NEW PERTURBATIVE APPROACH TO QUANTUM FIELD THEORY

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1. I will present a new approach to perturbation theory for renormalizable quantum fields theories (QFTs) which gives renormalization scheme (RS) independent predictions for observable and other quantities of interest (eg. Green's functions.) The resulting **RE**normalization **S**cheme **I**ndependent **PE**rturbation theory will be called RESIPE for short. I will illustrate how RESIPE works for a renormalizable QFT with one dimensionless coupling constant (see ref. 2). Applications of 2nd order RESIPE to some specific physical measurables, for massless QCD, are to be found in ref. 3. Generalization of the RESIPE formalism to QFT's with masses and more than one coupling constant and its connection with the renormalization group (RG) formalism is given in ref. 4. Here, in addition, a new scheme independent perturbation expansion, without reference to RG techniques, is given which is valid for the general case with masses, several kinematics variables and more than one coupling constant. These and references 5 and 6 may be consulted for more detail.

2. **RESIPE Formalism for a Renormalizable QFT with one Coupling constant**. Consider a QFT wich is renormalizable and has one dimensionless bare coupling constant g_0 (eg. QCD). For simplicity, consider a physical quantity wich depends on only one external energy scale Q. Corresponding to it one can always construct a dimensionless measurable quantity R, such that its regularized unrenormalized perturbation expansion is of the form

$$R = a_0 + r_{10}a_0^2 + r_{20}a_0^3 + \cdots \tag{1}$$

Here the bare couplant $a_0 \equiv g_0^2 / 4\pi^2$ and the subscript '0' denotes bare or unrenormalized quantities. The bare perturbation series is not well defined since the coefficients of expansion are infinite. In a renormalizable theory finite results are extracted by absorbing the infinities in the base parameters (coupling constants, masses, etc.) and the fields present in the Lagrangian. The definitions of the renormalized fields and parameters in terms of the corresponding bare quantities are, however, not unique because of the posibility of finite renormalizations. After renormalization, since the measurable R has no anormalous dimensions, equation (1) becomes

$$R = a + r_1a^2 + r_2a^3 + \cdots, \tag{2}$$

where the renormalized couplant $a \equiv g^2 / 4\pi^2$ and g = renormalized contant. The coefficients r_n are finite but their values depend on the RS used to define g. Consequently, finite order predictions for R in the renormalized procedure gives predictios for R which, although finite, are still ambiguous. Can this problem of RS-dependent perturbative predictions (present for all QFT's) be solved? Does the fact that the perturbative predictions base on equation (1) or equation (2) are not well defined mean that R itself is not directly computable in the theory, but instead the theory predicts some function f(R) of R uniquely? How and in what form does the theory determine f (R)? RESIPE provides the answers. We will see that for a renormalizable QFT with a single dimensionless coupling constant g_0 the theory, at best, determines the Q dependence of R through the differential equation.

$$Q\frac{dR}{dQ} \equiv R'(Q) = f(R(Q)) = -f_0R^2(1+f_1R+f_2R^2+\cdots). \tag{3}$$

The last term expresses f(R) as a series in R with finite RS-invariant coefficients f_0, f_1, ... Each term in this series is RS-invariant and therefore so is any finite order truncation. The convergence of perturbative approximations to f(R) is nowcontrolled by magnitude of R itself. For practical applications, one may approximate the r.h.s. by the first 2 or 3 terms if $|f_n R^n| << 1$ for $n \geq 2$ or 3. These would give the second or third order RESIPE prediction. Since these finite order predictions are RS-independent, their confrontation with eperiment provides an unambiguous probe for higher order corrections.

2a Determination of RS-invariants f_n's. Since the coefficients r_{n0} depend on Q through the regularization scale (eg., an ultraviolet cut off), equation (1) gives

$$R' = r'_{10}a_0^2 + r'_{20}a_0^3 + \ldots \qquad r'_{n0} \equiv Q\frac{\partial r_{n0}}{\partial Q}. \tag{4}$$

Eliminate a_0 between equations (1) and (4) to express R´ as a in series R and compare with equation (3). Or equivalently, substitute equation (1) in equation (3) and compare the resulting series in a_0 for R´ with equation (4). The resulting expressions for f_n´s in terms of r'_{n0} and r_{n0} are given in equation (6) below.

