law obtained in this way is:

$$\delta_\xi \lambda_\alpha(x) = \xi_\alpha - 2i \lambda(x) \sigma^m \tilde{\xi} \partial_m \lambda_\alpha(x) \quad (7)$$

This transformation law is simple because  $\lambda$  transforms into  $\lambda$  and  $\partial_m \lambda$ . Of course, this is not an independent nonlinear realization of supersymmetry: the fields  $\lambda$  and  $\tilde{\lambda}$  are related by a field redefinition:

$$\lambda(x) = \tilde{\lambda}(y) \quad y^m = x^m - i \tilde{\lambda}(y) \sigma^m \tilde{\lambda}(y) \quad (8)$$

or more explicitly

$$\begin{aligned} \lambda(x) = \tilde{\lambda}(x) &- i \{ \tilde{v}^m - i \tilde{v}^n \partial_n \tilde{v}^m - \tilde{v}^n \partial_n \tilde{v}^\ell \partial_\ell \tilde{v}^m - \frac{1}{2} \tilde{v}^n \tilde{v}^\ell \partial_n \partial_\ell \tilde{v}^m \} \partial_m \tilde{\lambda}(x) \\ &- \frac{1}{2} \tilde{v}^m \tilde{v}^n \partial_m \partial_n \tilde{\lambda}(x) \end{aligned}$$

$$\text{with } \tilde{v}^m(x) = \tilde{\lambda}(x) \sigma^m \tilde{\lambda}(x).$$

The corresponding inverse transformation is:

$$\begin{aligned} \tilde{\lambda}(x) = \lambda(x) &+ i \{ v^m + i v^n \partial_n \lambda \sigma^m \bar{\lambda} - \lambda \sigma^m \partial_n \bar{\lambda} \} + v^n \partial_n v^\ell \partial_\ell v^m \\ &+ \frac{1}{2} v^n v^\ell \partial_n \partial_\ell v^m \} \partial_m \lambda - \frac{1}{2} v^n v^\ell \partial_n \partial_\ell \lambda \end{aligned}$$

$$\text{with } v^m(x) = \lambda(x) \sigma^m \bar{\lambda}(x).$$

The corresponding transformation law for a SM-field is

$$\delta_\xi C_I(x) = 2i \lambda(x) \sigma^m \tilde{\xi} \partial_m C_I(x) \quad (9)$$

### 3. NONLINEAR AKULOV-VOLKOV SUPERFIELDS.

Linear and nonlinear realizations of Supersymmetry transformations are most conveniently formulated in terms of superfields [1]. These are functions of the variables  $x^m, \theta_\alpha, \bar{\theta}_\alpha : A(x, \theta, \bar{\theta})$ . If the super-

symmetry transformation law is such that the change of the spacetime quantum field  $a(x)$  is

$$\delta_{\xi} a(x) = (\xi \delta + \bar{\xi} \bar{\delta}) x a(x)$$

is known to lead to

$$[\delta_{\eta}, \delta_{\xi}] x a(x) = -2i (\eta \sigma^m \bar{\xi} - \xi \sigma^m \bar{\eta}) \partial_m a(x)$$

then it is possible to show that

$$A(x, \theta, \bar{\theta}) = \exp(\theta \delta + \bar{\theta} \bar{\delta}) x a(x) \quad (10)$$

is a superfield. A linear superfield transforms as follows:

$$\begin{aligned} \delta A &= (\xi \Omega + \bar{\xi} \bar{\Omega}) A \\ &= \left\{ \xi^{\alpha} \left[ \frac{\partial}{\partial \theta^{\alpha}} - i \sigma_{\alpha\dot{\alpha}}^m \cdot \bar{\theta}^{\dot{\alpha}} \partial_m \right] + \bar{\xi}_{\dot{\alpha}} \left[ \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i \theta^{\alpha} \sigma_{\alpha\dot{\beta}}^m \cdot \bar{\epsilon}^{\dot{\beta}} \partial_m \right] \right\} A \end{aligned} \quad (11)$$

the superfields corresponding to  $\tilde{\lambda}(x)$ ,  $\lambda(x)$  and  $C_I(x)$  are:

$$\begin{aligned} \tilde{\Lambda}_{\alpha}(x, \theta, \bar{\theta}) &= \exp(\theta \delta + \bar{\theta} \bar{\delta}) x \tilde{\lambda}_{\alpha}(x) \\ \Lambda_{\alpha}(x, \theta, \bar{\theta}) &= \exp(\theta \delta + \bar{\theta} \bar{\delta}) x \lambda_{\alpha}(x) \\ C_I(x, \theta, \bar{\theta}) &= \exp(\theta \delta + \bar{\theta} \bar{\delta}) x C_I(x) \end{aligned} \quad (12)$$

