FIELD THEORY APPROACH TO K^0 - $\overline{K}^0$ AND B^0 - $\overline{B}^0$ SYSTEMS

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ABSTRACT
We present an approach based on field theory to describe the production and decay of unstable $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ mixed systems. Applications to describe the time evolution amplitudes of K^0 and $\overline{K}^0$ at DAPHNE and CPLEAR are presented.

1. INTRODUCTION

Neutral strange and beauty pseudoscalar mesons, $K^0\overline{K}^0$ and $B^0\overline{B}^0$, are systems of two unstable mixed states of special interest for the study of weak interactions. They are particularly suited to study the phenomena of CP violation together with the oscillations in their-time-dependent decay probabilities [1].

The traditional description of unstable neutral kaons is based on the Wigner-Weisskopf (WW) formalism [2]. In this approach, the time evolution of decaying states is governed by a Schrödinger-like equation based on a *non-hemitian* hamiltonian [3] that allows particle decays. As a result, the diagonalizing transformations, in general, are not unitary and the corresponding eigenstates are not orthogonal.

Beyond these unsatisfactory features of the WW formalism, one faces other difficulties. Projected factories of K and B mesons [4, 5] are expected to measure the CP violation and oscillation parameters to a higher accuracy than present experiments. While it is not clear whether the approximations involved in the WW formalism are valid for both the K and B systems, a consistent scheme is certainly required to compute these observables to an arbitrary degree of accuracy.

In this paper we adopt the view that the quantum mechanical behavior of a complete process involving the production and decay of unstable states can only be consistently described in the framework of quantum field theory. In QFT, the S-matrix amplitude becomes the basic object that describes the properties of a physical process among particles. This amplitude is taken between *in*- and *out*- asymptotic states which are defined as non-interacting states (stable particles) existing far away the interaction region. Therefore, as a general rule, unstable particles cannot be considered as asymptotic states.

Under these conditions, unstable particles appear only as intermediate states to which we associate Green functions (propagators) to describe the propagation amplitudes from their production to their decay spacetime locations. The form of these propagators, which are consistent with special relativity and causality, determine the time evolution of its decay probability. Since Lorentz covariance is implicit to the field theory approach, neither boost transformations nor the choice of a specific frame are required to define the time parameter in the amplitude.

In this paper we will also address some questions related to the usual treatment of CP violating parameters. As is well known, the $K^0\overline{K}^0$ (and $B^0\overline{B}^0$) system requires of two parameters to account for CP violation in the propagation (*indirect*) and decay (*direct*) of neutral kaons, usually related to ε and ε', respectively [6]. The description based on the WW formalism is not valid beyond order ε because of the aforementioned difficulty in the normalization of non-orthogonal states. Since $\varepsilon' \sim \mathcal{O}(\varepsilon^2)$ for the $K^0\overline{K}^0$ system, it becomes necessary to establish a correct formalism [7] to account consistently for terms of order ε^2. Furthermore, this is necessary because the usual approximations for neutral kaons in the WW formalism, might fail in the case of B mesons.

On the other hand, ε and ε' can be related to the observable parameters that measure CP violation in the $K^0\overline{K}^0$ system by assuming isospin symmetry and the factorization of strong rescattering effects [6]. These approximations are rather strong assumptions in view of the smallness of direct CP violating effects [8, 9]. Without involving the isospin decomposition of the amplitudes, in this paper we shall parametrize CP violation in terms of the mixing of CP eigenstates $K_1 - K_2$ (indirect CP violation) and the parameters χ_{+-} and χ_{∞} which describe the CP violating 2π decays of K_2 in our approach.

This paper is organized as follows. In section II we discuss the diagonalization of mixed propagators in momentum space for the system of unstable neutral pseudoscalar K and B mesons. In section III we focus on the space-time representations of these propagators. Section IV is devoted to the applications of our formalism to compute the time-dependent distributions of neutral kaon decays as adapted to CPLEAR and DAPHNE experiments. Our conclusions are presented in section V.

2. UNSTABLE PARTICLE PROPAGATOR IN MOMENTUM SPACE

As previously discussed, the propagator is the basic object in the S-matrix amplitude that describes the propagation of an unstable state from its production at space-time point x through its decay at point x′. In this section we study the momentum space representation of the propagator for the neutral kaon system, which will be needed to compute the S-matrix amplitudes.

