LOGARITHMIC POTENTIAL BRANEWORLD IN LIGHT OF THE RECENT EXPERIMENT OBSERVATION

POTENCIAL LOGARÍTMICO DE MUNDO-BRANA A PARTIR DE OBSERVACIÓN EXPERIMENTAL RECIENTE

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In the loop inflation standard case, the spectral index n_s depends only on the e-folds number N, and it is larger than the value given by observational measurements. The ratio of tensor to scalar perturbations r is very small so that the gravitational waves are negligible. Therefore, we reconsider this model in the framework of the Randall-Sundrum II braneworld scenario. In this context, we have computed and discussed the perturbation of the scalar curvature and determined the energy scale $V_{0,}$ and the brane tension value λ. We showed that the various inflationary spectrum perturbation parameters (n_s, r and $\frac{dn_s}{d\ln k}$) depend on N and on the coupling constant α, and we reproduce the central value of the first parameter $n_s \sim 0.96$. Also, the ratio r and the running $\frac{dns}{d\ln k}$ are in perfect agreement with the experimental observations (Planck 2015).

En el caso estándar de inflación de bucle, el índice espectral n_s solo depende del número de pliegues N, y es más grande que el valor dado por las mediciones de observación. La relación r de tensor a perturbaciones escalares es muy pequeña, de modo que las ondas gravitatorias son insignificantes. Por lo tanto, reconsideramos este modelo en el marco del escenario del Mundo-Brana de Randall-Sundrum II. En este contexto, hemos calculado y discutido la perturbación de la curvatura escalar y hemos determinado la escala de energía V_0; y el valor de la tensión de brana λ. Mostramos que los diversos parámetros de perturbación del espectro inflacionario (n_s, r y $dn_s/d(\ln[k])$) dependen de N y de la constante de acoplamiento α, y reproducimos el valor central del primer parámetro $n_s \sim 0.96$. Además, la relación r y el desplazamiento $dn_s/d(ln[k])$ están en perfecto acuerdo con las observaciones experimentales (Planck 2015).

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I. INTRODUCTION

The inflation model provides a theoretical description of an accelerated expansion phase of our universe, and was introduced to solve the several problems associated with standard Big Bang cosmology, namely flatness, the horizon problem, the homogeneity and the numerical density of monopoles [1–3], and eventually to give an alternative explanation to some issues related to particle physics [4,5].

In that context, the inflation is supposed to be generated by some scalar degree of freedom, whose potential energy produces the required accelerated expansion of the early universe [6]. The key feature of a variety of inflationary models resides in the fact that the shape of the potential form in those models is not known, except that they must be related to a flat universe [7]. Further, the inflationary mechanism is supposed to take place at very high energies, in a regime where the corresponding particle physics comprehension of this domain is not well known [8]. Moreover, corresponding theoretical assertions have not been tested in accelerators. The inflationary models are based on scalar potentials, which must describe a flat universe and should oscillate around a minimum [9]. Since these potentials are considered a form of configurational or binding energy, they measure the amount of internal energy associated with a particular field value, and should also have a minimum in which inflation can end [10,11]. In fact, the most potentials that describe the cosmological inflation, have been proposed to describe the standard or supersymmetric model of particle physics. These potentials are constructed in the context of particle physics, as models that take place from a form of energy (Action, Lagrangian...) [12].

In the last years, the braneworld scenarios have attracted a lot of attention as a novel extra dimension theory of inflation [13], especially the standard model of particles is confined to the brane, while gravitation propagates into the bulk space-time, where the extra dimensions introduced in the Friedmann equation like higher-dimensional objects (branes), higher curvature corrections to gravity [14].One of the brane inflation scenarios, is the Randall–Sundrum II model [15]. This model has attracted a lot of interests to describe early inflation and recent acceleration of the universe in which our four dimensional universe is considered as a 3-brane embedded in five-dimensional anti-de Sitter space-time [16,17].

