OPTICAL MODEL OF THE UNACCOMMODATED HUMAN EYE

MODELO ÓPTICO DEL OJO HUMANO SIN ACOMODACIÓN

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Modern techniques for corrective eye surgery require an accurate knowledge of the refractive properties of the eye, personalized for the each patient [1]. With this information the surgeon can, for instance, calculate the shape and extension of the corneal ablations required for attaining the desired correction. Then, if she/he has an adequate model of the eye, the optical power and magnitude of the aberrations of the eye after the surgery can be predicted. To that end, it is necessary firstly to have a biomechanical model that, knowing the size and shape of the ablations, gives the new geometrical parameters of the cornea and, secondly, to introduce these parameters into an optical model that allows following the ray paths all the way from the anterior surface of the cornea to the retina. Although first or third order approximations could be used, exact ray tracing is best. The resulting model could be numerically implemented on a computer, easing the task.

In this contribution we develop an exact ray tracing model for an unaccommodated eye. Through it, accommodation is easily introduced changing the curvature of the surfaces of the crystalline. The final objective is to obtain a computing program that could be used in the ophthalmology services of Cuban hospitals, for a better planning of Refractive Laser Surgery. As far as we know, they currently do not have that possibility.

There are several ways to model the eye. Differences arise from different mathematical representations of the refractive surfaces, the law of dispersion of the media, and the model used for the crystalline. We will model the eye using four refractive surfaces (Fig 1). 1 and 2 represent anterior and posterior corneal surfaces, while 3 and 5 are anterior and posterior crystalline surfaces. Each is described by the equation of an aspherical revolution surface, of the form:

$$x^2 + y^2 + (Q+1)z^2 - 2zR = 0 \,, \tag{1}$$

where Q is the asphericity parameter and R is the radius at the center of the surface. These parameters could be experimentally

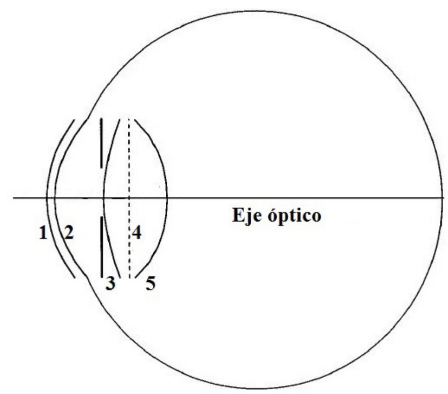

Figure 1: Simplified diagram of the eye. 1 and 2 represent anterior and posterior corneal surfaces respectively, while 3 and 5 are anterior and posterior crystalline surfaces, respectively. Surface 4 represents an auxiliary plane introduced for calculation purposes.

determined by the physician or, in case of a service without such capabilities, can be taken from a database. The values used in this work are given in Table I, and are averages from the best experimental data in literature [2]. Surface 4 is an artificial plane, introduced in order to help in the calculation process.

The algorithm for calculating ray paths is described as:

1. Select the incoming ray and represent it by the equation of a straight line.
2. Intersect this line with the first surface, and obtain the coordinates of the incidence point.
3. Determine the normal to the surface at this point and calculate the trajectory of the refracted ray by the refraction law. Refraction index (after the corresponding surface) and other optical parameters used are listed in Table 1.
4. With the equation of the refracted ray, obtain the incident point in the next surface.
5. Repeating this procedure, the position of the ray in the retina or the point of its intersection with the optical axis is obtained.

Table I
Parameters of the optical model referred to Fig. 1.

S	R (mm)	Q	n (0,555 μm)
1	7.77	-0.18	1.376
2	6.40	-0.60	1.336
3	12.40	-0.94	GRIN
5	-8.10	0.96	1.336

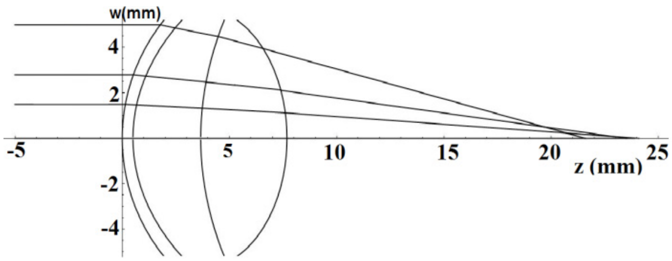

Figure 2: Graphical results of the analytic calculations for four different rays.