Since the weak interaction couples the flavor states K^0 and $\overline{K}^0$, the renormalized propagator for these two unstable particles is a non diagonal 2×2 matrix. By imposing the CPT symmetry, we can parametrize the inverse propagator for unstable kaons of four-momentum p as follows [1]:

$$D^{-1}(p^2) = \begin{pmatrix} d & a+b \\ a-b & d \end{pmatrix} \tag{1}$$

where

$$d \equiv p^2 - m_0^2 + im_0\Gamma_0 , \tag{2a}$$

$$a \equiv r^2 + is^2, \tag{2b}$$

$$b \equiv \mu^2 + i\nu^2 , \tag{2c}$$

and m_0, Γ_0, r^2, s^2, μ^2, ν^2 are real quantities.

We define the CP eigenbasis as

$$\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} \equiv S\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} \tag{3}$$

where $S = S^{-1}$. The corresponding inverse propagator is

$$\overline{D}^{-1}(p^2) \equiv SD^{-1}(p^2)S^{-1} = \begin{pmatrix} d+a & -b \\ b & d-a \end{pmatrix} . \tag{4}$$

CP conservation implies that b vanishes. For the $K^0 - \overline{K}^0$ system, b is small compared to the diagonal terms and it is predominantly imaginary [10].

Now, if we introduce the complex parameter $\hat{\varepsilon}$ as

$$\frac{\hat{\varepsilon}}{1+\hat{\varepsilon}^2} \equiv \frac{b}{2a}, \tag{5}$$

we can derive the diagonal form of the inverse propagator because Equation (4) can be rewritten as,

$$\overline{D}^{-1}(p^2) \equiv \frac{1}{1-\hat{\varepsilon}^2}\begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix}\begin{pmatrix} d + a\dfrac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2} & 0 \\ 0 & d - a\dfrac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2} \end{pmatrix}\begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix}. \tag{6}$$

Therefore, the physical basis of neutral kaons consists of two states $K_{L,S}$, of definite masses $m_{L,S}$ and decay widths $\Gamma_{L,S}$, such that

$$d_S \equiv p^2 - m_S^2 + i m_S \Gamma_S = d + a\frac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2} \tag{7a}$$

$$d_L \equiv p^2 - m_L^2 + i m_L \Gamma_L = d + a\frac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2}, \tag{7b}$$

and the propagator $\overline{D}(p^2)$ can be written as follows

$$\overline{D}(p^2) \equiv \frac{1}{1-\hat{\varepsilon}^2}\begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix}\begin{pmatrix} d_S^{-1} & 0 \\ 0 & d_L^{-1} \end{pmatrix}\begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix}. \tag{8}$$

As already anticipated, the diagonalization of the non-hermitian matrix given in Equation (4) involves a non-unitary matrix. Furthermore, according to Equation (8), we can obtain a proper orthogonal and normalized physical basis if we define independent *ket* (in-) and *bra* (out-) states, respectively, as:

$$\begin{pmatrix} |K_S\rangle \\ |K_L\rangle \end{pmatrix} \equiv \frac{1}{\sqrt{1-\hat{\varepsilon}^2}}\begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix}\begin{pmatrix} |K_1\rangle \\ |K_2\rangle \end{pmatrix} \tag{9}$$

and

$$\begin{pmatrix} \langle K_S| \\ \langle K_L| \end{pmatrix} \equiv \frac{1}{\sqrt{1-\hat{\varepsilon}^2}}\begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix}\begin{pmatrix} \langle K_1| \\ \langle K_2| \end{pmatrix} \tag{10}$$

Notice that bra states do not correspond to simply hermitian conjugate of ket states.

The quantities $m_{S,L}$, $\Gamma_{S,L}$ and $\hat{\varepsilon}$ can be measured experimentally, while the parameters a, b, m_0 and Γ_0 can be in principle computed from the theory. The relationships between these two sets of parameters are:

$$a = \frac{1}{2}\left(\frac{1+\hat{\varepsilon}^2}{1-\hat{\varepsilon}^2}\right)\left\{m_L^2 - m_S^2 - i(m_L\Gamma_L - m_S\Gamma_S)\right\}, \tag{11a}$$

$$m_0^2 - i m_0 \Gamma_0 = \frac{1}{2}\left\{m_L^2 - m_S^2 - i(m_L\Gamma_L - m_S\Gamma_S)\right\}, \tag{11b}$$

$$b = \frac{\hat{\varepsilon}}{1-\hat{\varepsilon}^2}\left\{m_L^2 - m_S^2 - i(m_L\Gamma_L - m_S\Gamma_S)\right\} . \tag{11c}$$

Since b is predominantly imaginary for the $K^0 - \overline{K}^0$ [10] and $\hat{\varepsilon} \sim \mathcal{O}(10^{-3})$, we can compute the phase of the CP violation parameter $\hat{\varepsilon}$ which is given by:

$$\begin{aligned}
\phi(\hat{\varepsilon}) &= \operatorname{arctg}\left(\frac{m_L^2 - m_S^2}{m_S\Gamma_S - m_L\Gamma_L}\right) \\
&= \operatorname{arctg}\left(\frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}\right) + O\left(\frac{\Gamma_S}{m_S}, \frac{m_L - m_S}{m_L + m_S}\right) \qquad (12) \\
&= (43.49 \pm 0.08)^\circ
\end{aligned}$$

where we have used $\Gamma_S/m_S \approx O(10^{-14})$ and $(m_L - m_S)/(m_L + m_S) \approx O(10^{-14})$. As is well known, this result is in excellent agreement with experimental data [11].