Furthermore, the validation of any model requires experimental data on cosmic fossils that are known by the cosmic microwave background (CMB) [18], which is considered an essential source of information about all epochs of the Universe. In this context, to investigate this model, we consider recent experiment results given by observation Planck [19], which are based on the measurement of the properties of CMB temperature fluctuations, and gives the inflationary parameters, the scalar spectral index $n_s = 0.9653 \pm 0.0048$ (68 % CL, TT+lowP+lensing), the fractional contribution of tensor modes is limited to $r \lesssim 0.11$ (95 %CL), the running of spectral index $\frac{dn_s}{d\ln k} = -0.002 \pm 0.013$ (95 % CL, TT+lowP+lensing), and the power spectrum of the curvature perturbations given by Planck, $P_R(k) = (2.130 \pm 0.053) \times 10^{-9}$. If we assume that all of these data sets are well-described by their published uncertainties, then these parameters provide a precise and accurate description of our universe.

Therefore, the simple and well motivated inflationary potentials are distinctively welcomed, such as the logarithmic potential, which has been introduced in the first time by Coleman and Weinberg (CW), in the context of the spontaneous symmetry breaking generated by radiative corrections of masses [20]. This considered potential has been used in different context, it has been found as a candidate to drive the inflation [21]. Thus, it was used to investigate the branewold inflation [22]. As well, it has taken a logarithmic form to investigate of the tachyon [23]. Furthermore, a simple inflationary model: $V = V_0\left(1 + \alpha \log(\frac{\phi}{Q})\right)$ was proposed to the conversion F-term and D-term [24, 25]. However, in the standard case, the last model does not lead to red tilt of the spectral index, this means $n_s > 0.98$ for the frequent value of the e-folds number $N \sim 50$, and the ratio r is negligible.

Basing on the previous results of the standard model, which is not fully compatible with observational data, we revisit the various inflationary spectrum perturbation parameters of the logarithmic model, in framework braneworld RSII. Therefore, for the brane tension value in the range $\lambda \sim (1-10) \times 10^{57} GeV^4$, and the natural value of the coupling α, we show that the spectral index can take the central value $n_s \sim 0.96$ and the ratio r becomes important and can reach the value 0.11.

This work is organized as follows. In section 2, we first recall the perturbation spectrum expressions of RSII model and the brane tension effect at high energy approximation. In section 3, we present our results concerning the different parameter values of loop inflation model, and we show that they are compatible with the recent experimental observation. The last section is devoted to a conclusion.

II. BRANEWORLD INFLATION FORMALISM

The braneworld cosmology offered another new approach to our understanding of the universe. It has proposed that our universe is a three dimensional surface (3-brane) embedded in a higher dimensional space. Taking for example the ekpyrotic model, which postulate that the universe did not start in a singularity, but came about from the collision of two branes colliding with each other, causing the exchange of energy which generated our universe [26, 27]. Generally, in braneworld scenario, the observable universe can be considered as 3-brane, embedded in $4 + d$ dimensional spacetime, particles and fields are trapped on the brane while gravity is free to access the bulk. Furthermore, the string theory includes the brane model, contains the basic ingredients, such as extra dimensions, higher dimensional objects, which are the branes. Since, the aim of the string theory is to give us a basic description of the nature, it is important to study this kind of cosmology predictions. Thus, the inflationary models have been analyzed in 4-dimensional cosmology; it is challenging to discuss them in alternative gravitational theories as well. The string theory proposes that the ordinary matter is confined to brane of four-dimensional, while gravity propagates in the whole spacetime [28].

In this paper, we consider the Randall-Sundrum II model, where $d = 1$, which means that a single brane embedded in five-dimensional AdS space with a Z_2 reflection symmetry imposed on the bulk. Consequently, the action of the field equation in RSII model is written in Refs. [15, 28] as

$$S_{RSII} = \int dx^5 \sqrt{-g_5}\left[\frac{m_5^3}{2}\hat{R} + \Lambda_5\right] + \int d^4x \sqrt{-g_4} L_{brane}, \tag{1}$$

where $\hat{R}$ is the Ricci curvature scalar of the bulk spacetime, with the bulk metric g_5. m_5 is the five-dimensional Planck mass and Λ_5 is the five dimensional cosmological constant. The second term is the action of the Lagrangian density of the matter on the brane, which given by

$$L_{brane} = L_{matter} + \lambda = |\partial\Phi|^2 - V + \lambda, \tag{2}$$

where V is the scalar potential and λ is the brane tension.

