SIMPLIFICATION OF POCKLINGTON'S EQUATION KERNEL FOR ARBITRARY SHAPED THIN WIRES

Víctor Barrera Figueroa[1], Jorge Sosa Pedroza[2], José Luis López Bonilla[3]
Instituto Politécnico Nacional, Escuela Superior de Ingeniería Mecánica y Eléctrica
Sección de Estudios de Postgrado e Investigación, UPALM, Lindavista, México D.F.

ABSTRACT
The simplification of Pocklington's equation kernel and the software implementation of the arbitrary shaped thin wire problem is presented here in order to get a useful antenna engineering tool that could be used in determining the current distribution in arbitrary shaped thin wires. The software is based in the widely used method of moments and is implemented in programming language C++. The work presented in this paper is concerned about how to simplify the kernel of the Pocklington's integro-differential equation in order to be more efficient computationally. We obtained an integral equation that reduces not only the programming efforts, but also the time spent in calculations. The integral equation presented in this paper was thought not only for arbitrary shaped thin wires, but also for the case of straight thin wires.

Key words: method of moment, Pocklington's equation, point matching technique, current distribution.

RESUMEN
La simplificación del grano de la ecuación de Pocklington y la aplicación del software del problema del alambre delgado formado arbitrario se presenta aquí para conseguir una antena útil que diseñe una herramienta que podría usarse determinando la distribución actual en alambres delgados formados arbitrarios. El software está basado en el método ampliamente usado de momentos y se lleva a cabo programando idioma C++. El trabajo presentado está interesado sobre cómo simplificar el grano de la ecuación del integro-diferencial de Pocklington para ser computacionado con más eficiencia. Nosotros obtuvimos una ecuación íntegra que no sólo reduce los esfuerzos de la programación, sino también el tiempo de gasto en cálculos. La ecuación íntegra presentada en este trabajo no sólo se pensó para los alambres delgados formados arbitrarios, sino también para el caso de recta de los alambres delgados.

Palabras clave: método de momento, la ecuación de Pocklington, punto que empareja técnica, distribución actual.

1. INTRODUCTION

Knowing the current distribution in any conductor is having the total knowledge about its electromagnetic behavior. For the electrical engineer this is the key for determining the electromagnetic field at any point of the surrounded space (through the magnetic and electric potential), and hence all the electric characteristics would be determined, i.e. input impedance, radiation pattern, directional gain, etc. [1].

The theoretical analysis is based upon two philosophies. The first one, the classical analysis, makes a great effort in guessing how the current is distributed along the antenna. For simple structures this method brings accurate results, but in complex structures (such as helical antennas or cross antennas) we don't have a priory knowledge about the current distribution itself, so the obtained results could be wrong. However, in order to improve the results, the second philosophy brings a better solution. This is based upon the well known method of moments (MM), which was introduced by Galerkin and was popularized by Roger F. Harrington in 1967 [2]. This method, when applied in an electromagnetic problem, brings results whose accuracy is as good as the engineer needs or the computer could provide [3].

The MM solution uses the Pocklington's equation, which is derived from the electric field integral equation. The solution can be found in the applied literature for straight thin wires [4,5]. For arbitrary shaped wires is necessary other integral equation formulation, starting with Mei's analysis and successors [6,7]. Although we are working in the procedure for helical and cross antennas, we present in this paper the well known solution for straight and loop antennas in order to compare results with the analytical solution of the same problem.

E-mail: [1]victorbarreraf@hotmail.com,
[2]jsosa@ipn.mx,
[3]jlopezb@ipn.mx

In this paper we show the simplification we made to the Pocklington's equation. Such simplification transforms the integro-differential equation in an integral one which can be easily evaluated through some numerical integration algorithm. This simplification saves time used by the computer for calculating the impedance and current matrices which is an important issue for being considered when we are looking for a better solution for the considered problem.