3. SPACE-TIME EVOLUTION OF RESONANCE PROPAGATORS

In this section we are interested in the time dependent properties of the propagation of unstable particles for the purposes of studying CP violation and the time oscillations in the kaon system. We shall therefore focus on the properties of the unstable state propagator in configuration space.

Let us first consider the propagator for a stable spin zero particle:

$$\Delta_F(x' - x) = \int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip.(x'-x)}}{p^2 - m^2 + i\varepsilon} . \tag{13}$$

To manifest the time dependence in the amplitude, it is necessary to put this expression into another form showing a separate time evolution for the particle and the antiparticle. A contour integration in the complex p^0 plane gives:

$$\begin{aligned}
\Delta_F(x' - x) = &-i\int \frac{d^3p}{(2\pi)^3}\,\frac{e^{-i\vec{p}.(\vec{x}'-\vec{x})}e^{-iE(t'-t)}}{2E}\,\theta(t'-t) \\
&i\int \frac{d^3p}{(2\pi)^3}\,\frac{e^{-i\vec{p}.(\vec{x}'-\vec{x})}e^{iE(t'-t)}}{2E}\,\theta(t-t') \qquad (14)
\end{aligned}$$

with $E \equiv \sqrt{\vec{p}^2 + m^2}$.

Depending of the specific process, the first (second) term in Equation (14) will survive in the time-dependent amplitude and will describe a particle (antiparticle) propagating forward in time.

Let us now consider the propagator of a spin zero resonance. The Dyson summation of self-energy graphs leads to the following renormalized propagator in momentum space representation:

$$\frac{1}{p^2 - m^2 + i\sqrt{p^2}\,\Gamma(p^2)\theta(p^2 - p_{th}^2)} , \tag{15}$$

where p_{th}^2 is the threshold for the vanishing of the imaginary part of the self-energy in the case that we consider only one decay channel.

In order to justify the constant width approximation used in Equation (7), let us consider the 2π decay width of kaons. A direct computation of this decay width, for a kaon of squared four-momentum s, gives:

$$\Gamma(s) = \frac{m^2}{s}\left(\frac{s - s_{th}}{m^2 - s_{th}}\right)^{1/2}\Gamma \tag{16}$$

where m is the kaon mass, $s_{th} = 4m_\pi^2$ and $\Gamma \equiv \Gamma(s = m^2)$. The influence of the kaon width in the propagator is felt only for $\sqrt{s}$ values near the kaon mass, i.e., for $m - x\Gamma \le \sqrt{s} \le m + x\Gamma$, with x an arbitrary number of order 1 such that $x\Gamma/m << 1$.

Since Γ_S/m_S, $\Gamma_S/(m_S - 2m_\pi) \sim O(10^{-14})$, the form of the propagator with a constant width

$$\frac{1}{p^2 - m^2 + im\Gamma\theta\left(p^2 - p_{th}^2\right)}, \tag{17}$$

turns out to be an extremely good approximation for the renormalized propagator.

Therefore, the space-time representation of the spin zero propagator for the unstable particles can be written as:

$$\Delta_R(x' - x) = \int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-i\vec{p}.(x'-x)}}{p^2 - m^2 + im\Gamma\theta(p^2 - p_{th}^2)}. \tag{18}$$

Similarly as done above for the stable particle propagator, we would like to express explicitly the time dependence of $\Delta_R(x'-x)$. It becomes convenient to separate the propagator into two pieces:

$$\Delta_R(x' - x) = \Delta_R^{(1)}(x' - x)\Delta_R^{(2)}(x' - x) \tag{19}$$

with

$$\Delta_R^{(1)}(x' - x) = \int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-i\vec{p}.(x'-x)}}{p^2 - m^2 + im\Gamma}$$

$$\Delta_R^{(2)}(x' - x) = \int \frac{d^3p}{(2\pi)^3}\int_{-p_{th}^0}^{p_{th}^0}\frac{dp^0}{2\pi}\,e^{-i\vec{p}.(x'-x)}\left\{\frac{1}{p^2 - m^2} - \frac{1}{p^2 - m^2 + im\Gamma}\right\} \tag{20}$$

where $p_{th}^0 = \sqrt{\vec{p}^2 + p_{th}^2}$.