The dynamics was determined and developed more precisely in reference [29, 30]. It was originally proposed as an alternative resolution of the hierarchy problem arising from the large difference between the Plank scale and the electroweak scale. This model is based on a brane which has positive tension. In this theory, the metric projected on the brane is a spatially flat, and the Friedmann equation is generalized as in Ref. [31]

$$H^2 = \frac{8\pi}{3m_p^2}\rho(1 + \frac{\rho}{2\lambda}), \tag{3}$$

where H is the Hubble parameter, ρ is the energy density, and $m_p = 1.2 \times 10^{19} GeV$ is the four-dimensional Planck. In high energy limit approximation $V >> 2\lambda$, one has to use slow roll parameters, which are given by [31]

$$\epsilon = \frac{m_p^2}{4\pi}\frac{\lambda V'^2}{V^3}, \quad \eta = \frac{m_p^2}{4\pi}\frac{\lambda V''}{V^2}, \quad \xi^2 = \frac{m_p^4}{16\pi^2}\frac{\lambda^2 V' V'''}{V^4}, \tag{4}$$

where $V'' = \frac{d^2V}{d\phi^2}$ and $V' = \frac{dV}{d\phi}$.

During inflation $\epsilon \ll 1$ and $|\eta| \ll 1$, so that Inflationary phase will terminate when the universe heats up when the condition $\epsilon = 1$ (or $|\eta| = 1$) is satisfied.

The small quantum fluctuations in the scalar field lead to fluctuations in the energy density, which is known to be related to the scalar curvature perturbation. For that reason, and in approximation $V >> 2\lambda$, the power spectrum of the curvature perturbations is given as

$$P_R(k) \simeq \frac{16\pi}{3m_p^6}\frac{V^6}{V'^2\lambda^3}. \tag{5}$$

One can also define the the various inflationary spectrum perturbation parameters as

$$n_s \simeq -6\epsilon + 2\eta + 1, \quad r \simeq 24\epsilon, \quad \frac{dn_s}{d\ln k} \simeq 16\epsilon\eta - 18\epsilon^2 - 2\xi^2. \quad (6)$$

In high energy limit approximation $V >> 2\lambda$, the number of e-folds of inflation generated since the modes that are entering the observable Universe now left the horizon (at ϕ_*) till the end of inflation (at ϕ_{end}) is given by [31]

$$N \simeq -\frac{4\pi}{m_p^2\lambda}\int_{\phi_*}^{\phi_{end}} \frac{V^2}{V'}d\phi, \quad (7)$$

Brane tension effect

Braneworld inflation models have been proposed from extra dimensional theories, to give solutions to the problems of the standard Big-Bang model. One finds among these model is the Randal-Sundrum-II, which is one of the simplest phenomenological models. For that, it was suggested by some works, in order to explain the hierarchy problem. This model is characterized by the parameter λ, which its value depends on the model or type of the problem addressed. Hence the brane tension λ is not a generic result and does not lead to a unique value, so that certain works suggest the brane tension value vary from $10^{52}GeV^4$ to $10^{64}GeV^4$. For instance the paper [32] discusses the validity of chaotic braneworld model from the power spectrum of the curvature perturbations P_R and the scalar spectral index n_s with values of m_5 up to $2.4 \times 10^{17}GeV$ implies $\lambda \sim 10^{64}GeV^4$. To reduce the values of coupling constant in order to take the natural values, and to refine the phase of inflation field in Affleck–Dine inflation model, the brane tension was found about $\lambda \sim 10^{60}GeV^4$ [33]. Moreover, in work [34] the authors investigate the aspects of thermal leptogenesis in braneworld, and they found that the thermal equilibrium is expected at $m_5 \leq 10^{16}GeV$ that is $\lambda \leq 10^{58}GeV^4$.

