ON THE ONSET OF INSTABILITY OF A VISCOELASTIC FLUID SATURATING A POROUS MEDIUM IN ELECTROHYDRODYNAMICS

SOBRE LA APARICIÓN DE INESTABILIDAD PARA UN FLUIDO VISCOELÁSTICO QUE SATURA UN MEDIO POROSO EN LA ELECTROHIDRODINÁMICA

G. C. Rana[a†], R. Chand[b], V. Sharma[c]

a) Department of Mathematics, NSCBM Govt. P. G. College, Hamirpur-177 005, Himachal Pradesh, India; drgcrana15@gmail.com†
b) Department of Mathematics, Govt. College Nurpur, District Kangra, Himachal Pradesh, India
c) Department of Mathematics and Statistics, Himachal Pradesh University, Shimla – 171 005, Himachal Pradesh, India
† corresponding author

Recibido 5/7/2016; Aceptado 12/10/2016

In this paper, we study the effect of AC electric field on the onset of electrohydrodynamic instability in a viscoelastic fluid layer saturating a porous medium caused by dielectrophoretic force due to variation in the dielectric constant with temperature. Walters' (model B') fluid model is used to describe the behaviour of a viscoelastic dielectric fluid and for porous medium, Darcy model is employed. The fluid layer is induced by the dielectrophoretic force due to the variation of dielectric constant with temperature. We derive the dispersion relation describing the influence of viscoelasticity, Darcy number and AC electric field by applying linear stability theory and normal mode analysis method. It is observed that Walters' (model B') fluid behaves like an ordinary Newtonian fluid in the stationary convection whereas Darcy number and AC electric field have destabilizing effect on the stationary convection. The present results are in good agreement with the earlier published results.

En este artículo estudiamos el efecto de un campo eléctrico alterno en la aparición de la inestabilidad electrohidrodinámica en una capa de un fluido viscoelástico que satura un medio poroso, causada por la fuerza dielectroforética debido a la variación de la constante dieléctrica con la temperatura. Se usa el modelo de Walter (modelo B') para describir el comportamiento de un fluido dieléctrico viscoelástico y, para el medio poroso, el modelo de Darcy. La capa fluída es inducida por la fuerza dielectroforética debida a la variación de la constante dieléctrica con la temperatura. Derivamos la relación de dispersión que describe la influencia del número de Darcy de la viscoelasticidad y el campo eléctrico alterno aplicando la teoría de estabilidad lineal y el método de análisis de modo normal. Se observa que el fluido que sigue el modelo B' se comporta como un fluido Newtoniano ordinario en la convección estacionaria, mientras que el número de Darcy y el campo eléctrico alterno tienen un efectos desestabilizante sobre la convección estacionaria. Estos resultados están en concordancia con resultados previamente publicados.

PACS: Electrohydrodynamics, 47.65.-d, non-Newtonian, 47.50.-d, fluid dynamics, 47.53.+n, through porous media, 47.56.+r

I. INTRODUCTION

The study of Newtonian fluid heated from below saturating a porous medium has attracted many researchers for the last few decades as it has various applications in geophysics, food processing, oceanography, soil sciences, ground water hydrology, astrophysics etc. Chandrasekher [1] discussed thermal instability of Newtonian fluid under the various assumptions of hydrodynamics and hydromagnetics. A good account of thermal instability problems in a porous medium is given by Wooding [2], Ingham and Pop [3], Vafai and Hadim [4] and Nield and Bejan [5].

Recently, the study of electrohydrodynamic instability in dielectric fluid attracts many researchers because it has various applications in climatology, oceanography, EHD enhanced thermal transfer, EHD pumps, EHD in microgravity, micromechanic systems, drug delivery, micro-cooling system, nanotechnology etc. Chen *et al.* [6] discussed the applications of electrohydrodynamics in brief. They said that EHD heat transfer came out as an alternative method to enhance heat transfer, which is known as electrothermohydrodynamics (ETHD). Many researchers have been studied the effect of AC or DC electric field on natural convection in a horizontal dielectric fluid layer by taking different types of fluids. The onset of electrohydodynamic convection in a horizontal layer of dielectric fluid was studied by Landau [7], Robert [8], Castellanos [9], Lin [10], Gross and Porter [11], Turnbull [12], Maekawa *et al.* [13], Smorodin and Velarde [14], Galal [15], Rudraiah and Gayathri [16] and Chang *et al.* [17]. Takashima and Ghosh [18] studied the electrohydrodynamic instability in a viscoelastic liquid layer and found that oscillatory modes of instability exist only when the thickness of the liquid layer is smaller than about 0.5 mm and for such a thin layer the force of electrical origin is much more important than buoyancy force while Takashima and Hamabata [19] studied the stability of natural convection in a vertical layer of dielectric fluid in the presence of a horizontal AC electric field.