Using again the condition $\Gamma/\left(m - \sqrt{p_{th}^2}\right) << 1$, we can show that

$$\Delta_R^{(2)}(x'-x) \sim \mathcal{O}\left(\frac{\Gamma}{m - \sqrt{p_{th}^2}}\right),$$

which allows to write

$$\Delta_R(x'-x)=\int\frac{d^4p}{(2\pi)^4}\;\frac{e^{-ip.(x'-x)}}{p^2-m^2+im\Gamma}\left[1+\mathcal{O}\left(\frac{\Gamma}{m-\sqrt{p_{th}^2}}\right)\right]. \tag{21}$$

In order to made explicit the time dependence of the unstable propagator let us use the following pole decomposition

$$p^2-m^2+i\Gamma m=\left(p_0-E+\frac{i\Gamma m}{2E}\right)\left(p_0+E-\frac{i\Gamma m}{2E}\right)\left(1+\mathcal{O}\left(\frac{\Gamma^2}{m^2}\right)\right) \tag{22}$$

where $E=\sqrt{\vec{p}^2-m^2}$.

Therefore, by neglecting very small terms of order 10^{-14}, the contour integral in the complex p^0 plane with the poles located at $\pm$ (E - imΓ/2E) gives

$$\Delta_R(x'-x)=-i\int\frac{d^3p}{(2\pi)^3}\;\frac{e^{i\vec{p}.(\vec{x}'-\vec{x})}e^{-iE(t'-t)}}{2E}e^{-\frac{\Gamma}{2}\frac{m}{E}(t'-t)}\theta(t'-t)$$

$$-i\int\frac{d^3p}{(2\pi)^3}\;\frac{e^{-i\vec{p}.(\vec{x}'-\vec{x})}e^{-iE(t-t')}}{2E}e^{-\frac{\Gamma}{2}\frac{m}{E}(t-t')}\theta(t-t'). \tag{23}$$

The interpretation is similar to the one for the stable particle, except for the decay constant Γ which expresses the unstability of the particle and antiparticle. The case of $K^0\overline{K}^0$ system considered in this paper is more involved, because the propagator is a 2 $\times$ 2 matrix. This problem is circumvented by performing the diagonalization before the contour integration in the complex plane or p^0.

Notice that $\tau = t' - t$ is the time elapsed between the production and decay locations of the resonance. Note also that, contrary to non-relativistic approaches, the factor m/E naturally appears in the exponential decay factor. Therefore, no boost transformations are required to relate the *proper* time to the time parameter of a moving particle. Of course, the exponential decay gets its usual form $e^{-\Gamma\tau/2}$ [1] in the rest frame of the resonance.

4. APPLICATIONS

In this section we compute the full S-matrix amplitudes for the production and decay of neutral kaons as studied at CPLEAR and DAPHNE experiments. Then, we derive the time evolution of these transition amplitudes and introduce the CP violation parameters intrinsic to our description.

4.1. CPLEAR experiments

At the CPLEAR experiment [12], K^0 and $\overline{K}^0$ are produced at point x in the strong interaction annihilation of $p\bar{p}$, and subsequently decay at point x'to $\pi^+\pi^-$ by the effects of weak interactions. The production mechanisms of K^0 and $\overline{K}^0$ are $p\bar{p}\to K^0K^-\pi^+, \overline{K}^0K^+\pi^-$, thus neutral kaons can be tagged by identifying the accompanying charged kaon [2]. After their production, both K^0 and $\overline{K}^0$ oscillates between their two components K_L and K_S before decaying to the 2π final states. We would therefore be interested in the description of the time evolution of the full decay amplitude and its interference phenomena. It is interesting to note that despite the fact that charged kaons and pions have similar lifetimes as K_L, they can be treated as asymptotic particles in the present case.

In order to relate the different S-matrix amplitudes, let us first consider the production mechanism of $K^0\overline{K}^0$. Since strong interactions conserve strangeness, we have

$$\mathcal{M}\left(p\bar{p} \to K^-\pi^+\overline{K}^0\right) = \mathcal{M}\left(p\bar{p} \to K^+\pi^-K^0\right) = 0\,, \tag{24}$$

which, according to equation (3), implies

$$\mathcal{M}\left(p\bar{p} \to K^-\pi^+K_1\right) = \mathcal{M}\left(p\bar{p} \to K^-\pi^+K_2\right) \equiv A \tag{25a}$$

$$\mathcal{M}\left(p\bar{p} \to K^+\pi^-K_1\right) = -\mathcal{M}\left(p\bar{p} \to K^+\pi^-K_2\right) \equiv B. \tag{25b}$$

