BCS-LIKE FORMALISM TO INCLUDE THREE NUCLEON CORRELATIONS IN HEAVY NUCLEI

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ABSTRACT
Starting with the approach of considering a part of the residual hamiltonian corresponding to the three nucleon interaction, to be a term due to the interaction between Cooper pairs and unpaired nucleons, we develop a formalism which makes possible to calculate the excited states for heavy nuclei, as well, as their energies. A system of 3 coupled equations, similar to that of the BCS is obtained in such a way that all the parameters introduced in the model can be calculated.

RESUMEN
En este trabajo se considera un modelo de tipo BCS para calcular niveles de energía para núcleos pesados. Se tienen en cuenta, además de la interacción por pares, la interacción entre los pares de Cooper y las partículas desapareadas, lo cual se interpreta como una parte del Hamiltoniano residual de tres nucleones. Se obtiene un sistema de 3 ecuaciones acopladas del cual se pueden determinar los parámetros introducidos y así obtener los estados que tienen en cuenta la interacción de tres cuerpos.

INTRODUCTION

The study of exotic nuclear configurations, including nuclei in extreme conditions, has been a topic of great interest during the last few years. There has been considerable efforts both from the experimental and theoretical point of view looking for a more comprehensive approach of such phenomena. In particular, with this aim, several models has been developed, and other well known, as the shell model, has been modified trying to explain the characteristics of such exotic configurations [1, 2].

Particular interest has been devoted to some structural characteristics of the above mentioned nuclei. This is the case of the nuclear halos and the superdeformed states. There has been attempts to explain these phenomena in the framework of single particle and mean field models with some good results [3, 4].

Nevertheless several works show the importance of considering terms in the nuclear Hamiltonian beyond the mean field approximation (the so called residual Hamiltonian). Taking one of these terms into account the Interacting Boson Model (IBM) has been applied to describe the inertia moments and other parameters of superdeformed nuclei [5]. Also the Hartree-Fock model together with the Bardeen, Cooper and Schrieffer (BCS) one has been used to obtain ground states for nuclei close to the proton drip line [6]. Other pairing models have also been successfully applied to explain some characteristics of exotic nuclei [7].

The approach of considering the pairing in the nuclear Hamiltonian is better to that of considering only the mean field, but the pairing interaction is still a part of the residual Hamiltonian. Other terms have proved to be important. For example, the four nucleon interaction could play an important role in the calculation of inertia moments for heavy nuclei [8]. In this regard, the three nucleon interaction could be an important contribution for the nuclear structure even for heavy nuclei [9]. At this stage one should mention that three body interaction is interpreted as a correlation beyond the mean field approach, which take into account some kind of collective behavior of the system.

The three nucleon interaction was recognized long ago [11] and now is still under study mainly for light nuclei [12, 13]. This could be due to the fact that the problem of a many body Hamiltonian including three nucleon interactions is hardly solvable. Even with approximate methods is very difficult to solve such systems.

This paper intend to develop a formalism that include the three nucleon interaction in the frame work of a BCS approach for heavy nuclei taking into consideration the success of such method in the treatment of nuclei in extreme conditions and far off nuclear stability.

THREE NUCLEON INTERACTION POTENTIAL

In order to include the three nucleon interaction into the nuclear Hamiltonian we need to set up a particular form for this potential. In the present paper we propose the potential as an interaction between Cooper pairs and unpaired nucleons.

In the BCS method it is introduced a canonical transformation to quasiparticles, which are called

Cooper pairs, for these quasiparticles are pairs of nucleons. According to this model [14] an excited state of the system correspond to the creation of certain number of quasiparticles. On the other hand there could be some unpaired nucleons. The BCS model does not consider interaction neither between the Cooper pairs nor between pairs and unpaired nucleons. In this paper we go beyond the free quasiparticle approach by considering the interaction between Cooper pairs and unpaired nucleons. Such an interaction in this representation takes the form of a two body one, but there are really three nucleons interacting, thus it contains a part of the three nucleon interaction.

Let $\hat{V}_2(\bar{r}^q,\bar{r})$ be the interaction potential between a quasiparticle and an unpaired nucleon. Then the potential due to this interaction for the whole nucleus is:

$$\hat{V}_{qp} = \sum_{i=1}^{n_q}\sum_{j=1}^{n}\hat{V}_2(\vec{r}_i^{\,q},\vec{r}_j) \tag{1}$$

where n_q and n are the numbers of quasiparticles and unpaired nucleons respectively.