2. THE ELECTRIC FIELD INTEGRAL EQUATION

The electric field integral equation describes the strong relationship between the electric field and the current distribution on the conductor. This equation can be derived from Maxwell's equations [8] through the use of the concept of magnetic potential vector and electric potential scalar. Mathematically:

$$E^t(r) = -\frac{j\eta}{k}\int_{v'} J(r')[k^2+\nabla^2]\frac{e^{-jk|r-r'|}}{4\pi|r-r'|}dv' \qquad (1)$$

Where $E^t(r)$ is the total electric field vector measured at the observation point r; J(r') is the current distribution (the unknown of the problem) located at the source point r'; $[k^2 + \nabla^2]$ is the Helmholtz's operator [9] dealing on Green's function for the free space $e^{-jk|r-r'|}/4\pi|r-r'|$; k and η is the wave number and the intrinsic impedance for the free space, respectively:

$$k = \omega\sqrt{\mu_0\varepsilon_0} \qquad \eta = \sqrt{\mu_0/\varepsilon_0} \qquad (2)$$

Where ω is the angular frequency of the current and μ_0 and ε_0 is the permeability and permittivity of the free space, respectively. Notice the use of the volume integral over all source points in the structure that contribute to the total electric field. As we can see in equation (1), the unknown is inside the integral operator.

3. POCKLINGTON'S INTEGRAL EQUATION FOR THIN WIRES

Pocklington's integral equation can be derived from (1) by applying the electric field boundary condition over the structure of the arbitrary shaped thin wire. This condition states that the total electric field vanishes over the surface of any perfectly conducting metal. In practice, the metal from which the antennas are made is almost as perfect as an ideal conductor (i.e. cupper) and hence this condition models accurately the current behavior on the wire's surface. If the wire's surface is represented by $r = r_s$, then:

$$E^t(r_s) = E^i(r_s) + E^s(r_s) = 0 \qquad (3)$$

Where $E^i(r_s)$ is the impressed electric field and $E^s(r_s)$ is the scattered electric field. For high frequencies, due the skin effect, the current will be located all over the wire's surface. The more the frequency increases, the better the current will be confined in an infinitesimal layer that cover the wire. The skin depth δ is obtained from:

$$\delta = \sqrt{2/\omega\mu\sigma} \qquad (4)$$

Where σ is the electrical conductivity of the metal from which the antenna was made. By using (1), (3) and considering that the current is located only on the wire's surface we can get the impressed electric field on the wire's surface, as is shown in Figure 1:

$$E^s(r = r_s) = -\frac{j\eta}{k}\int_s\oint_\varphi J_s(r' = r_s)\left[k^2+\nabla^2\right]\frac{e^{-jk|r-r'|}}{4\pi|r-r'|}a\,d\varphi' ds' \qquad (5)$$

Notice that the integration is over the wire's surface. The variable s' represents the arch length over the wire and φ' the azimuthal angle around the cross section of the wire. We have supposed that the wire is much thinner than the wavelength of the current. Under this assumption it is valid to suppose that the current distributes itself without circumferential variations; this is called the thin-wire approximation.

If the wire's radius is represented by a, then the superficial current distribution can be expressed as:

$$2\pi a J_S = I_S(s') \quad \Rightarrow \quad J_S = \frac{I_S(s')}{2\pi a} \qquad (7)$$

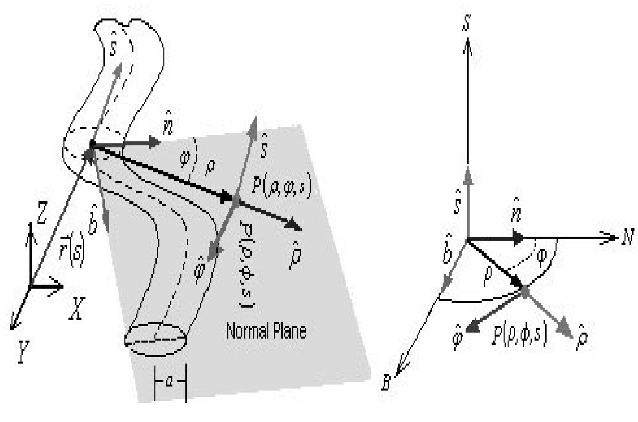

Figure 1. Relations among the vectors of the arbitrary shape wire.