The AV superfields  $\tilde{\Lambda}$ ,  $\Lambda$  and the standard matter superfield  $C_I$  can also be defined through constraints. Actually this method of definition has the advantage of being immediately extended to curved superspace. To derive these constraints one uses the fundamental identity [1].

$$\begin{aligned} D_{\alpha} \exp(\theta \delta + \bar{\theta} \bar{\delta}) x &= \exp(\theta \delta + \bar{\theta} \bar{\delta}) \delta_{\alpha} x \\ \bar{D}_{\dot{\alpha}} \exp(\theta \delta + \bar{\theta} \bar{\delta}) x &= \exp(\theta \delta + \bar{\theta} \bar{\delta}) \bar{\delta}_{\dot{\alpha}} x \end{aligned} \quad (13)$$

The differential operators  $D$ ,  $\bar{D}$  are:

$$\begin{aligned} D_{\alpha} &= \frac{\partial}{\partial \theta^{\alpha}} + i \sigma_{\alpha\dot{\alpha}}^m \cdot \bar{\theta}^{\dot{\alpha}} \partial_m \\ \bar{D}_{\dot{\alpha}} &= \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i \theta^{\alpha} \sigma_{\alpha\dot{\beta}}^m \partial_m \end{aligned} \quad (14)$$

This leads to the following constraint eqs. for the superfields  $\tilde{\Lambda}$ ,  $\Lambda$  and  $\tilde{C}_I$ :

$$D_{\beta} \tilde{\Lambda}_{\alpha} = \epsilon_{\alpha\beta} + i \sigma_{\beta\dot{\rho}}^m \cdot \bar{\Lambda}^{\dot{\rho}} \partial_m \tilde{\Lambda}_{\alpha} \quad (15)$$

$$\bar{D}_{\dot{\beta}} \tilde{\Lambda}_{\alpha} = -i \tilde{\Lambda}^{\dot{\rho}} \sigma_{\rho\dot{\beta}}^m \partial_m \tilde{\Lambda}_{\alpha}$$

$$D_{\beta} \tilde{\Lambda}_{\dot{\alpha}} = i \sigma_{\beta\dot{\beta}}^m \cdot \tilde{\Lambda}^{\dot{\beta}} \partial_m \tilde{\Lambda}_{\dot{\alpha}} \quad (16)$$

$$D_{\dot{\beta}} \bar{\Lambda}_{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}} - i \tilde{\Lambda}^{\beta} \sigma_{\beta\dot{\beta}}^m \partial_m \bar{\Lambda}_{\dot{\alpha}}$$

$$D_{\beta} \tilde{C}_I = i \sigma_{\beta\rho}^m \tilde{\Lambda}^{\rho} \partial_m \tilde{C}_I$$

$$\bar{D}_{\dot{\beta}} \tilde{C}_I = -i \tilde{\Lambda}^{\rho} \sigma_{\rho\dot{\beta}}^m \partial_m \tilde{C}_I$$

The constraints for the new superfields  $\Lambda$ ,  $\bar{\Lambda}$  and  $C_I$  look simpler:

$$D_{\beta} \Lambda_{\alpha} = \epsilon_{\alpha\beta}$$

$$\bar{D}_{\dot{\beta}} \Lambda_{\alpha} = -2i \Lambda^{\rho} \sigma_{\rho\dot{\beta}}^m \partial_m \Lambda_{\alpha}$$

$$D_{\beta} \bar{\Lambda}_{\dot{\alpha}} = 2i \sigma_{\beta\dot{\beta}}^m \bar{\Lambda}^{\dot{\beta}} \partial_m \bar{\Lambda}_{\dot{\alpha}}$$

$$\bar{D}_{\dot{\beta}} \bar{\Lambda}_{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}} \quad (17)$$

$$D_{\beta} C_I = 0$$

$$\bar{D}_{\dot{\beta}} C_I = -2i \Lambda^{\rho} \sigma_{\rho\dot{\beta}}^m \partial_m C_I$$

These constraint eqs. can be taken as the definition of the nonlinear superfields  $\Lambda$ ,  $\bar{\Lambda}$  and  $C_I$ . If we solve them we will find that all the higher components of these superfields are functions of the lowest component and its derivatives. This is exactly the same superfield defined in eq. (11).

