THE EFFECT OF TRAP ANHARMONICITY ON THE CRITICAL TEMPERATURE FOR BOSE-EINSTEIN CONDENSATION

Augusto González*, Instituto de Cibernética, Matemática y Física, Ciudad de La Habana, Cuba

ABSTRACT

The anharmonicity of a magnetic atomic trap at long distances from its center (forty times the radius of the first atomic orbit along the elongated axis, i.e. hundreds of microns) is shown to increase up to 45 % the temperature for Bose-Einstein condensation. This effect is perhaps small in the traditional traps, but should certainly be taken into account in magnetic microtraps, which characteristic dimensions are of order 1 mm .

PACS numbers: 03.75.Fi

RESUMEN

El anharmonicity de una trampa atómica magnética a las distancias largas de su centro (cuarenta veces el radio de la primera órbita atómica a lo largo del eje largo, es decir ciento de micras) se muestra para aumentar por encima de un 45 % la temperatura para la condensación de Bose-Einstein. Este efecto es quizás pequeño en las trampas tradicionales, pero debe tenerse en cuenta ciertamente en microtraps magnético que las dimensiones características son de orden 1 mm .

Unlike the situation in liquid Helium, atomic vapors undergoing Bose-Einstein condensation (BEC) in magnetic traps are very rarefied[1]. As a consequence, the free-boson model works extremely well. Interaction effects, computed in mean-field approximation, are shown to decrease the critical temperature for BEC in a few percent[2]. Still lower corrections come from the approximation of the one-particle discrete spectrum by a continuum of states, the so-called “finite-N” corrections[2], which also decrease the critical temperature.

In the present paper, we show that a relatively important increase of the critical temperature (up to 45 %) could be related to trap anharmonicities at "long" distances from the center. By long distances, we mean around forty times the radius of the first atomic orbit along the elongated direction in the trap potential, i.e. around 200 μm. In the commonly used traps, which characteristic dimensions are a few centimeters, the belief is that anharmonic effects should be very weak at distances of 200 μm. Nevertheless, we present an example of a recent experiment[3], in which the measured values of the total number of atoms in the trap and the critical temperature are not consistent unless anharmonicity (or other) effects raising T_c are included. On the other hand, the trap potential should certainly be anharmonic at 200 μm in the recently developed atomic microtraps[4], which characteristic dimensions are of order 1 mm.

E-mail: *agonzale@icmf.inf.cu

In the continuum free-boson model, the critical temperature is simply estimated from the conditions $N_0 = 0$, $\mu = 0$ in:

$$N - N_0 = \int_0^\infty \frac{g(\epsilon)\,d\epsilon}{\exp\left(\frac{\epsilon - \mu}{k_B T}\right) - 1} \qquad (1)$$

N_0 is the number of atoms in the condensate, and $\epsilon \geq 0$ is the one atom excitation energy. The chemical potential takes values $\mu \leq 0$. g is the density of states.

Magnetic traps with cylindrical symmetry are very common. The trap potential, coming from the interaction of an hyperfine atomic species with the magnetic field, is proportional to the latter which, near the trap center, has a minimum and is written as:

$$B(\rho, z) = B_0 + b_\rho \rho^2 + b_z z^2. \qquad (2)$$

The corresponding harmonic oscillator frequencies are ω_ρ and ω_z. It is useful to define $\omega_0 = (\omega_\rho^2 \omega_z)^{1/3}$, in terms of which the level density is written $g(\epsilon) = ½\, \epsilon^2/(\hbar\omega_0)^3$, and the critical temperature becomes:

$$k_B T_c^0 = \left(\frac{N}{\zeta(3)}\right)^{1/3} \hbar\omega_0 = 0.94\, N^{1/3} \hbar\omega_0, \qquad (3)$$

where $\zeta(x) = \sum_{n=1}^{\infty} 1/n^x$ is the Riemann Zeta function.

The corrections to T_c due to interaction or ifnite-N effects take, respectively, the form[2,5]:

$$\frac{T_c - T_c^0}{T_c^0} = -1.33\frac{a}{r_0}N^{1/6}, \qquad (4)$$

$$\frac{T_c - T_c^0}{T_c^0} = -0.73\frac{\overline{\omega}}{\omega_0}N^{-1/3}, \qquad (5)$$

where $r_0 = \sqrt{\hbar/(m\omega_0)}$, a is the s-wave scattering length, and $\overline{\omega} = (\omega_z + 2\omega_\rho)/3$. Typical values $N \sim 10^6$, $a/r_0 \sim 10^{-2}$ lead to small numbers in the r.h.s. of Equations (4,5). As mentioned, the corrections (4) are computed within mean-field theory. More elaborated variational calculations based on diffusion Monte-Carlo[6] or hypernetted chain theory[7] support the validity of mean-field approximations at the relevant particle densities. Notice that both corrections (4) and (5) are small and make T_c decrease. In a general power-law trap, the corrections may be of any sign and their relative magnitudes may be still greater[5]. However, for usual magnetic traps one expects a harmonic potential coming from (2), at least in the vicinity of the trap center.

Let us suppose that at long distances, $r >> r_0$, anharmonic terms are relevant in (2). In fact, one expects a saturation of the magnitude of B, which should approach the bias field value as the distance increases. In a trap potential that saturates at long distances, the density of energy levels decreases at high energies[8]. It means that the occupation of the ground state is increased, and thus the critical temperature is raised.