Due the azimuthal independence, the current distribution only depends of the arch length; in other words, we can think that the current is confined in a very thin current filament $I_S(s')$ over the wire's surface. If the axis of the wire is represented by the following vectorial equation:

$$r(s) = x(s)i + y(s)j + z(s)k \quad (8)$$

Then the curve which represents the current filament has the next vectorial equation:

$$r'(s') = r(s') + an(s') \quad (9)$$

Where n(s) is the unit normal vector for the wire's axis; in other words, the curve which represents the current filament is a parallel curve to the curve which represents the wire's axis. Considering the circular cross section, the wire's axis will have an infinite collection of parallel curves, although in practice, we select the one which could make the estimations easier.

Applying the former considerations we can get an expression for the impressed electric field over the wire's surface, known as the Pocklington's equation:

$$E_S^I = -\frac{j}{\omega\varepsilon}\int_{s'} I_S(s')\left[k^2 s \bullet s' + \frac{\partial^2}{\partial s \partial s'}\right]\frac{e^{-jk|r-r'|}}{4\pi|r-r'|}ds' \quad (10)$$

Where E_S^I is the tangential impressed electric field. Notice that due the thin-wire approximation and the skin effect, we could express the electric field as a linear integration over the arch length s'.

The equation (10) is the general Pocklington's equation which is valid for any possible geometry that the wire could have; nevertheless, Pocklington's equation form found in the applied literature is restricted only for straight wires [Balanis, op. cit].

The wire's geometry is expressed by the dot product $s \bullet s'$, where $s(s)$ is the unit tangential vector for the wire's axis and $s'(s')$ is the unit tangential vector for the parallel curve which represents the current filament, as is shown in Figure 2. These vectors are calculated from:

$$s(s) = \frac{dx(s)}{ds}i + \frac{dy(s)}{ds}j + \frac{dz(s)}{ds}k$$
$$s'(s') = \frac{dx'(s')}{ds'}i + \frac{dy'(s')}{ds'}j + \frac{dz'(s')}{ds'}k \quad (11)$$

The geometry is also expressed by the difference between the vectors $|r - r'|$ which can be expressed as:

$$R = |R| = |r - r'| = \sqrt{[x(s) - x'(s')]^2 + [y(s) - y'(s')]^2 + [z(s) - z'(s')]^2} \quad (12)$$

In this way, the work just consists in finding the vectors which represent the parallel curve and the axis curve for the considered wire.

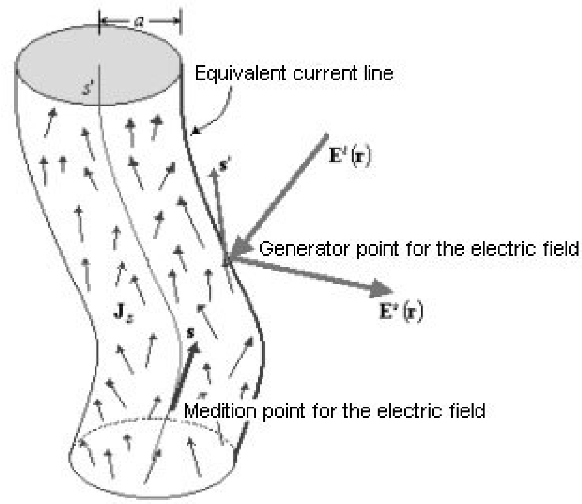

Figure 2. Unit tangential vectors for the thin wire.

4. THE MM SOLUTION TO POCKLINGTON'S EQUATION

The MM is a numerical technique which transforms an operational equation into a matrix equation. The fact that an operational equation could be discretized is the base in which the equation could be solved through a computational procedure [10].