In these fluids, an applied temperature gradient produces non-uniformities in the electrical conductivity and the variation of the electrical conductivity of the fluid with temperature produces free charges in the bulk of the fluid. These free charges interacting with applied or induced electric field produce a force that causes fluid motion. On the other hand, when there is variation in dielectric permitivity and the electric field is intense then the polarization force which is induced by the non-uniformity of the dielectric constant causes fluid motion. In either case, convection can occur in a dielectric fluid layer even when the temperature gradient is stabilizing.

Reiner [20] and Walters' [21] developed the non-linear constitutive equations for non-Newtonian compressible and incompressible fluid respectively. Green [22] was the first who studied the problem of convective instability of a viscoelastic fluid heated from below while Vest and Arpaci [23] studied the problem of overstability of a viscoelastic fluid. With the growing importance of non-Newtonian fluids having applications in geophysical fluid dynamics, chemical technology and petroleum industry attracted widespread interest in the study on non-Newtonian fluids. One such type of fluids is Walters' (model B') elastico-viscous fluid having relevance in chemical technology and industry. A good account of thermal instability problems of Walters' (Model B') fluid in porous medium has been studied by Sharma and Rana [24], Gupta and Aggarwal [25], Rana and Kumar [26], Rana and Jamwal [27], Rana *et al*. [28], Rana and Chand [29] and Rana *et al*. [30].

The main aim of this paper is to study the effect of uniform AC electric field and Darcy number on the onset of instability of viscoelastic Walters' (model B') fluid layer. To the best of my knowledge, this problem has not been studied as yet.

II. MATHEMATICAL MODEL

We consider an infinite horizontal layer of an incompressible Walters' (model B') viscoelastic fluid of thickness d saturating a porous medium, bounded by the planes $z = 0$ and $z = d$. The fluid layer is acted upon by a gravity force $g = (0, 0, -g)$ aligned in the z direction and the uniform vertical AC electric field applied across the layer. The lower surface is grounded and the upper surface is kept at an alternating potential (60 *Hz*) whose root mean square value is ϕ. The temperature T at the lower and upper boundaries is assumed to take constant values T_0 and T_1 ($< T0$) respectively. The Darcy law is assumed to hold and the Oberbeck-Boussinesq approximation is employed (see Figure 1).

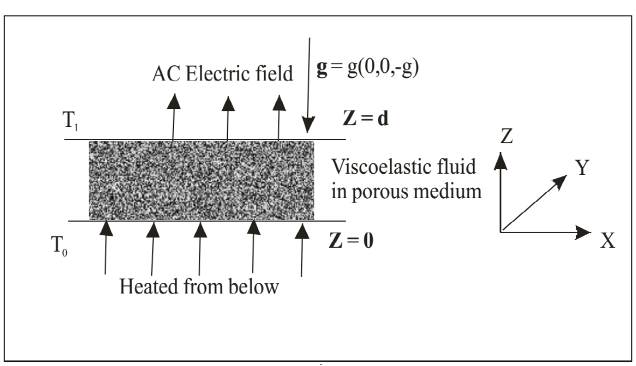

Figure 1. Physical configuration of the problem

III. GOVERNING EQUATIONS

Let ρ, μ, μ', ϕ, p, K, $\mathbf{q}(u, v, w)$, $\mathbf{g}$, T, κ, A and $\mathbf{E}$ denote respectively, the density, viscosity, viscoelasticity, medium porosity, pressure, dielectric constant, Darcy- velocity vector, acceleration due to gravity, temperature, thermal diffusivity, ratio of heat capacity and the root-mean-square value of electric field. The equations of conservation of mass, momentum and thermal energy for Walters' (model B') elastic-viscous fluid (Chandrasekhar [1], Walters' [21], Takashima and Ghosh [18], Sharma and Rana [24], Robert [8] and Rana *et al*. [30]) are

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\frac{\rho}{\phi}\frac{d\mathbf{q}}{dt} = -\nabla P + \rho\mathbf{g} - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\mathbf{q} - \frac{1}{2}(\mathbf{E}\cdot\mathbf{E})\nabla K, \tag{2}$$

$$A\frac{\partial T}{\partial t} + (\mathbf{q}\cdot\nabla)T = \kappa\nabla^2 T, \tag{3}$$

where $d/dt = (\partial/\partial t) + (1/\phi)(\mathbf{q}\cdot\nabla)$ stands for convection derivative and

$$P = p - \frac{\rho}{2}\frac{\partial K}{\partial\rho}(\mathbf{E}\cdot\mathbf{E}) \tag{4}$$

is the modified pressure.

