A FEW WORDS ABOUT COMPLEXITY AND THE NATIONAL INSTITUTE OF SCIENCE AND TECHNOLOGY FOR COMPLEX SYSTEMS OF BRAZIL[1]

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In dynamic systems, the next state is defined according to some rule applied to the current one. Normally, this rule is modeled by some differential equation, in general a stochastic one which also takes into account diverse random ingredients influencing the system under study such as noise, etc. For the sake of reasoning, instead of the continuous version we will consider here the discrete time evolution, in principle the extension to the continuous case is straightforward. Besides the internal functioning mechanism of the system itself, the dynamic rule includes also the influence of the environment. The paradigmatic example is the evolution of a population through heredity.

Also, we will consider a system with many individual components, say N, for which the universe of conceivable states is enormous, i.e. the size of this universe exponentially grows for increasing N. Given some current state, this whole universe is not reachable in the next time step, only a subset of it would be compatible with the current state. Therefore, if one follows the historic path, the effectively available states for the next time step generally corresponds only to a tiny subset of the quoted universe, from which one particular state is chosen to be the next one, and so on. In other words, the potential universe of states is always drastically shrank, if one considers only the next time step, a feature we will call here shrinking availability. Within an evolving population, for example, not all possible conceivable genes can be present in the next generation, only those already present in the current generation (except for extremely rare innovative random mutations).

On the other hand, for the long-term evolution, two cases are conceivable. First, in spite of the short term shrinking availability, eventually the whole universe may be effectively visited, for any initial state (except, perhaps, a null-measure set of the whole universe), within a ``finite" time T. The quotes mean that T may even grow with increasing N, provided this growing behavior is sub-exponential. In this case the system under study is said to be ergodic. Within the second conceivable, non-ergodic case, the whole universe of states is ``never" completely covered. The quotes here mean that T grows at least exponentially with increasing N. The system becomes ``forever" (quotes in the same sense) confined into small sub-sets of the whole universe of possible states.

Complex systems are those falling into this second case. They are dynamic systems whose long term behavior depends on the historic path effectively followed. If one is able to repeat the evolution starting from the same initial state, the long term result may be completely different from one realization to the other. In particular, minor contingencies occurred during this historic path eternally leave its own mark on the future evolution of the system, a long term memory behavior.

The well established classical Boltzmann-Gibbs Statistical Mechanics theory is based on the assumption that the system under study falls into the first case, i.e. it is ergodic, the so-called chaotic hypothesis adopted by Boltzmann. By the way, complexity should not be confounded with chaos, the long or short term memory is the main difference between these two cases, respectively. This difference can be quantitatively established by the so-called Lyapunov exponent, characteristic of the system's dynamic behavior: it is strictly positive for chaotic systems and exactly null for complex ones. The transient time necessary to reach equilibrium is the inverse of this exponent. Thus, complex systems evolve in eternal transients, ``never" reaching the thermodynamic equilibrium described by the Boltzmann-Gibbs theory.

Therefore, in principle the Boltzmann-Gibbs theory is applicable only to ergodic systems. An equivalent theory for complex systems is not available. Besides quantitative observations or experiments and the statistical analysis of the corresponding time series, the main instrument we have to study such systems is computer modeling simulation. In particular, the so-called agents- based models in which the behavior of each component of the systems is followed step by step as (computer) time goes by. To model means to impose some dynamic rule for the behavior of each component according to the current state of the others, also according to the influence of the environment and to the contingencies implemented through the use of (pseudo) random

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numbers generators. To simulate means to program these rules on a computer, following the resulting dynamic evolution.

Computer modeling simulation is a feasible approach due to a crucial detail described in the next paragraph. Usually, long term memory (in time) implies long range correlation (in space). It occurs even when each component directly interacts only with some neighboring others, not with the whole system (the so-called short range interactions, not the same concept as short or long range correlations). An important quantity is the correlation length xi, measuring how far from the position of some particular component a modification performed on its current state is felt by the others. If xi is much larger than the typical distance between neighboring components, approaching the linear size L of the system itself, the whole set of components are correlated to each other. Such a system is called critical. It behaves as a whole not as a simple superposition of its components. Each component is not free to occupy any of its individual states, but only those compatible with the others' current states, a space related feature similar to the shrinking availability already commented within the time evolution. Criticality enhances even more the shrinking availability of states. This behavior poses an extra difficulty in order to treat such a system, because it cannot be divided into smaller blocks, the so-called reductionist approach. Strategies where a small piece of the macroscopic system is first studied in isolation, and then the influence of the remainder parts is included as perturbation simply do not work for critical systems. Neither for complex systems, which present long term memory besides long range correlations.

However, this same characteristic makes the particular short range interaction between neighboring components unimportant for the global behavior. In other words, a critical (or complex) system global behavior is defined by its large scale properties, and not by the specific short range interaction between its neighboring components. Systems which are completely different in what concerns their microscopic properties may present the same global dynamic behavior. In this case, they belong to the same universality class. The researcher's task is to invent some artificial dynamic rule retaining only the essential ingredients governing the global behavior. In other words, the task is to invent a computer model which belongs to the same universality class as the real system of interest.

In short, a complex system evolves in time visiting only tiny fractions of the universe of possible states, and this fraction depends on the historical path effectively realized. The probability distribution for this restricted set of states, therefore, is not the same one would find in the case where the whole universe of states can be freely visited, as in the Boltzmann-Gibbs theory. One of the goals is just to propose alternative distribution probabilities adequate for each case.

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