The objective of the MM applied in Pocklington's equation is to get the current distribution $I_S(s')$ of the wire. Looking at (10) is clear that reaching that goal is a very difficult task because the unknown function $I_S(s')$ is inside the integral operator. The formulation of the MM is needed in order to get a numerical solution of (10) instead of an analytical one (in cases where the wire's geometry is simpler, it is possible to get an analytical solution for (10), but in other cases there isn't warranty for getting such one).

The MM states that the unknown function must be expressed in terms of a linear combination of linearly independent functions $i_n(s')$ called basis functions:

$$I_S(s') = \sum_{n=1}^{N} c_n i_n(s') \quad (13)$$

Where c_n are unknown coefficients that must be determined and N is the number of basis functions. By substituting (13) into (10) it results in one equation with N unknowns:

$$E_S^I = -\frac{j}{\omega\varepsilon}\sum_{n=1}^{N} c_n \int_{s'} i_n(s')\left[k^2 s \bullet s' + \frac{\partial^2}{\partial s \partial s'}\right]\frac{e^{-jk|r-r'|}}{4\pi|r-r'|}ds' \quad (14)$$

In order to get a consistent equation system we must find N linearly independent equations, which can be obtained by taking the inner product of (14) with another set of N linearly independent functions $w_m(s)$ chosen, called weighting function:

$$\langle w_m, E_S^I \rangle = -\frac{j}{\omega\varepsilon}\sum_{n=1}^{N} c_n \left\langle w_m, \int_{s'} i_n(s')\left[k^2 s \bullet s' + \frac{\partial^2}{\partial s \partial s'}\right]\frac{e^{-jk|r-r'|}}{4\pi|r-r'|}ds' \right\rangle$$

$$m = 1, 2, \Lambda, N \qquad (15)$$

Remembering the definition of inner product we can write the system (15) in the form:

$$\int_s w_m E_S^I ds = -\frac{j}{\omega\varepsilon}\sum_{n=1}^{N} c_n \int_s w_m \int_{s'} i_n(s')\left[k^2 s \bullet s' + \frac{\partial^2}{\partial s \partial s'}\right]\frac{e^{-jk|r-r'|}}{4\pi|r-r'|}ds'ds$$

$$m = 1, 2, \Lambda, N \qquad (16)$$

The system could be written in a matrix form:

$$\begin{bmatrix} Z_{11} & Z_{12} & \Lambda & Z_{1N} \\ Z_{21} & Z_{22} & \Lambda & Z_{2N} \\ M & M & O & M \\ Z_{N1} & Z_{N2} & \Lambda & Z_{NN} \end{bmatrix}\begin{pmatrix} c_1 \\ c_2 \\ M \\ c_N \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ M \\ v_N \end{pmatrix} \qquad (17)$$

$$[Z_{mn}](c_n) = (v_m)$$

Where the elements Z_{mn} are calculated from:

$$Z_{mn} = -\frac{j}{\omega\varepsilon}\int_s w_m \int_{s'} i_n(s')\left[k^2 s \bullet s' + \frac{\partial^2}{\partial s \partial s'}\right]\frac{e^{-jk|r-r'|}}{4\pi|r-r'|}ds'ds \qquad (18)$$

And the elements v_m are obtained from:

$$v_m = \int_s w_m E_S^I ds \qquad (19)$$

And c_n are, of course, the unknowns of the system. In the literature the matrices of the systems (18) have special names. These are the following: $[Z_{mn}]$, impedance matrix; (c_n), current matrix ; and (v_m), voltage matrix. Though their names remember us the Kirchhoff's equation of electric circuits, their real units are Ω, A/m and V/m, respectively. Nevertheless, if the length unit is taken as unitary, the units of (17) are then the same like the Kirchhoff's ones. So, the solution of (17) is written as:

$$(c_n) = [Z_{mn}]^{-1}(v_m) \qquad (20)$$

Where some numerical technique could be used in order to get the inverse matrix $[Z_{mn}]^{-1}$.

5. THE POINT MATCHING SIMPLIFICATION TECHNIQUE

As we can see, each element of the system (17) consists in a double integration over the Helmholtz's operator and the Green's function. This could be a very difficult task because the calculation time and programming efforts increases with N; then we must find some simplifications in order to reduce the computational needs [11].