Since the driving force (dielectrophoretic force) is strictly speaking periodic, the stationary state solution to the system 1-4 is not time independent. To keep it simple, neglecting the nonlinearities in the system 1-4 a rough estimate of the order of magnitude of the terms involving time derivatives gives

$$q \simeq q_0,\ \rho\frac{\partial q}{\partial t} = 2\rho\omega q_0,\ \mu'\frac{\partial q}{\partial t} = 2\mu'\omega q_0 \text{ and } A\frac{\partial T}{\partial t} = 2A\omega T_0,$$

where q_0 and T_0 are respectively, a characteristic velocity and temperature of the problem, and ω is the frequency of the forcing.

The Coulomb force term $\rho_e\mathbf{E}$, where ρ_e is the free charge density, is of negligible order as compared with the dielectrophoretic force term for most dielectric fluids in a 60-*Hz* AC electric field. Thus, we retain only the dielectrophoretic term, *i.e.*, last term in equation 2 and neglect the Coulomb force term. Furthermore, the electrical relaxation times of most dielectric liquids appear to be sufficient long to prevent the build up of free charge at standard power line frequencies. At the same time, dielectric loss at these frequencies is very low that it makes no significant contribution to the temperature field. It is also seen that the dielectrophoretic force term depends on $(\mathbf{E}\cdot\mathbf{E})$ rather than $\mathbf{E}$. As the variation of $\mathbf{E}$ is so speedy, the root-mean-square value of $\mathbf{E}$ is used as effective value in determining the motion of fluids. So we can consider the AC electric field as the DC electric field whose strength is equal to the root mean square value of the AC electric field.

A charged body in an electric field tends to move along the electric field lines and impart momentum to the surrounding fluid. The Maxwell equations are

$$\nabla \times \mathbf{E} = 0, \tag{5}$$

$$\nabla \cdot (K\mathbf{E}) = 0. \tag{6}$$

In view of Eq. 5, **E** can be expressed as

$$E = -\nabla V, \tag{7}$$

where V is the root mean square value of electric potential. The dielectric constant is assumed to be linear function of temperature and is of the form

$$K = K_0[1 - \gamma(T - T_0)], \tag{8}$$

where $\gamma > 0$, is the thermal coefficient of expansion of dielectric constant and is assumed to be small.

The equation of state is

$$\rho = \rho_0[1 - \alpha(T - T_0)], \tag{9}$$

where α is coefficient of thermal expansion and the suffix zero refers to values at the reference level $z = 0$.

IV. BASIC STATE

The basic state of the system is taken to be quiescent layer (no settling) and is given by

$$\begin{aligned} &\mathbf{q} = \mathbf{q}_b(z),\ P = P_b(z),\ T = T_b(z),\ \mathbf{E} = \mathbf{E}_b(z), \\ &K = K_b(z),\ \ \text{and}\ \ \rho = \rho_b(z), \end{aligned} \tag{10}$$

where the subscript b denotes the basic state.

Substituting equations given in 10 in Eqs. 1 – 9, we obtain

$$0 = -\nabla\frac{P_b}{\rho_0} + \frac{\rho_b(z)}{\rho_0}\mathbf{g} - \frac{1}{2\rho}\mathbf{E}^2\nabla K, \tag{11}$$

$$\frac{\partial^2 T_b(z)}{\partial z^2} = 0, \tag{12}$$

$$K_b(z) = K_0[1 - \gamma(T_b - T_0)], \tag{13}$$

$$\rho_b(z) = \rho_0[1 - \alpha(T_b - T_0)], \tag{14}$$

$$\nabla \cdot (K_b\mathbf{E_b}) = 0. \tag{15}$$

Solving Eq. 12 by using the following boundary conditions

$$T_b(z) = T_0 \text{ at } z = 0 \ \text{ and } \ T_b(z) = T_1 \ \text{ at } \ z = 1, \tag{16}$$

we obtain

$$T_b = T_0 - \Delta T\frac{z}{d}. \tag{17}$$

In view of Eq. 15 and noting that $E_{bx} = E_{by} = 0$, it follows that

$$K_b E_{bz} = K_0 E_0 = \text{constant (say)}. \tag{18}$$

Then

$$\mathbf{E} = \mathbf{E}_b(z) = \frac{E_0}{1 + \gamma\Delta T\frac{z}{d}}. \tag{19}$$