Assuming CPT invariance we obtain

$$\mathcal{M}\left(p\bar{p} \to K^-\pi^+K^0\right) = \mathcal{M}\left(p\bar{p} \to K^+\pi^-\overline{K}^0\right) \equiv C. \tag{25c}$$

Collecting all these constraints, we get

$$A = B = \frac{C}{\sqrt{2}}. \tag{26}$$

Now, let us first consider the complete process for the production of a K^0 decaying into $\pi^+\pi^-$

$$p(q) + \bar{p}(q') \to K^-(k) + \pi^+(k') + K^0(p) \to K^-(k) + \pi^+(k') + \pi^+(p_1) + \pi^-(p_2) \tag{27}$$

The full amplitude corresponding to this process can be written (the subscript + - refers to the charged of the two pions from K^0 decay):

$$T_{+-} = \int d^4x d^4x' e^{i(p_1+p_2).x'} \left(\mathcal{M}\left(K_1 \to \pi^+\pi^-\right), \mathcal{M}\left(K_2 \to \pi^+\pi^-\right)\right) \times \Delta_R^{K_1K_2}(x'-x)$$

$$\begin{pmatrix} \mathcal{M}\left(K^0 \to K_1\right) \\ \mathcal{M}\left(K^0 \to K_2\right) \end{pmatrix} \cdot \mathcal{M}\left(p\bar{p} \to K^-\pi^+K^0\right) e^{i(k+k'-q-q')\cdot x} \tag{28}$$

where $\Delta_R^{K_1K_2}(x'-x)$ is the propagator matrix for the coupled $K_1 - K_2$ system in configuration space.

With the help of equations (3), (8) and (25), this gives:

$$\begin{aligned} T_{+-} &= \int d^4x d^4x' e^{i(p_1+p_2).x'} \left(\mathcal{M}\left(K_1 \to \pi^+\pi^-\right), \mathcal{M}\left(K_2 \to \pi^+\pi^-\right)\right) \int \frac{d^4p}{(2\pi)^4} e^{-ip.(x'-x)} \\ &\times \frac{1}{1-\hat{\varepsilon}^2} \begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix} \begin{pmatrix} d_S^{-1}(p) & 0 \\ 0 & d_L^{-1}(p) \end{pmatrix} \begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix} \begin{pmatrix} \mathcal{M}\left(K^0 \to K_1\right) \\ \mathcal{M}\left(K^0 \to K_2\right) \end{pmatrix} \\ &\times \sqrt{2}A \cdot e^{i(k+k'-q-q')\cdot x} \end{aligned} \tag{29}$$

$$= (2\pi)^{-4} \int d^4x d^4x' d^4p e^{i(p_1+p_2-p).x'} e^{i(k+k'-q-q'+p).x} \cdot A$$

$$\times \frac{1}{1-\hat{\varepsilon}} \left\{ \left[\mathcal{M}\left(K_1 \to \pi^+\pi^-\right) + \hat{\varepsilon}\mathcal{M}\left(K_2 \to \pi^+\pi^-\right) \right] \frac{1}{p^2 - m_S^2 + im_S\Gamma_S} \right.$$

$$\left. + \left[\hat{\varepsilon}\mathcal{M}\left(K_1 \to \pi^+\pi^-\right) + \mathcal{M}\left(K_2 \to \pi^+\pi^-\right) \right] \frac{1}{p^2 - m_L^2 + im_L\Gamma_L} \right\} \tag{30}$$

$$= (2\pi)^{-4}\delta^{(4)}(q + q' - k - k' - p_1 - p_2) \frac{1}{1+\hat{\varepsilon}} \cdot A\mathcal{M}\left(K_1 \to \pi^+\pi^-\right)$$

$$\times \left\{ \left(1 + \chi_{+-}\hat{\varepsilon}\right) \frac{1}{(p_1 + p_2)^2 - m_S^2 + im_S\Gamma_S} \right.$$

$$\left. + \left(\hat{\varepsilon} + \chi_{+-}\right) \frac{1}{(p_1 + p_2)^2 - m_L^2 + im_L\Gamma_L} \right\} \tag{31}$$

where

$$\chi_{+-} \equiv \frac{\mathcal{M}(K_2 \to \pi^+\pi^-)}{\mathcal{M}(K_1 \to \pi^+\pi^-)} \quad , \tag{32}$$

is the parameter describing direct Cp violation in our approach.