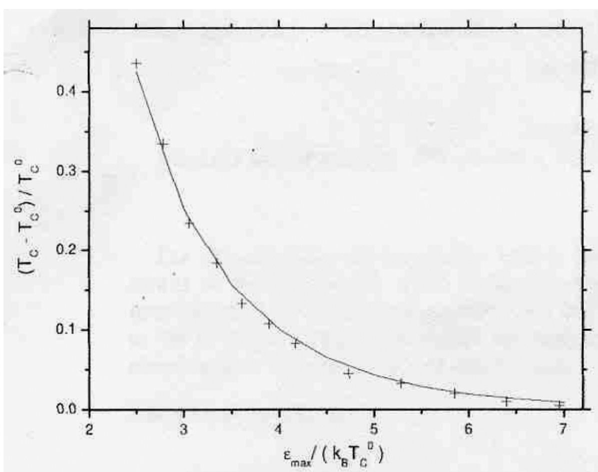

Figure 1. Shift in the critical temperature as a function of the energy cutoff. Solid line: corrections coming from Equation (6), crosses: finite-N calculations for the trap used in Reference 3.

We will model the density of levels of the (saturating) anharmonic potential in the simplest way: by truncating the one-particle spectrum. It means that an upper integration limit, ϵ_{max}, is set in (1). The critical temperature is thus determined from:

$$t^3\zeta(3) = \zeta(3) + Li_3(e^{-tx}) - tx\, Li_2(e^{-tx}) + \frac{(tx)^2}{2}\ln(1 - e^{-tx}), \qquad (6)$$

where we have defined $t = T_c^0/T_c$, $x = \epsilon_{max}/(k_B T_c^0)$, and $Li_k(z) = \sum_{n=1}^{\infty} z^n/n^k$ is the polylogarithm function.

The corresponding correction to T_c^0, i. e. 1/t - 1, is shown in Figure 1 as a function of x. It is around 10 % when $\epsilon_{max} \approx 4k_B T_c^0$. As $k_B T_c^0 \sim 10^3 \hbar\omega_z$ in many of the experiments, energies of order $4k_B T_c^0$ involve particle orbits of radius around 60 r_{oz}. On the other hand, when $\epsilon_{max}/k_B \approx 2.5\, T_c^0$, corrections to T_c^0 reach 40 %, and the cutoff distance is around 40 r_{0z}. A fit to the long-x tail of the curve leads to:

$$\frac{T_c - T_c^0}{T_c^0} = 22\left(\frac{k_B T_c^0}{\epsilon_{max}}\right)^4. \qquad (7)$$

In quality of example, let us consider the trap used in Reference 3, in which BEC of spin-polarized Helium was achieved. The example is quite interesting because independent measurements of N and T_c are reported. The authors-provided frequencies are v_z = 115 Hz, v_ρ = 1090 Hz. The critical temperature, number of atoms in the trap and the scattering length are estimated as $T_c = 4.7 \pm 0.5$ μK, $N = (5 \pm 2.5) \times 10^6$, $a \approx 16$ nm, respectively. Turning back to Equations (3-5), we obtain that to a temperature T_c = 4.7 μK, corresponds to a number of atoms $N = 1.4 \times 10^7$, well above the error bars. On the other hand, if we take the lowest value T_c = 4.2 μK, then the number of atoms is $N = 8 \times 10^6$, a value closer but still outside error bars. The reason for this apparent discrepancy may be either the rough estimation of N, or the anharmonicity corrections, which have not been included.

In conclusion, we have shown that the saturation of the magnetic field at distances around 200 - 300 μm from the trap center leads to a decrease of the density of energy levels, and thus to an increase of the ground-state occupation and to an increase of T_c. We have modeled the decrease of the level density simply by means of an energy cutoff in the single-particle spectrum. The corresponding correction to T_c^0 partially cancels out with interaction or finite-N effects, both of which lower the critical temperature. Although expected to be small, detailed calculations of the magnetic field configurations in the commonly used traps are needed in order to evaluate the role of anharmonicity corrections. With regard to the atomic microtraps, which characteristic dimensions are of order 1 mm, anharmonicities are expected to be very important and should be taken into account.

ACKNOWLEDGMENTS

Part of this work was carried out at the Abdus Salam ICTP. The author acknowledges the ICTP Associate and Federation Schemes for support.

REFERENCES

[1]LEGGET, A.J. (2001): **Rev. Mod. Phys**. 73, 307.

[2]DALFOVO, F. **et al**. (1999): **Rev. Mod. Phys**. 71, 463.

[3]PEREIRA DOS SANTOS, F. **et al**. (2001): **Phys. Rev. Lett**. 86, 3459.

[4]FEENSTRA, L.; L.M. ANDERSSON and J. SCHMIEDMAYER: condmat/0302059.

[5]SALASNICH, L. (2002): **Int. J. Mod. Phys**. B 16, 2185.

[6]GIORGINI, S.; J. BORONAT and J. CASULLERAS (1999): **Phys. Rev**. A 60, 5129.

[7]FANTONI, S. **et al**. (2002): **Phys. Rev**. A 66, 033604.

[8]We assume that the confinement potential decays abruptly above r_0. The density of energy levels refers to states in which the particle motion is confined inside the trap volume, i.e. the discrete spectrum. The contribution of states in the continuum spectrum (in which the particle is free to move in all the space) to the trap thermodynamic properties is proportional to the probability of finding the particle inside the trap, i.e. equal to zero.