Transforming this expression to the second quantization representation we get:

$$\hat{V}_{qp} = \sum_{k_1k_2k_3k_4} V_{qp}^{1234}\hat{\alpha}^+_{k_1}\hat{\alpha}_{k_2}\hat{a}^+_{k_3}\hat{a}_{k_4} \tag{2}$$

where:

$$V_{qp}^{1234} = \left\langle BCS_{k_1}k_3\middle|\hat{V}_2\middle|BCS_{k_2}k_4\right\rangle \tag{3}$$

This is the most general possible expression for a matrix element of the interaction between a Cooper pair and an unpaired nucleon and could be understood as follows. A pair k_2, which is a pair built up from two particles of momentum k_2 and $-k_2$ respectively, annihilate with a particle of momentum k_4 and are created a pair k_1 and a particle with momentum k_3. Nevertheless, the momentum has to be conserved in the interaction, then we consider as usual constant matrix elements for $k_3 = k_4$ and we get:

$$\hat{V}_{qp} = T\sum_{k_1k_2k_3}\hat{\alpha}^+_{k_1}\hat{\alpha}_{k_2}\hat{a}^+_{k_3}\hat{a}_{k_3} \tag{4}$$

Using the Bogolyubov canonical transformations [14] and organizing the operators in normal form, the potential yields:

$$\hat{V}_{qp} = \hat{V}^{D}_{qp} + \hat{V}^{ND}_{qp} + \hat{V}^{int}_{qp} \tag{5}$$

where:

$$\hat{V}^{D}_{qp} = T\sum_{k_1k_2}(1+v_{k_2}^2)\hat{\alpha}^+_{k_1}\hat{\alpha}_{k_2} \tag{6}$$

$$\hat{V}^{ND}_{qp} = T\sum_{k_1k_2}2u_{k_2}v_{k_2}\hat{\alpha}^+_{k_1}\hat{\alpha}^+_{-k_2} \tag{7}$$

The term $\hat{V}^{D}_{qp}$ is diagonal with respect to the number of quasiparticles and $\hat{V}^{D}_{qp}$ isn't. The term $\hat{V}^{int}_{qp}$ is a set of terms, every one having four annihilation and creation operators, thus, it is a term of interaction between quasiparticles, which is neglected in the same way as in the BCS method.

In order to go forward is necessary to diagonalize the Hamiltonian including all the terms of the BCS method and those corresponding to the interaction between quasiparticles and free nucleons (6) and (7).

DIAGONALIZATION OF THE TOTAL HAMILTONIAN

The Hamiltonian to be diagonalized is:

$$\hat{H}_{MBCS} = \hat{H}'_{BCS} + \hat{V}_{qp} \tag{8}$$

where $\hat{H}'_{BCS}$ is the BCS Hamiltonian, which already includes the chemical potential λ.

In order to fix the number of quasiparticles we introduce the Lagrange multiplier λ_1 in the following way:

$$\hat{H}'_{MBCS} = \hat{H}_{MBCS} - \lambda_1\hat{N}_q \tag{9}$$

Where $\hat{N}_q$ is the quasiparticle number operator.

The ground state of the nucleus (the quasiparticle vacuum) is not modified because of the introduction of the potential $\hat{V}_{qp}$ for it doesn't have a constant term. This could also be understood from the fact that $\hat{V}_{qp}$ is an interaction between free particles and quasiparticles, thus, in the absence of quasiparticles it vanishes. The ground state then will be the same as in BCS, then we have to assure that the first excited level is a minimum. For these reasons it has to satisfy the following variational principle.

$$\delta\left\langle 01_{k_n}0\middle|\hat{H}'_{MBCS}\middle|01_{k_n}0\right\rangle = 0 \tag{10}$$

Here the vector $|01_{k_n}0$ represents a state with one quasiparticle in the single particle level k_n and all the other single particle states empty. We obtain an expression very similar to the gap equation.