Hence

$$V_b(z) = -\frac{E_0 d}{\gamma\Delta T}\log(1 + \gamma\Delta T\frac{z}{d}), \tag{20}$$

$$\text{where } E_0 = -\frac{V_1\gamma\Delta T/d}{\log(1 + \gamma\Delta T)}, \tag{21}$$

is the root-mean-square value of the electric field at $z = 0$.

V. PERTURBATIONS SOLUTIONS

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, so that

$$\begin{aligned} &\mathbf{q} = \mathbf{q}',\ T = T_b + T',\ \mathbf{E} = \mathbf{E}_b + \mathbf{E}', \\ &\rho = \rho_b + \rho'\ K = K_b + K',\ \ \text{and}\ \ P = P_b + P', \end{aligned} \tag{22}$$

where $q', T', \mathbf{E}', \rho', K'$ and P' be the perturbations in $q, T, \mathbf{E}, \rho, K$ and P respectively. Substituting Eq. 10 in Eqs. 1 – 9, linearizing the equations by neglecting the product of primed quantities, eliminating the pressure from the momentum Eq. 2 by operating curl twice and retaining the vertical component and non-dimensionalizing the resulting equations by

$$(x^*, y^*, z^*) = \left[\frac{x, y, z}{d}\right],\ q^* = \frac{d}{\kappa}q,\ t^* = \frac{\kappa}{d^2}t,$$

$$T^* = \frac{1}{\Delta T}T \text{ and } V^* = \frac{1}{\gamma E_0 \Delta T d}V.$$

Neglecting the asterisk for simplicity, we obtain the linear stability equations in the form

$$\left[\frac{1}{Pr}\frac{\partial}{\partial t} + \frac{1}{Da}\left(1 - F\frac{\partial}{\partial t}\right)\nabla^2\right]\nabla^2 w = Ra_t\nabla_h^2 T + Ra_e\nabla_h^2\left(T - \frac{\partial V}{\partial z}\right), \tag{23}$$

$$\left[\frac{\partial}{\partial t} - \nabla^2\right]T = w, \tag{24}$$

$$\nabla^2 V = \frac{\partial T}{\partial z}, \tag{25}$$

where we have used dimensionless parameters as:

$$Pr = \frac{\nu\phi}{\kappa},\ F = \frac{\mu'}{\mu}\ \text{ and }\ Da = \frac{k_1}{d^2} \tag{26}$$

$$Ra_t = \frac{g\alpha\Delta T d^3}{\nu\kappa} \tag{27}$$

$$Ra_e = \frac{\gamma^2 K_0 E_0^2 (\Delta T)^2 d^2}{\mu\kappa} \tag{28}$$

The parameter Pr is the Prandtl number, F is the viscoelasticity parameter, Da is the Darcy number, Ra_t is the familiar thermal Rayleigh number and Ra_e is the AC electric Rayleigh number.

Now we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries. The boundary conditions appropriate (Chandrasekhar [2], Takashima and Ghosh [18], Rana and Jamwal [27] and Rana *et al.* [30] to the problem are

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial V}{\partial z} = 0,\ T_0\ \text{ or }\ DT = 0. \tag{29}$$

VI. LINEAR STABILITY ANALYSIS

Using normal mode analysis method, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w, T, V] = [W(z), \Theta(z), \Phi(z)] \exp(\imath lx + \imath my + \sigma t), \tag{30}$$

where l and m are the wave numbers in the x and y direction, respectively, and σ is the complex growth rate of the disturbances.