In addition, in the work [35], authors have shown that for some value of brane tension $\lambda \sim (10^{57}GeV^4)$, the fine tuning problem is eliminated, and the value of FI-term ξ is reduced leading to the resolution of cosmic string problem. Else, to solve the problem related to the topological defects, caused by the instability of the magnetic monopoles, the brane tension value has been found $\lambda \sim (1-10)\times 10^{52}GeV^4$ [36], and in other work $\lambda \sim (1-10)\times 10^{50}GeV^4$ [37]. Moreover, in other models, the brane tension value λ is variable or it is dependent on other parameters [17, 22].

III. LOOP CORRECTION POTENTIAL IN BRANEWORLD

The main goal of this section is to reassess within the framework of the approximation high energy limit RSII braneworld model, the inflationary predictions correspond the loop correction potential [24, 25]

$$V(\phi) = V_0\left[1 + \alpha\log(\frac{\phi}{Q})\right], \quad (8)$$

where V_0 is the constant determining the energy scale, Q is the renormalization scale and α is a dimensionless parameter that tunes the strength of the radiative effects.

In fact, this inflationary potential was invented in the context hybrid inflation in supersymmetric theories (F-term and D-term), in which the two real scalar fields ψ and ϕ play crucial rule. The first field ψ provides the vacuum energy density that drives inflation and the other field varies slowly and represent inflaton field. Inflation ends by a rapid rolling of the field ψ so-called waterfall, triggered by the slow rolling of the field ϕ. In this type of scenarios, the energy scale Λ is in order of GUT scale so $\Lambda \sim 10^{16}GeV$. The parameter α corresponds the coupling of the inflaton ϕ to the waterfall field ψ, in such way that α equal the fraction $g^2/8\pi^2$ [24, 25], where $g \lesssim 1$ so that $\alpha \lesssim 10^{-2}$, and $V_0 = g^2\Lambda^4$ so that $V_0 \sim 10^{60}GeV^4$.

In this work, we study the inflationary consequence of the braneworld RSII model of this potential, in which we adjust the coupling α (so that g) between the inflaton and the waterfall field to be strong, and evaluate the brane tension value λ in order to validate this model.

In high energy limit approximation $V >> 2\lambda$. From Eqs. 4 and 8, the slow-roll parameters are given as

$$\epsilon = \frac{\alpha^2}{4\pi}\frac{m_p^2\lambda}{V_0\phi^2}, \quad \eta = \frac{\alpha}{4\pi}\frac{m_p^2\lambda}{V_0\phi^2} \quad \xi = \frac{\alpha^2}{8\pi^2}\frac{m_p^4\lambda^2}{V_0^2\phi^4}. \quad (9)$$

The end of inflation is determined either by the failure of the slow-roll conditions when $\epsilon = 1$, which gives the following expression of the scalar field ϕ_{end} at the end of inflation as

$$\phi_{end} = \frac{1}{2}\sqrt{\frac{\alpha\lambda}{\pi V_0}}m_p. \quad (10)$$

Eqs. 7 and 8 give the number of e-folds, as the integrated expansion from ϕ_* to ϕ_{end}

$$N = \frac{2\pi V_0}{\alpha m_p^2\lambda}(\phi_*^2 - \phi_{end}^2). \quad (11)$$

From the Eqs. 11, 10, we calculate the inflaton field ϕ_*

$$\phi_* \simeq \left(\frac{\alpha(2+N)\lambda}{2\pi V_0}\right)^{1/2} m_p. \quad (12)$$

On the other hand, taking into account Eqs. 5, 8 and 12, the expression of the amplitude of the power spectrum becomes

$$P_R(k) \simeq \frac{8}{3m_p^4}\frac{V_0^3(2+N)}{\alpha\lambda^2}. \quad (13)$$

From Eqs. 13, we can easily see that, the power spectrum, of the curvature perturbations, depends on the following variables : V_0, λ, α and N. Then, from the Eqs. 6 and 9 the expression of the spectral index n_s, the ratio tensor to scalar r and the running the spectral index $\frac{dn_s}{d\ln k}$ are given by

$$n_s - 1 = -\frac{3\alpha+1}{2+N}, \quad r = \frac{12\alpha}{2+N}, \quad \frac{dn_s}{d\ln k} = -\frac{3\alpha+1}{(2+N)^2}. \quad (14)$$

One can be easily see from the above equation 14 that the inflationary parameter values are dependent on the values of e-fold number N and the parameter α. However, we see that brane tension λ does not be appear in the expressions of the n_s, r and $\frac{dns}{d\ln(k)}$, but is responsible for the appear of the parameter α.