An exception is the ray path though crystalline. There is conclusive evidence [3] in favor of a distribution of refraction indices inside the crystalline, indicating that it behaves like a GRIN lens. We choose a parabolic model of GRIN lens [3] frequently used in literature:

$$n(w,z) = n_{0.0} + n_{0,1}z + n_{0,2}z^2 + n_{2,0}w^2 . \qquad (2)$$

Here $n_{i,j}$ are parameters of the model, z is the coordinate along the optical axis and w is perpendicular to it ($w^2 = x^2 + y^2$). This model is the best fit to the experimental data. It is possible to demonstrate that in an axis-symmetric GRIN model, if an incident ray is coplanar with optical axis, it keeps in this plane. n_{ij} has been chosen to provide a refraction index in the experimental range (1.368 – 1.404) [2,3]. Sometimes a simplified model of the crystalline is constructed, assuming an equivalent refraction index around 1.44 [4].

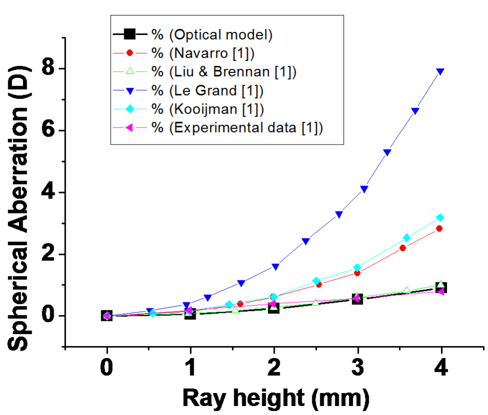

Figure 3: Spherical aberration calculated by our model, compared with other models and experiment.

In most of cases, the shell model [5] is used to calculate ray paths through the crystalline. Here we use Fermat's principle. We did not find any previous report of its use in this task. Minimizing the functional of travel time, the solution of the Euler -Lagrange differential equation gives the trajectory. In this case the equation is highly no linear, of the form:

$$w''(z) = f_1\left[w(z),z\right]\left\{1+\left[w'(z)\right]^2\right\} - f_2\left[w(z),z\right]\left\{w'(z)+\left[w'(z)\right]^3\right\}, \qquad (3)$$

where f_1 and f_2 are known functions. This equation was solved numerically, dividing the crystalline in two parts, with different parameters into eq. (2). The surface dividing the two parts is a plane (surface 4 in Figure 1), so it does not have refracting power.

An example of ray paths is shown in figure 2. Spherical aberrations are clearly present.

The maximum of the spherical aberration is calculated as the difference between the optical power for two rays: one very close and parallel to the optical axis, and the other parallel to the first one, but passing by the border of the iris. The results are very close to those in [6], and agree with experiments for rays that are less than 4 mm apart from the optical axis (see Fig. 3).

The model also reproduces well the chromatic aberration at every wavelength. This is calculated assuming normal dispersion for all the components of the eye, with a dependence of refraction index with wavelength equal to that of water, as [2]:

$$n(\lambda) = n(555\,nm) + 0.0512 - 0.1455\,\lambda + 0.0961\,\lambda^2 \qquad (4)$$

Values of chromatic aberration relative to the wavelength of 555 nm are shown in figure 4. This distribution compares very well with experiments.

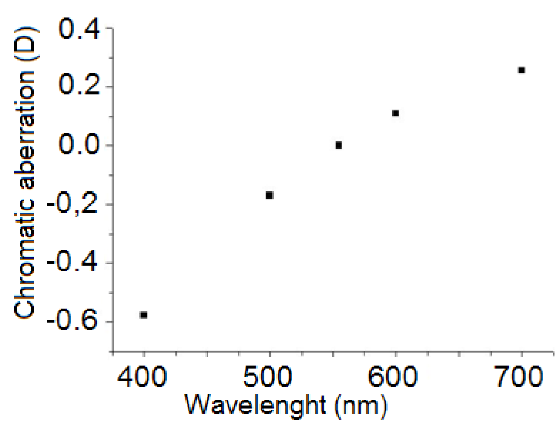

Figure 4: Chromatic aberration calculated by our model.

The model allows to calculate the axial length of the eye. Our result is 23.96 mm, very close to the average value obtained by MRI studies [7].

All in all, the model developed here gives very good results for calculating exact ray paths through the eye, and the optical power (60.32 D) as well as the axial length of the eye. Aberrations are near experimental results.

In the future we expect to develop a different method for the calculation of ray paths through the crystalline, because the one presented here, in spite of its high precision, consumes a lot of computing resources. The present method could then be used as a standard for comparison and evaluation of other models. We also expect to extend the model of crystalline to include accommodation and changes in refraction index with age.

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