The first simplification is based upon the method of moments' technique known as *Point Matching Technique*. This technique uses the Dirac's delta function as weighting function. The properties of this function are the followings:

$$\int_{\Delta s} \delta(s - s_m)ds = \begin{cases} 1 & \text{if} \quad s_m \in \Delta s \\ 0 & \text{elsewhere} \end{cases}$$

$$\int_{\Delta s} f(s)\delta(s - s_m)ds = \begin{cases} f(s_m) & \text{if} \quad s_m \in \Delta s \\ 0 & \text{elsewhere} \end{cases} \qquad (21)$$

By applying (21) into (18) and (19) we get:

$$Z_{mn} = -\frac{j}{\omega\varepsilon}\int_{s'} i_n(s')\left[k^2 s \bullet s' + \frac{\partial^2}{\partial s \partial s'}\right]\frac{e^{-jk|r-r'|}}{4\pi|r-r'|}ds'\Bigg|_{s=s_m}$$

$$v_m = E_S^I(s = s_m) \qquad (22)$$

Using Dirac's delta means that the boundary electromagnetic conditions are being applied only at discrete points on the wire's structure, exactly in the places where the Dirac's functions have their roots s_m. Choosing the places where the roots must be located is an important problem [12].

In this paper, the roots will be found in the axis of the wire, which means that we try to find the electric field in this place. A second simplification is to choose some basis function that could simplify the integration (22).

The one chosen is the pulse function that is defined by:

$$i_n(s') = \begin{cases} 1 & \text{if} \quad (n-1)\Delta s' \le s' < n\Delta s' \\ 0 & \text{elsewhere} \end{cases} \qquad (23)$$

The pulse function divides the wire's structure into N segments of length $\Delta s'$, producing an stair representation of the structure current. The size of each segment must be freely chosen, however, in order to keep the linear independence of each equation of (17), usually the whole structure is equally divided, so the length of each segment is $\Delta s' = L/N$. By applying (23) into (22) we get:

$$Z_{mn} = -\frac{j}{\omega\varepsilon}\int_{(n-1)\Delta s'}^{n\Delta s'}\left[k^2 s \bullet s' + \frac{\partial^2}{\partial s \partial s'}\right]\frac{e^{-jk|r-r'|}}{4\pi|r-r'|}ds'\Bigg|_{s=s_m} \qquad (24)$$

6. THE KERNEL SIMPLIFICATION

Another simplification can be made. We note in (24), that the integral operator deals with an iterated differential operator; instead that the software makes such operation numerically each time for the impedance element, it is possible to develop the derivative operation. By using (12) we find the iterated derivative for Green's function:

$$\frac{\partial^2}{\partial s \partial s'}\frac{e^{-jkR}}{R} = -\frac{\partial}{\partial s'}\left[\frac{(1+jkR)e^{-jkR}\frac{\partial R}{\partial s}}{R^2}\right]$$

$$= e^{-jkR}\,\frac{\left(\frac{\partial R}{\partial s}\right)\left(\frac{\partial R}{\partial s'}\right)\left[2-k^2R^2+2jkR\right]-R(1+jkR)\frac{\partial^2 R}{\partial s\partial s'}}{R^3} \tag{25}$$

As we can see, we must find the value of the multiple derivatives of R that appears in the last equation. According to (11) and (12), we have:

$$\frac{\partial R}{\partial s'} = \frac{1}{2R}\frac{\partial}{\partial s'}\left\{[x(s)-x'(s')]^2+[y(s)-y'(s')]^2+[z(s)-z'(s')]^2\right\} = -\frac{1}{R}R\bullet s'$$

$$\frac{\partial R}{\partial s} = \frac{1}{2R}\frac{\partial}{\partial s}\left\{[x(s)-x'(s')]^2+[y(s)-y'(s')]^2+[z(s)-z'(s')]^2\right\} = \frac{1}{R}R\bullet s$$