#### 4. THE STANDARD FORM FOR NONLINEAR SUPERFIELDS.

The general theory of nonlinear realizations of internal symmetries tells us that any linear realization of a given group can be converted into a direct sum of nonlinearly transforming fields by means of a group transformation with the Goldstone field as a parameter. The analogous relationship for the case of supersymmetry was established by Ivanov and Kapustnikov [5]: it is always possible to decompose a linear superfield into SM-fields and inversely, to build a linear superfield from SM-fields. Starting from a linear superfield it is possible to combine the component fields in such a way that they transform like a standard form: it is enough to perform certain field dependent changes of variables:

$$\phi_I(x, \theta, \bar{\theta}) \rightarrow \tilde{\phi}_I(x, \theta, \bar{\theta}) = \phi_I(x^m + i(\lambda(x) \sigma^m \bar{\theta} - \theta \sigma^m \bar{\lambda}(x))),$$

$$\phi = \lambda(x), \quad \bar{\theta} = \bar{\lambda}(x) \quad (18)$$

From the transformation laws (1) and (11) it follows the transformation law of  $\tilde{\phi}_I$ :

$$\delta \tilde{\phi}_I = -i (\lambda(x) \sigma^m \bar{\xi} - \xi \sigma^m \bar{\lambda}(x)) \partial_m \tilde{\phi}_I \quad (19)$$

All the components of  $\tilde{\phi}_I$  transform according to the standard form (3), in particular its lowest component:

$$\phi_I|_{\theta} = \bar{\theta} = 0 = \theta_I(x, -\lambda(x), -\bar{\lambda}(x)) \quad (20)$$

It is then possible to parametrize any realization or representation of supersymmetry, except of course the AV-transformation law (1), in such a way that it transforms in the standard form: we first construct the corresponding superfield using (10) and then use the formula (20):

$$\tilde{a}(x) = \exp(\theta \delta + \bar{\theta} \bar{\delta}) x a(x) \Big|_{\theta = -\lambda(x), \bar{\theta} = -\bar{\lambda}(x)} \quad (21)$$

## 5. THE AKULOV-VOLKOV EFFECTIVE LAGRANGIAN.

Once we have a superfield formalism we can build corresponding supersymmetric Lagrangians taking the highest component of products, sums and spacetime derivatives of superfields. For the goldstino superfield, a candidate Lagrangian would be

$$\mathcal{L}(x) = \int d^2\theta d^2\bar{\theta} \Lambda^2 + \text{h.c.}$$

where

$$\begin{aligned} \Lambda^2 = \Lambda^\alpha \Lambda_\alpha = & \lambda \lambda + \frac{2}{k} \lambda \theta - 4i k w^m (\lambda \partial_m \lambda) + \frac{1}{k^2} \theta \theta \\ & - 2i (2w^m \partial_m \lambda \theta + j^m (\lambda \partial_m \lambda)) + 4k^2 \bar{\theta} \bar{\theta} (\partial_n \lambda \sigma^{mn} \partial_m \lambda) \lambda \lambda \\ & - \frac{2i}{k} j^m \partial_m \lambda \theta - 4k \left[ j^m \partial_m (w^n (\lambda \partial_n \lambda)) + 2w^n (\partial_n w^m) \partial_m \lambda \theta \right. \\ & \quad \left. + w^m w^n \partial_m \partial_n (\lambda \theta) \right] \\ & + \theta \theta \bar{\theta} \bar{\theta} (\partial_n (2\lambda \sigma^{mn} \partial_m \lambda) - \frac{1}{2} \lambda \partial^n \lambda) \end{aligned}$$

From this expression we immediately appreciate that the last component of  $\Lambda^2$  is a total derivative and therefore such Lagrangian is trivial. Another possibility is the highest component of

$$\mathcal{L}(x) = -\frac{1}{2} k^2 \int d^2\theta d^2\bar{\theta} (\Lambda \Lambda) (\bar{\Lambda} \bar{\Lambda}) \quad (22)$$

This gives the original AV Lagrangian

$$\mathcal{L}_\lambda = -\frac{1}{2k^2} - i\lambda \sigma^m \partial_m \bar{\lambda} + \text{interaction terms} \quad (23)$$

Notice the positive vacuum energy  $1/2k^2$  which also signals supersymmetry breaking [1]. Its magnitude is a measure for supersymmetry breaking. Since the dimension of  $k$  is  $m^{-2}$ , we have

$$m_{ssb} = k^{-\frac{1}{2}} \quad (24)$$

The Akulov-Volkov nonlinear realization shows that supersymmetry can be realized entirely in terms of a single fermion field without additional scalar or vector fields.

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