$$\Delta' = \frac{1}{2}\Delta'|G|\sum_{k>0}\left(\tilde{\varepsilon}_k^2 + \Delta'^2\right)^{-1/2} + T \qquad (11)$$

where the modified gap is defined as:

$$\Delta' = |G|\sum_{k>0} u_k v_k + T = \Delta + T \qquad (12)$$

and

$$\tilde{\varepsilon}_k = \varepsilon_k - (\bar{\varepsilon}_{k_n} + T)\delta_{kk_n} \qquad (13)$$

$$\bar{\varepsilon}_{k_n} = \bar{\varepsilon}^0_{k_n} - \lambda - \lambda_1 \qquad (14)$$

Then the Hamiltonian is diagonalized and the new quasiparticle levels are:

$$\tilde{E}_k = \left(\tilde{\varepsilon}_k^2 + \Delta'^2\right)^{1/2} + \frac{1}{2}T\left[3 - \varepsilon_k\left(\tilde{\varepsilon}_k^2 + \Delta'^2\right)^{-1/2}\right] \qquad (15)$$

These new levels are modified for the three nucleon interaction. As expected in the case of no interaction between quasiparticles and free nucleons ($T = 0$) $\Delta' = \Delta$ and the expression (15) becomes the corresponding expression for the quasiparticle states of the BCS approach.

The expressions obtained above have the same form as those from the BCS method, then we may use similar approximations and reduce the sum in (11) to the shell Ω containing the Fermi surface.

$$\Delta' = \frac{1}{2}\Delta'|G|\sum_{\substack{k>0\\ k\in\Omega}}\left(\tilde{\varepsilon}_k^2 + \Delta'^2\right)^{-1/2} + T \qquad (16)$$

For the complete solution of (16) we have to consider the presence of the parameters λ and λ_1 which can be obtained from the conservation equations of the nucleons and quasiparticle numbers in the state $|01_{k_n}0\rangle$.

$$\langle 01_{k_n}0|\hat{N}|01_{k_n}0\rangle = N \qquad (17)$$

and

$$\langle 01_{k_n}0|\hat{N}_q|01_{k_n}0\rangle = 1 \qquad (18)$$

We finally obtain a set of three coupled equations.

$$\Delta' = \frac{1}{2}\Delta'|G|\sum_{\substack{k>0\\ k\in\Omega}}\left\{\left[\varepsilon_k^0 - \lambda - (\varepsilon_k^0 - \lambda - \lambda_1 + T)\delta_{kk_n}\right]^2 + \Delta'^2\right\}^{-1/2} + T \qquad (19)$$

and:

$$N = 1 + \frac{1}{2}\left\{1 + (T - \lambda_1)\left[(T - \lambda_1)^2 + \Delta'^2\right]^{-\frac{1}{2}}\right\} \qquad (20)$$

and:

$$1 = \frac{1}{2}\sum_{\substack{k>0\\ k\in\Omega}}\left\{1 - \left[\varepsilon_k^0 - \lambda - (\varepsilon_k^0 - \lambda - \lambda_1 + T)\delta_{kk_n}\right]\cdot\left[\left(\varepsilon_k^0 - \lambda - (\varepsilon_k^0 - \lambda - \lambda_1 + T)\delta_{kk_n}\right)^2 + \Delta'^2\right]\right\}^{-1/2} \qquad (21)$$

From the solution of this system we obtain the values of the parameters λ, λ_1 and Δ' and then substituting them into (15), (14) and (13) we obtain the energy levels including a part of the three nucleon interaction.

The formalism developed in this paper could be applied to the study of the structure of nuclei in extreme conditions, were the three nucleon forces has shown to play an important role.

CONCLUSIONS

In this paper we proposed an interaction potential which include a part of the residual Hamiltonian corresponding to the three nucleon interaction, as an interaction potential between Cooper pairs and unpaired nucleons. This form of the potential has the advantage that it can be included into the BCS Hamiltonian.

The total Hamiltonian including the two and three nucleon potential was diagonalized using a variational principle, from which we obtained a system of three coupled equations similar to the BCS ones. From the solutions of the system all the variational parameters can be obtained.

In the equation system there's still unknown the parameter T, but it isn't a simple adjustment parameter without any physical meaning, for it is the matrix element which in this paper is considered to be constant. On the other hand this approximation can be removed and then we will get an equation system similar to that of the Hartree-Fock-Bogolyubov one.

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