$$\frac{\partial^2 R}{\partial s\partial s'} = -\frac{1}{R}\left\{\frac{\partial x'(s')}{\partial s'}\frac{\partial x(s)}{\partial s}+\frac{\partial y'(s')}{\partial s'}\frac{\partial y(s)}{\partial s}+\frac{\partial z'(s')}{\partial s'}\frac{\partial z(s)}{\partial s}\right\}+R\bullet s'\frac{1}{R^2}\frac{\partial R}{\partial s}$$

$$= -\frac{1}{R}s\bullet s' + \frac{1}{R^3}(R\bullet s)(R\bullet s') \tag{26}$$

By substituting (26) in (25) we have:

$$\frac{\partial^2}{\partial s\partial s'}\frac{e^{-jkR}}{R} = e^{-jkR}\,\frac{R^2(1+jkR)s\bullet s' - (R\bullet s')(R\bullet s)(3-k^2R^2+3jkR)}{R^5} \tag{27}$$

By substituting (27) in (24) we get the simplified kernel of Pocklington's equation:

$$\left[k^2 s\bullet s' + \frac{\partial^2}{\partial s\partial s'}\right]\frac{e^{-jkR}}{4\pi R} = \left[R^2(1+jkR-k^2R^2)s\bullet s' - (3+3jkR-k^2R^2)(R\bullet s)(R\bullet s')\right]\frac{e^{-jkR}}{4\pi R^5} \tag{28}$$

So the elements of the impedance matrix are expressed for:

$$Z_{mn} = -\frac{j}{4\pi\omega\varepsilon}\int_{(n-1)\Delta s'}^{n\Delta s'} R^2(1+jkR-k^2R^2)s\bullet s'\frac{e^{-jkR}}{R^5}ds'\Bigg|_{s=s_m} + \frac{j}{4\pi\omega\varepsilon}\int_{(n-1)\Delta s'}^{n\Delta s'}(3+3jkR-k^2R^2)(R\bullet s)(R\bullet s')\frac{e^{-jkR}}{R^5}ds'\Bigg|_{s=s_m} \tag{29}$$

For the numerical integration of (29) the well known trapezoid rule could be used [13,14].

7. MODELING THE SOURCE

As we can see in (22) the elements of the matrix are the value of the impressed electric field in the roots of Dirac's delta. It means that we must know the value of the impressed electric field all over the structure. This could be as hard as knowing the current distribution in the conductor. The solution is to choose some source which will produce some known impressed electric field distribution on the conductor. Although there are known sources used for modeling the voltage matrix [Stutzman, op. cit], the most used, is the delta gap generator; this source supposes that the impressed electric field is different from zero only in the place where it is connected. Mathematically:

$$E_S^I(s) = \begin{cases} V/\Delta s & \text{if} \quad s\in\Delta s_m \\ 0 & \text{elsewhere}\end{cases} \tag{30}$$

Where V is the value of the voltage fasor of the source connected in the *m*th segment of the structure. The delta gap generator is a very idealized generator that could not be obtained in practice, however its use simplifies the solution and produces results with enough accuracy. In this way, if the antenna uses only one source connected at the *m*th segment, the voltage matrix will be:

$$(v) = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \\ \vdots \\ v_N \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ V/\Delta s \\ \vdots \\ 0 \end{pmatrix} \tag{31}$$

The value of V is usually taken like 1, however, other choices could be made.

For considering impedances, we must apply the Kirchhoff's voltage law. If the impedance Z is connected at the *m*th segment, the potential difference in its ends is

$$V_m = v_m - c_m Z \tag{32}$$

Where c_m is the current in such segment. In this way, by substituting in (17) we have

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1m} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2m} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mm} & \cdots & Z_{mN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nm} & \cdots & Z_{NN} \end{bmatrix}\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \\ \vdots \\ c_N \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_m - c_m Z \\ \vdots \\ v_N \end{pmatrix} \tag{33}$$