Substituting Eq. 30 in Eqs. 23 – 25 and 29, we get

$$\left[\frac{\omega}{Pr} + \frac{1}{Da}(1 - F\sigma)\right](D^2 - a^2)W = -Ra_t a^2\Theta + Ra_e a^2(\Theta - D\Phi), \tag{31}$$

$$[A\omega - (D^2 - a^2)]\Theta = W, \tag{32}$$

$$(D^2 - a^2)\Phi = D\Theta, \tag{33}$$

$$W = D^2W = D\Phi = 0, \;\; \Theta = 0, \;\; \text{or} \;\; D\Phi = 0, \tag{34}$$

where $a^2 = l^2 + m^2$ and $D = d/dz$.

Eqs. 31 – 33 form an eigenvalue problem for Ra_t or Ra_e and σ with respect to the boundary conditions 34.

We assume the solution to W, Θ, Φ and Z of the form

$$W = W_0 \sin \pi z, \;\; \Theta = \Theta_0 \sin \pi z \;\; \text{and} \;\; \Phi = \Phi_0 \sin \pi z, \tag{35}$$

which satisfy the boundary conditions of Eq. 34. Substituting Eq. 35 into Eqs. 31 – 33, we obtain the following matrix equation

$$\begin{bmatrix} \left[\frac{\omega}{Pr} + \frac{1}{Da}(1 - F\sigma)\right]J^2 & -a^2(Ra_t + Ra_e) & -Ra_e a^2\pi \\ -1 & A\sigma + J^2 & 0 \\ 0 & \pi & J^2 \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \;\; \text{where } J^2 = \pi^2 + a^2 \text{ is the total wave number.} \tag{36}$$

The linear system 36 has a non-trivial solution if and only if

$$\begin{vmatrix} \left[\frac{\omega}{Pr} + \frac{1}{Da}(1 - F\sigma)\right]J^2 & -a^2(Ra_t + Ra_e) & -Ra_e a^2\pi \\ -1 & A\sigma + J^2 & 0 \\ 0 & \pi & J^2 \end{vmatrix} = 0,$$

which yields

$$Ra_t = \frac{J^2(J^2 + A\sigma)}{a^2}\left[\frac{\sigma}{Pr} + \frac{1}{Da}(1 - \sigma F)\right] - \frac{a^2}{J^2}Ra_e. \tag{37}$$

Eq. 37 is the dispersion relation accounting for the effect of Prandtl number, electric Rayleigh number, Darcy number and kinematic viscoelasticity parameter in a layer of Walters' (model B') viscoelastic dielectric fluid.

VII. STATIONARY CONVECTION

For stationary convection, putting $\sigma = 0$ in equation 37 reduces it to

$$Ra_t = \frac{(\pi^2 + a^2)^2 Da^{-1}}{a^2} - \frac{a^2}{\pi^2 + a^2}Ra_e. \tag{38}$$

Eq. 38 expresses the thermal Rayleigh number as a function of the dimensionless resultant wave number a, the parameters electric Rayleigh number and Darcy number Da. It is found that the kinematic viscoelasticity parameter F vanishes with ω and the Walters' (model B') viscoelastic dielectric fluid behaves like an ordinary Newtonian dielectric fluid. Eq. 38 is in good agreement with the equation obtained by Roberts [8].

In the absence of AC electric field (i. e., when $Ra_e = 0$), Eq. 38 reduces to

$$Ra_t = \frac{(\pi^2 + a^2)^2 Da^{-1}}{a^2}. \tag{39}$$

To study the effect of AC electric field on electrohydrodynamic stationary convection, we examine the behaviour of $\partial Ra_t/\partial Ra_e$ analytically. From Eq. 38, we obtain

$$\frac{\partial Ra_t}{\partial Ra_e} = -\frac{a^2}{\pi^2 + a^2}, \tag{40}$$

which is negative implying thereby AC electric field hastens the electroconvection, implying thereby AC electric field has destabilizing effect on the system which is in an agreement with the results derived by Takashima and Ghosh [18] and Rana *et al*. [30]. Also Eq. 38 yields

$$\frac{\partial Ra_t}{\partial Da} = -\frac{(\pi^2 + a^2)^2 Da^{-2}}{a^2}, \tag{41}$$

which is negative implying thereby Darcy number hastens the electroconvection, implying thereby Darcy number has destabilizing effect on the system which is in good agreement with the results derived by Sharma and Rana [24], Gupta and Aggarwal [25], Rana and Kumar [26], Rana and Jamwal [27] and Rana *et al*. [30].

The dispersion relation 38 is analysed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.