To obtain the time dependence of the full amplitude where an originally pure K^0 state decays to $\pi^+\pi^-$, we must insert Equation (23) into Equation (28) and we get:

$$T_{+-} = -i(2\pi)^4\delta^{(4)}(q + q' - k - k' - p_1 - p_2) \frac{1}{1+\hat{\varepsilon}} A\mathcal{M}\left(K_1 \to \pi^+\pi^-\right)$$

$$\times \left\{ \left(1 + \chi_{+-}\hat{\varepsilon}\right) \int dt \frac{1}{2E_S} \left[e^{-i(E_S - E)t} e^{-\frac{1}{2}\Gamma_S \frac{m_S}{E_S} t} \theta(t) + e^{i(E_S - E)t} e^{\frac{1}{2}\Gamma_S \frac{m_S}{E_S} t} \theta(-t) \right] \right.$$

$$\left. + \left(\hat{\varepsilon} + \chi_{+-}\right) \int dt \frac{1}{2E_L} \left[e^{-i(E_L - E)t} e^{-\frac{1}{2}\Gamma_L \frac{m_L}{E_L} t} \theta(t) + e^{i(E_L - E)t} e^{\frac{1}{2}\Gamma_L \frac{m_L}{E_L} t} \theta(-t) \right] \right\} \tag{33}$$

where $E = p_1^0 + p_2^0$ is the total energy of the $\pi^+\pi^-$ system,

$$E_S = \sqrt{(\vec{p}_1 + \vec{p}_2)^2 + m_S^2} \; ,$$

$$E_L = \sqrt{(\vec{p}_1 + \vec{p}_2)^2 + m_L^2} \; ,$$

and we have defined the time t in Equation (33) as the time elapsed from the production to the decay locations of K^0. Thus, the transition amplitude $T(t)$ describing the time evolution of the system for $t > 0$ is given by the integrand proportional to $\theta(t)$ in Equation (33), namely

$$T_{+-}(t) = -i(2\pi)^4\delta^{(4)}(q + q' - k - k' - p_1 - p_2) \frac{1}{1+\hat{\varepsilon}} A\mathcal{M}\left(K_1 \to \pi^+\pi^-\right) e^{iEt}$$

$$\times\left\{\frac{1}{2E_S}e^{-iE_S t}e^{-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}(1+\chi_{+-}\hat{\varepsilon})+\frac{1}{2E_L}e^{-iE_L t}e^{-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}(\hat{\varepsilon}+\chi_{+-})\right\}. \quad (34)$$

Let us now consider the analogous process where a pure $\overline{K}^0$ state is initially produced and the decay to $\pi^+\pi^-$, i.e. $p\bar{p}\to K^+\pi^-\overline{K}^0\to K^+\pi^-\pi^+\pi^-$ [12]. Following the same procedure as in the case of K^0 production and decay, we can get the following expression for the time evolution of $\overline{K}^0$ decays:

$$T'_{+-}(t)=(-i)(2\pi)^4\delta^{(4)}(q+q'-k-k'-p_1-p_2)\frac{1}{1-\hat{\varepsilon}}A\mathcal{M}(K_1\to\pi^+\pi^-)e^{iEt}$$

$$\left\{\frac{1}{2E_S}e^{-iE_S t}e^{-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}(1+\chi_{+-}\hat{\varepsilon})-\frac{1}{2E_L}e^{-iE_L t}e^{-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}(\hat{\varepsilon}+\chi_{+-})\right\}. \quad (35)$$

Let us notice that if we were interested in the $\pi^0\pi^0$ decay mode of neutral kaons, we would have to replace in Equations (34) and (35) $\mathcal{M}(K_1\to\pi^+\pi^-)$ by $\mathcal{M}(K_1\to\pi^0\pi^0)$ and χ_{+-} by χ_{00} where

$$\chi_{00}\equiv\frac{\mathcal{M}(K_2\to\pi^0\pi^0)}{\mathcal{M}(K_1\to\pi^0\pi^0)} \quad (36)$$

Using Equations (9(and (10), we can express the ratio of CP-violating to CP-conserving decay amplitudes of K_L, K_S states in terms of the CP-violating parameters in our approach:

$$\eta^{+-}\equiv\frac{\mathcal{M}(K_L\to\pi^+\pi^-)}{\mathcal{M}(K_S\to\pi^+\pi^-)}=\frac{\hat{\varepsilon}+\chi_{+-}}{1+\chi_{+-}\hat{\varepsilon}}, \quad (37)$$

and

$$\eta^{00}\equiv\frac{\mathcal{M}(K_L\to\pi^0\pi^0)}{\mathcal{M}(K_S\to\pi^0\pi^0)}=\frac{\hat{\varepsilon}+\chi_{00}}{1+\chi_{00}\hat{\varepsilon}}. \quad (38)$$

As is well known, the parameters η^{+-} and η^{00} are commonly used to express the violation of CP in the two pion decays of K_L (see for example pages 422-425 in [11]). Note that the above relations between measurable quantities and the parameters that quantify direct and indirect violation of CP, are derived without relying on assumptions based on isospin symmetry, contrary to the relations obtained for the η parameters in terms of the usual parameters ε and ε'. Since the parameters $\chi_{+-,00}$ are expected to be very small, we can neglect terms of $\mathcal{O}(\chi_{ij}\varepsilon)$ in the above equations and use isospin symmetry to show that in that limit,

$\chi_{+-}=\epsilon'$

$\chi_{00}=-2\varepsilon'$.