We can note that, by using the definition of matrix product, the *m*th element of the voltage matrix can be expressed like:

$$c_1 Z_{m1} + \ldots + c_m Z_{mm} + \ldots + c_N Z_{N1} = v_m - c_m Z \tag{34}$$

$$c_1 Z_{m1} + \ldots + c_m \left(Z_{mm} + Z \right) + \ldots + c_N Z_{N1} = v_m$$

So, the equation (33) can be written as:

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1m} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2m} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mm}+Z & \cdots & Z_{mN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nm} & \cdots & Z_{NN} \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \\ \vdots \\ c_N \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \\ \vdots \\ v_N \end{pmatrix} \tag{35}$$

Therefore, we conclude that connecting an impedance at the *m*th segment is the same than adding its value to the corresponding Z_{mm} matrix element.

8. CURRENT DISTRIBUTION IN A STRAIGHT LINEAR DIPOLE

The dipole antenna is very used in communication systems and is the most simple antenna ever made. The first step is to find the equation that describes its axis. If the antenna axis lies in the Z axis, as shown in Figure 3, its equation and its unit tangential vector is:

$$r(s) = sk \qquad s(s) = k \tag{37}$$

The next step is to choose the parallel curve that describes the current filament. For simplicity we choose the curve that lies over the YZ plane. Its equations are:

$$r'(s') = s'k + aj \quad s'(s') = k \tag{38}$$

The dipole is feeding in its central segment, so the voltage matrix will be:

$$(v_m) = \begin{pmatrix} 0 \\ \vdots \\ V/\Delta s \\ \vdots \\ 0 \end{pmatrix} \tag{39}$$

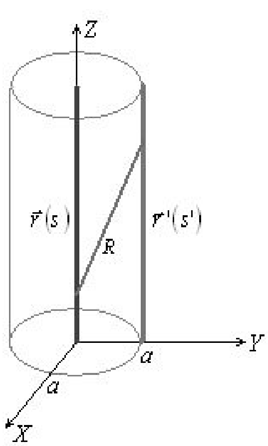

Figure 3. Dipole antenna and its associated vectors.

By using (37), (38) and (39) for a 2λ length dipole with the following characteristics:

f = 1GHz   N = 41 segments   L = 2λ

a = 0.005λ   V = 1 volt

The C++ program we developed produces the current distribution of Figure 4.

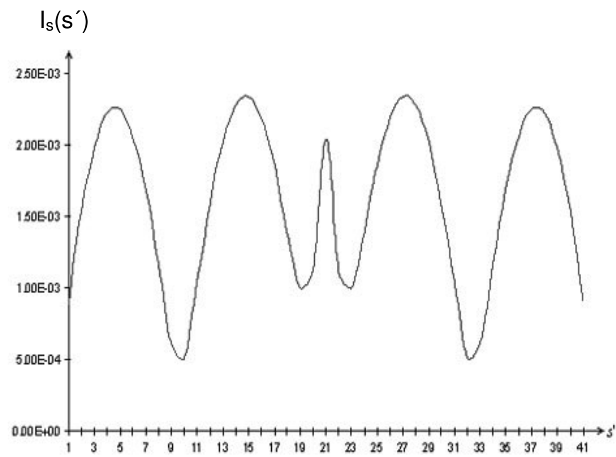

Figure 4. Current distribution for a 2λ length dipole antenna.

We can note that the current's magnitude at the 21th segment is 2.04099 mA, so the input impedance is:

$$\left| Z_{in} \right| = \frac{\left| V_{21} \right|}{\left| I_{21} \right|} = \frac{1\,V}{2.04099\,mA} = 489.95\Omega \tag{40}$$

The results are consistent with the expected ones cited by literature [Balanis, op. cit.].