In Figure 2, the thermal Rayleigh number Ra_t is plotted against dimensionless wave number a for different values of electric Rayleigh number Ra_e as shown. This shows that as Ra_e increases the thermal Rayleigh number Ra_t decreases. Thus AC electric field has destabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. 40.

In Figure 3, the thermal Rayleigh number Ra_t is plotted against dimensionless wave number a for different values of Darcy number Da as shown. This figure depicts that as Darcy number Da increases the thermal Rayleigh number Ra_t

decreases. Therefore, Darcy number has destabilizing effect on the stationary convection which is in good agreement with the result obtained analytically from Eq. 41.

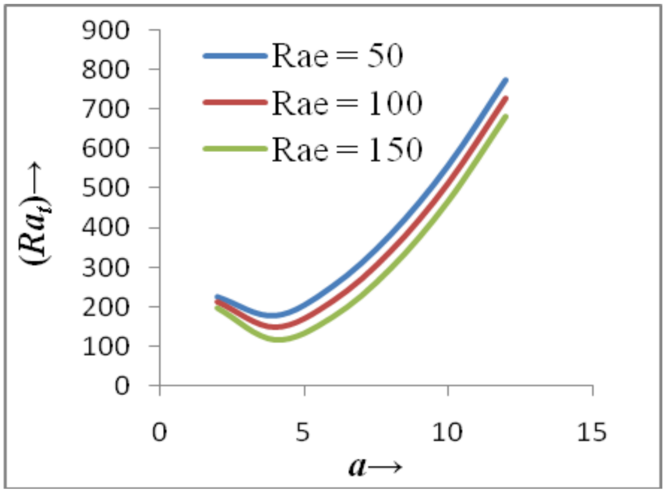

Figure 2. Dependence of the thermal Rayleigh number Ra_t on the wave number a while varying Ra_e.

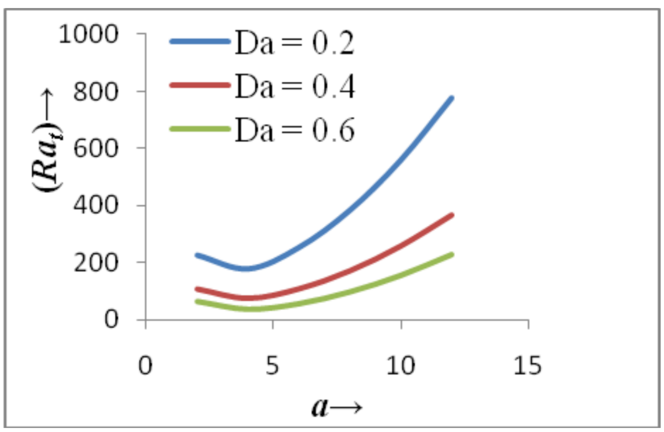

Figure 3. Dependence of the thermal Rayleigh number Ra_t on the wave number a varying Da.

VIII. CONCLUSIONS

The effect of AC electric field on the onset of instability of Walters' (model B') viscoelastic dielectric fluid layer heated from below saturating a porous medium has been studied for the case of free-free boundaries by using linear stability analysis based on normal modes. For the case of stationary convection, the non-Newtonian electrohydrodynamic Walters' (model B') viscoelastic dielectric fluid behaves like an ordinary Newtonian fluid. AC electric field and Darcy number both hastened the onset of electrohydrodynamic stationary convection as $\partial Ra_t/\partial Ra_e$ and $\partial Ra_t/\partial Da$ indicating that the thermal Rayleigh number Ra_t is decreasing function of both electric Rayleigh number Ra_e and Darcy number Da. Hence, AC electric field and Darcy number both have destabilizing effect on the stationary convection.

IX. ACKNOWLEDGEMENTS

Authors would like to thank the learned referees for their critical comments and suggestions for the improvement of quality of the paper. Authors also thank the worthy editor for his valuable suggestions.