One would conclude that the inflationary trajectory is not affected with brane tension which is far from being correct. In fact, if we look at the expression of the scalar curvature perturbation 13, we can plot the variation of the brane tension with respect of the other parameters adjust the value of $P_R(k)$. Furthermore its effect is altered in the calculation, since the standard expressions of the same potential is significantly different from what we have found: in the standard case $n_s = 1 - \frac{1}{N}$, $r \simeq \frac{4\alpha}{N}$, where $\alpha = g^2/8\pi^2$ ($g \leq 1$) [24, 25]. In the last case, for the most frequent value of e-folds $N = 50 - 60$, the scalar curvature perturbation is $n_s \geq 0.98$ and the ratio r is very small, so that the gravitational wave is negligible. These are constraints prevent this model to be a good candidate to drive inflation in the standard case.

Thus it is of great interest to adjust this situation, and look at braneworld context predictions. Keeping that in mind, we pursue the computation of the perturbation spectrum parameters in the brane world case.

Numerical results

Firstly, we set the value of the number folding in themost commonly used value $N = 50$, and we fix it at $V_0 = 10^{60}GeV^4$ to be in the order of Grand Unified Theories GUT so that $V_0^{1/4} = 10^{15}GeV$, which is perfectly in agreement with the observations [19]. The by using the central value of the power spectrum $P_R(k) \simeq 2.13 \times 10^{-9}$, we will look for the brane tension λ and the constant α, to discuss the inflationary parameter values, in order to validate this model.

Let's begin by some values of α : for $\alpha = 0.005$ gives $\lambda = 7 \times 10^{57}GeV^4$. If we set $\alpha = 0.1$ we get $\lambda = 2 \times 10^{57}GeV^4$. Then, if we take $\alpha = 0.48$, we get $\lambda = 1 \times 10^{57}GeV^4$. Focus now our attention on the values of the scalar curvature perturbation. Planck measurement indicate that it should be $n_s \in [0.96 - 0.97]$. This will implies that the parameter α should be restricted to take any values in the following interval $[0.18 - 0.35]$. As a consequence the brane tension must be in the range $\lambda \sim (2.8 - 3.2) \times 10^{57}GeV^4$.

We can do more computation, by considering at the same time the obtained spectrum of the inflationary parameter (n_s, r and $\frac{dn_s}{d\ln k}$). Thus, for $\alpha = 0.01$ gives $n_s \simeq 0.98$, $r \simeq 0.0023$ and $\frac{dn_s}{d\ln k} \simeq -0.38\times10^{-3}$. Then, if we take $\alpha = 0.1$ we get $n_s \simeq 0.975$, $r \simeq 0.023$ and $\frac{dn_s}{d\ln k} \simeq -0.48 \times 10^{-3}$. Now, if we set $\alpha = 0.4$, we get: $n_s \simeq 0.957$, $r \simeq 0.093$ and $\frac{dn_s}{d\ln k} \simeq -0.82 \times 10^{-3}$.

These different values of the parameter α don't belong to the ranges that we determined above. Despite that fact, we have obtained tiny ratio of the scalar to tonsorial perturbation amplitude. This behavior is similar to that of some successfully developed models like the starobinsky model [38]. Apart from that, the scalar curvature perturbation is not consistent with Planck measurement. The central value of the first inflationary parameter $n_s \sim 0.965$ can be reproduced when the brane tension λ is near to$10^{57}GeV^4$, and $\alpha = 0.27$.