Finally, let us mention that Equations (34) and (35) reduce to the two well known expressions for the time evolution used in the analysis of the CPLEAR collaboration [12], when we choose the center of mass frame of the two pion produced in $K^0-\overline{K}^0$ decays.

4.2. Neutral kaon production ad DAPHNE

In this section we consider the oscillations of the pair of neutral kaons produced in e^+e^- annihilations at DAPHNE [4]. The results obtained in the present formalism for the $K^0\overline{K}^0$ system can be straightforward generalized to describe the same phenomena in pair production of neutral B mesons in the $\Upsilon(4s)$ region [5].

Neutral and charged kaons will be copiously produced (~ 10^9 pairs $K^0\overline{K}^0$/year) in e^+e^- collisions operating at a center of mass energy around the mass of the $\phi(1020)$ meson [4]. The ϕ mesons produced in e^+e^- annihilations decay at point x into $K^0\overline{K}^0$ pairs, and subsequently each neutral kaon oscillates between its K_L–K_S components before decaying to final states $f_1(p)$ and $f_2(p')$ at spacetime points y and z:

$$\phi(q) \to K^0\overline{K}^0 \to f_1(p) f_2(p') \tag{39}$$

where q, p and p′ are the corresponding four-momenta.

Since each final state can be produced by either K^0 or $\overline{K}^0$, we must add coherently the two amplitudes arising from the exchange of K^0 or $\overline{K}^0$ as intermediate states. Conservation of angular momenta forces the system of neutral kaons to be in a p-wave. Taking into account the charge conjugation properties of the electromagnetic current, the pair of neutral kaons are found to be in a total antisymmetric wavefunction [13]. Thus, the relative sign of the two contributions to $\phi \to f_1 f_2$ decays must be negative. The S-matrix amplitude for the process indicated in Equation (39) is:

$$\begin{aligned} T_{f_1 f_2} = & \int d^4x\, d^4y\, d^4z\, e^{ip.y+ip'.z}\, \mathcal{M}(\phi \to K^0\overline{K}^0) e^{-iq.x} \\ & \times \left\{ \left(\mathcal{M}(K_1 \to f_1), \mathcal{M}(K_2 \to f_1)\right) \Delta_R^{K_1K_2}(y-x) \begin{pmatrix} \mathcal{M}(K^0 \to K_1) \\ \mathcal{M}(K^0 \to K_2) \end{pmatrix} \right\} \\ & \times \left\{ \left(\mathcal{M}(K_1 \to f_2), \mathcal{M}(K_2 \to f_2)\right) \Delta_R^{K_1K_2}(z-x) \begin{pmatrix} \mathcal{M}(\overline{K}^0 \to K_1) \\ \mathcal{M}(\overline{K}^0 \to K_2) \end{pmatrix} \right\} \end{aligned} \tag{40}$$

Let us define $\mathcal{M}_{ij} \equiv \mathcal{M}(K_i \to f_j)$. With the help of Equations (3) and (8), we can reexpress the previous amplitude as:

$$\begin{aligned} T_{f_1 f_2} = & \int d^4x\, d^4y\, d^4z\, e^{ip.y+ip'.z-iq.x} \int \frac{d^4k}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4} e^{-ik.(y-x)-ik'.(z-x)} \frac{1}{2(1-\hat{\varepsilon}^2)} \\ & \Big\{ \left[(\mathcal{M}_{11} + \hat{\varepsilon}\mathcal{M}_{21}) d_S^{-1}(k) + (\hat{\varepsilon}\mathcal{M}_{11} + \mathcal{M}_{21}) d_L^{-1}(k) \right] \\ & \times \left[(\mathcal{M}_{12} + \hat{\varepsilon}\mathcal{M}_{22}) d_S^{-1}(k') - (\hat{\varepsilon}\mathcal{M}_{12} + \mathcal{M}_{22}) d_L^{-1}(k') \right] \\ & - \left[(\mathcal{M}_{11} + \hat{\varepsilon}\mathcal{M}_{21}) d_S^{-1}(k) - (\hat{\varepsilon}\mathcal{M}_{11} + \mathcal{M}_{21}) d_L^{-1}(k) \right] \\ & \times \left[(\mathcal{M}_{12} + \hat{\varepsilon}\mathcal{M}_{22}) d_S^{-1}(k') - (\hat{\varepsilon}\mathcal{M}_{12} + \mathcal{M}_{22}) d_L^{-1}(k') \right] \Big\} \\ & \cdot \mathcal{M}(\phi \to K^0\overline{K}^0) \\ = & (2\pi)^4 \delta^{(4)}(q-p-p') \frac{1}{1-\hat{\varepsilon}^2} \times \mathcal{M}(\phi \to K^0\overline{K}^0) \end{aligned} \tag{41}$$