Since the theory is renormalizable, one can start with equation (2) to obtain

$$R' = r'_1a^2 + r'_2a^3 + \ldots, \qquad r'_n \equiv Q\frac{\partial r_n}{\partial Q}. \tag{5}$$

Manipulating equations (2), (3) and (5) as indicated above yields expressions for f_n's in terms of r_n and r'_n. Note the algebra is the same whether one starts with equation (1) or equation (2). Thus, we find:

$$-f_0 = r'_{10} = r'_1, \tag{6.1}$$

$$-f_0f_1 = r'_{20} - 2r'_{10}r_{10} = r'_2 - 2r'_1r_1, \tag{6.2}$$

$$-f_0f_2 = r'_{30} - 3r'_{20}r_{10} - 2r'_{10}r_{20} + 5r'_{10}r_{10}^2 = r'_3 - 3r'_2r_1 - 2r'_1r_2 + 5r'_1r_1^2, \tag{6.3}$$

etc. Since r_{n0} and r'_{n0} are RS-independent, while r_n and r'_n are finite (by definition) equation (6) proves that f'_n are both finite and RS-invariant. These properties for the f_n's are, in a sense, obvious from equation (3), since both R and R' posses these two properties being measurables. Note that f_0, f_1,... etc. can be directly calculated from the combinations of the bare series coefficients (in equation (6)) without having to renormalize them. The finiteness of f_n's is guaranteed by renormalizability of the theory. Note that f_0 and f_1 are universal in the sense that they are independent of the process under consideration. Of course, f_n, $n \geq 2$, do depend on the process, that is R, though this has not been explicity indicated in equation (3) for notational simplicity.

2b Testing RESIPE. Equation (3) requires the knowledge of R at some $Q = Q_0$ (which has to be obtained from experiment) to predict it at any other Q. This boundary condition on equation (3) provides the process dependent scale Λ_R for R to have a non trivial dependence on Q. Dependence of R on the RS-independent scale Λ_R (undetermined by the theory) is consistent with the fact that the starting Lagrangian contained the undetermined parameter g_0. The dependence of R on the dimensionless g_0 has now appeared, by "dimensional transmutation" (ref. 7), through Λ_R. In RESIPE different physical quantities R, $\overline{R}, \cdots$ will automatically have scales $\Lambda_R, \Lambda_{\overline{R}} \cdots$ which are specific to them. Does that mean the theory has many independent scales? The aswer is no (ref. 2). For the massless case, one can integrate equation (3) for process R and the corresponding equation

$$\tilde{R}' = -f_0\tilde{R}^2(1 + f_1\tilde{R} + \tilde{f}_2\tilde{R}^2 + \cdots), \tag{7}$$

for the process $\tilde{R}' = a + \tilde{r}a^2 + \cdots$, since the RS-invariant f_n's and $\tilde{f}_n$'s are constants independent of Q. One can show that the two scales Λ_R and $\Lambda_{\tilde{R}}$ are related:

$$\Lambda_{\tilde{R}} = \Lambda_R \exp\left[f_0^{-1}(\tilde{r}_{10} - r_{10})\right] \text{ and } \Lambda_R = \Lambda \exp\left[f_0^{-1}(r_1)_{\mu=Q}\right], \tag{8}$$

where Λ is the usual RS-dependent scale parameter and μ is the renormalization point. Note r_n's and $\tilde{r}_n$'s are functions of Q/μ only and $\tilde{r}_{10} - r_{10} = \tilde{r}_1 - r_1$. To test the theory using RESIPE one can extract Λ_R and $\Lambda_{\tilde{R}}$ to a given order and see how well Equation (8) is satisfied. Alternatively, one can compare the value of Λ obtained in the two cases.

CONCLUSION

The central idea of RESIPE is to use some observable quantity as the perturbation expansion parameter instead of the usual RS-dependent coupling constant, as is normally done in conventional renormalized perturbation theory (CRPT). This central idea can be implemented in different ways depending on the techniques used (ref. 8). RESIPE can be considered as a full-fledged RS-independent substitute for CRPT.

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REFERENCES

[1] DHAR, A. (1983): **Phys. Lett.**, 128 B, 407.

[2] ________ and V. GUPTA (1984): **Phys. Rev.**, D29, 2822.

[3] ________ and ________ (1983): **Pramana**, 21, 207.

[4] GUPTA, V.; D.V. SHIRKOV and O.V. TARASOV (1991): **International Journal of Modern Physics**, A6, 3381.

[5] The formulation of the RESIPE program based on the RG is the equivalent to the RG formalism developed by N.N. Bogoliubov and D.V. Shirkov, **Nuovo Cimento** 3, 845 (1956); Introduction to the theory of Quantized Fields, 3rd Ed. Wiley Interscience, N.Y. (1980) Chap. 9.

[6] For an explicit example of the emergence of RS-invariants when masses are present see S.G. Gorishny, A.L. Kataev, S.A. Larin and L.R. Surguladze, **Phys. Rev.** D 43, 1633 (1991).

[7] COLEMAN, S. and E. Weinberg (1973): **Phys. Rev.** D7, 1888; D.J. Gross and A. Neveu (1974), ibid 10, 3235.

[8] GRUNBERG, G. (1980): **Phys. Lett** 95 B, 70; S.J. Maximov and V.I. Vovk, **Phys. Lett**. 199B, 443 (1987).