9. CURRENT DISTRIBUTION IN A CIRCULAR LOOP ANTENNA

Following the same procedure for the circular loop antenna, we must find the vectorial equation for the axis and the filament curve. If the antenna has a radius A, as in Figure 5, then the equations are:

$$r(s) = A\cos\left(\frac{s}{A}\right)i + A\mathrm{sen}\left(\frac{s}{A}\right)j$$

$$s(s) = -\sin\left(\frac{s}{A}\right)i + \cos\left(\frac{s}{A}\right)j \tag{41}$$

Among the possible parallel curves we choose the simpler for the current filament, which is the one located at a distance a from the axis of the antenna; then the equations are:

$$r'(s') = A\cos\left(\frac{s'}{A}\right)i + A\mathrm{sen}\left(\frac{s'}{A}\right)j + ak$$

$$s'(s') = -\sin\left(\frac{s'}{A}\right)i + \cos\left(\frac{s'}{A}\right)j \tag{42}$$

It is clear that the parallel curve has the same length than the axis curve.

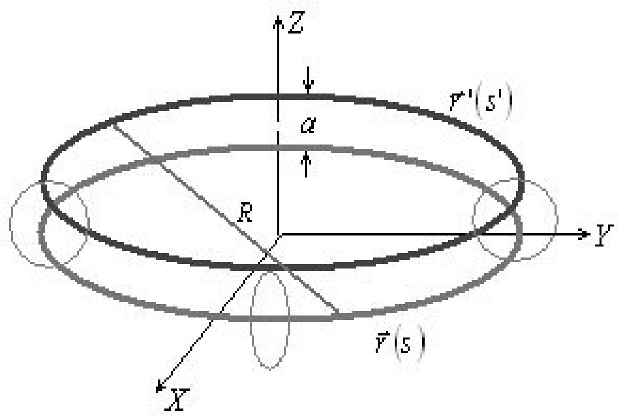

Figure 5. Circular loop antenna and its associated vectors.

The antenna is feeding in its ends, so the voltage matrix will be:

$$(v_m) = \begin{pmatrix} V/2\Delta s \\ 0 \\ 0 \\ \vdots \\ 0 \\ -V/2\Delta s \end{pmatrix} \qquad (43)$$

By using (41), (42) and (43) for a 3λ length circumference loop antenna with the following characteristics:

f = 1 GHz N = 41 segments $2\pi A = 3\lambda$

$A = 0.005\lambda$ V = 1 volt

The C++ program gives the curve of Figure 6. By looking at Figure 6 we can note that the current's magnitude in the ends has the same value, equal to 0.8015937 mA, so the input impedance is:

$$\left|Z_{in}\right| = \frac{\left|V_1\right|}{\left|I_1\right|} = \frac{1\,V}{0.8015937\ mA} = 1247.5147\Omega \qquad (44)$$

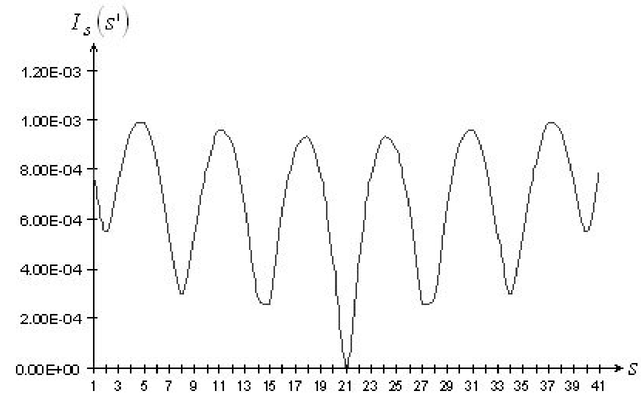

Figure 6. Current distribution for a 3λ length circumference loop antenna.

10. CONCLUSION

In this paper has been shown the general procedure for getting the current distribution in an arbitrary shaped wire. Its geometry has been taken into account in order to get a good representation of the wire through the relations among the vectors that describe the whole wire. There is not doubt that Pocklington's equation describes correctly the electromagnetic problem, so that the results have as good accuracy as the size of the programming efforts. The simplification of the integral equation's kernel brings computational simplification, reducing the computation time and the source code of the program. For any problem, the engineer just needs to specify the geometry of the antenna, the source and the load impedance.

In this first effort, the program only calculates the current distribution, but we are working for getting the current distribution graph and the radiation partner plot, and a mathematical syntax analyzer for interpreting the vectorial equations of the antenna.

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