In the following, we plot the curves describing the relative variation of the perturbation spectrum parameters (n_s, r and $\frac{dn_s}{d\ln k}$).

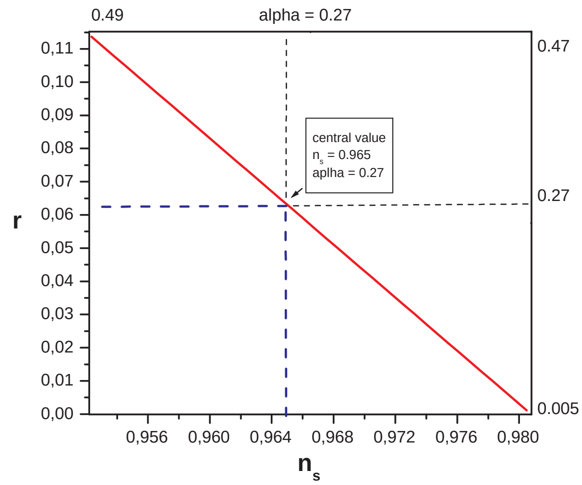

Figure 1. The evolution of the r versus n_s for various values of α.

Fig. 1 shows that the variation of r versus the scalar spectral index n_s, which behaves as a decreasing linear function with the variation of the coupling α. The central value of the spectral index $n_s \sim 0.96$ is reproduced when the coupling constant is about $\alpha \sim 0.27$. We can see also that the compatibility of the ratio r is realized with respect to the recent Planck data for $0.005 < \alpha < 0.48$.

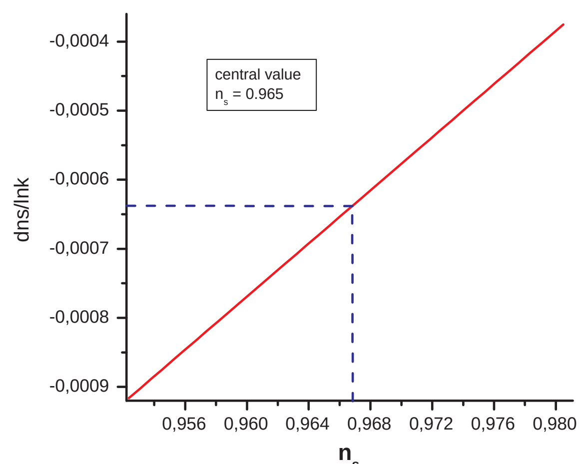

Figure 2. The evolution of the $\frac{dn_s}{d\ln k}$ versus n_s for various values of α.

Fig. 2 shows that the running of the scalar index $\frac{dn_s}{d\ln k}$ is a increasing function with respect to n_s. The central value of the scalar spectral index corresponds to $\frac{dn_s}{d\ln k} = -0.00067$. Also, the running takes the extremely weak values, which are widely consistent with recent observations.

To summarize this section, one can note that our computation is based on the addition the fraction term $\frac{2\lambda}{V}$ in the expression of inflationary parameters. In the high energies limit, the brane effect is to reproduce the expression of perturbation spectrum parameters by introduction the coupling constant α. This variation in the formulas of the perturbation spectrum results (n_s, r and $\frac{dn_s}{d\ln k}$) gives the compatibility with the recent observations, and consequently validating this model.

IV. CONCLUSION

In this work, we have studied the loop inflation model in the framework of the RSII braneworld model. We have used a logarithmic potential to evaluate various perturbation spectrum parameters. From the observed value of the power spectrum of the curvature perturbation $P_R(k)$, we have determined the energy scale $V_0 \sim 10^{60} GeV^4$, and we have evaluated the brane tension value $\lambda \sim (1-8) \times 10^{57} GeV^4$. Thus, for a suitable choice of the coupling constant α, which makes it to take much more natural value, we have obtained the central value of the spectral index $n_s = 0.96$. The values of the tensor to scalar ratio reaches the maximum value given by the observation $r \sim 0.11$, so that the gravitational waves becomes important, and running of the spectral index $\frac{dns}{d\ln k}$ is in excellent agreement with the latest observations of the Planck observations.

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