$$\left\{ -(\mathcal{M}_{11} + \hat{\varepsilon}\mathcal{M}_{21})(\hat{\varepsilon}\mathcal{M}_{12} + \mathcal{M}_{22}) \frac{1}{p^2 - m_S^2 + i m_S \Gamma_S} \cdot \frac{1}{p'^2 - m_L^2 + i m_L \Gamma_L} \right.$$

$$+(\hat{\varepsilon}\mathcal{M}_{11}+\mathcal{M}_{21})(\mathcal{M}_{12}+\hat{\varepsilon}\mathcal{M}_{2})\frac{1}{p^2-m_L^2+im_L\Gamma_L}$$

$$\cdot\frac{1}{p'^2-m_S^2+im_S\Gamma_S}\Bigg\}\,. \tag{42}$$

As anticipated, the relative sign of the two contributions is negative.

As in the previous subsection, in order to obtain a time evolution of the amplitude, we can insert the explicit time-dependent propagator, Equation (23), into the amplitude (41). The result is

$$T_{f_1f_2}=\int dt\,dt'(\mathcal{T}(t,t')\theta(t)\theta(t')+\text{other terms in }\theta(\pm t),\theta(\pm t'))$$

where t and t′ are the times taken by unstable kaons to propagate from the common production point (x) up to their disintegration into f_1 at point y and f_2 at point z, respectively.

Thus, the explicit time evolution of the decaying amplitude is given by

$$\mathcal{T}(t,t')=\frac{1}{1-\varepsilon^2}(2\pi)^4\delta^{(4)}(q-p-p')e^{ip^0t+ip^{0\prime}t'}\frac{1}{4E_SE_L}\cdot\mathcal{M}\left(\phi\to K^0\overline{K}^0\right)$$

$$\Bigg\{-[\mathcal{M}_{11}+\hat{\varepsilon}\mathcal{M}_{21}]\,[\hat{\varepsilon}\mathcal{M}_{12}+\mathcal{M}_{22}]\times e^{-iE_S(p)t-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}\cdot e^{-iE_L(p')t'-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t'}$$

$$+[\hat{\varepsilon}\mathcal{M}_{11}+\mathcal{M}_{21}][\mathcal{M}_{12}+\mathcal{M}_{22}]\times e^{-iE_S(p')t'-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t'}\cdot e^{-iE_L(p)t-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}\Bigg\}$$

where $E_{S;L}(p)\equiv\sqrt{\vec{p}^2+m_{S;L}^2}$.

As we have already pointed out in the case of the CPLEAR experiment, no boost transformations are required to adequate the time evolution of the decay amplitude to a givem reference frame. Observe that, due to the initial antisymmetrisation of the $K^0\overline{K}^0$ system, $\mathcal{T}(t,t)=0$ if $f_1=f_2$ and $p=p'$ as noted in Reference [13].

5. DISCUSSIONS AND CONCLUSIONS

We have already discussed in the introduction the problems intrinsic to the Wigner-Weisskopf approximation. Other papers have appeared recently criticizing the old approach [14, 15, 16, 17] but they all present shortcomings which we will discuss in detail in an extended version of this paper. Their introduction of a proper time parameter for each particle forces them to use boosts to express the answer in a common frame. As we have shown in this article, relativistic quantum field theory yields results for the time evolution valid in any frame. An interference term showing time oscillation has then to be converted to a space evolution by the classical formula $t=\frac{E}{|\vec{p}|}x$. We remark here that this formula applies to particles observed in the detector and not to unstable K_S and K_L states.

A similar discussion based on field theory was considered sometime ago in Reference [3]. The field theory formalism, considering unstable particles as intermediate states between in and out stable states avoids asking questions without answers about these intermediate particles. Moreover the result is relativistically correct and valid in any frame. Finally, we have introduced the CP violation parameters $\hat{\varepsilon}$ and χ_{+-}, χ_{00} (cf equations (5), (32) and (36)) without relying on the Wigner-Weisskopf effective hamiltonian or approximations based on isospin symmetry.

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