REFERENCES

[1] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, (Dover Publication, New York 1981).
[2] R. A. Wooding, Rayleigh instability of a thermal boundary layer in flow through a porous medium. Journal of Fluid Mechanics, 9, 183-192 (1960).
[3] D. D. Ingham and I. Pop, Transport Phenomena in Porous Media, (Elsevier, New York 1981).
[4] K. Vafai and H.A. Hadim, Hand Book of Porous Media, (Mureel Decker, New York 2000).
[5] D. A. Nield and A. Bejan, Convection in Porous Medium, (Springer, New York 2006).
[6] X. Chen, J. Cheng and X. Yin, Advances and applications of electrohydrodynamics. Chinese Science Bulletin, 48, 1055-1063 (2003).
[7] L. D. Landau, Electrodynamics of Continuous Media, (New York, Oxford 1960).
[8] P.H. Roberts, Electrohydrodynamic convection. Quarterly Journal of Mechanics and Applied Mathematics, 22: 211–220 (1969).
[9] A. Castellanos, Electrohydrodynamics (Springer-Verlag Wien, New York 1998).
[10] H. Lin, Electrokinetic instability in microchannel flows: a review. Mech. Res. Comm., 36, 33-38 (2009).
[11] M. J. Gross and J.E. Porter, Electrically induced convection in dielectric liquids. Nature, 212, 1343-1345 (1966).
[12] R. J. Turnbull, Effect of dielectrophoretic forces on the Be'nard instability. Phys. Fluids, 12, 1809–1815 (1969).
[13] T. Maekawa, K. Abe and I. Tanasawa, Onset of natural convection under an electric field. Int. J. Heat Mass Trans., 35, 613–621 (1992).
[14] B.L. Smorodin, Stability of plane flow of a liquid dielectric in a transverse alternating electric field. Fluid Dynamics, 36, 548–555 (2001).
[15] M. M. Galal, Electrohydrodynamic instability if two superposed viscous miscible streaming liquids. J. Electrostatics, 40, 185-190 (1997).
[16] N. Rudraiah and M.S. Gayathri, Effect of thermal modulation on the onset of electrothermoconvection in a dielectric fluid saturated porous medium. ASME Journal of Heat Transfer, 131, 101009–101015 (2009).
[17] M. H. Chang, A.C. Ruo and F. Chen, Electrohydrodynamic instability in a horizontal fluid layer with electrical conductivity gradient subject to a weak shear flow. J. Fluid Mech., 634,191–215 (2009).
[18] M. Takashima and A.K. Ghosh, Electrohydrodynamic instability in a viscoelastic liquid layer. J. Phys. Soc. Japan, 47,1717-1722 (1979).

[19] M. Takashima and H. Hamabata, The stability of natural convection in a vertical layer of dielectric fluid in the presence of a horizontal AC electric field. J. Phys. Soc. Japan, 53,1728-1736 (1984).

[20] M. Reiner, A mathematical theory of dilatancy. American Journal of Mathematics, 67, 350 –362 (1945).

[21] K. Walters', The solution of flow problems in the case of materials with memory. J. Mecanique, 1, 469-778 (1962).

[22] P. Green, Convective instability of viscoelastic fluid heated from below. Phy. Fluids, 11, 1410, (1968).

[23] C. M. Vest and V. Arpaci, Overstability of viscoelastic fluid heated from below. J. Fluid Mech., 36, 613, (1969).

[24] V. Sharma and G.C. Rana, Stability of Walters' (Model B') superposed fluid in porous medium. Jnanabha, 29, 123-136 (1999).

[25] U. Gupta and P. Aggarwal, Thermal instability of compressible Walters' (model B') fluid in the presence of hall currents and suspended particles. Thermal Science, 15, 487-500 (2011).

[26] G. C. Rana and S. Kumar, Thermal instability of Walters' (Model B') elastico-viscous rotating fluid permitted with suspended particles and variable gravity field in porous medium. Engineering Transactions, 60, 55-68 (2012).

[27] G.C. Rana and H. S. Jamwal, Hydromagnetic thermal instability of Walters' (Model B') rotating fluid permeated with suspended particles in porous medium. Studia Geotechnica et Mechanica, 2013. XXXV, 75-88 (2013).

[28] G. C. Rana, S. K. Kango and K. Chadha, Magneto-thermal convection in Walters'(model B') elastico-viscous fluid saturated by a Darcy-Brinkman porous medium. Engineering Mechanics, 21, 425–435 (2014).

[29] G. C. Rana and R. Chand, Rayleigh-Be'nard convection in an elastico-viscous Walters' (model B') nanofluid layer. Bulletin of the Polish Academy of Sciences-Technical Sciences, 63, 235-244 (2015).

[30] G. C. Rana, R. Chand and D. Yadav, The onset of electrohydrodynamic instability of an elastico-viscous Walters' (Model B') dielectric fluid layer, FME Transactions, 43,154-160 (2015).