ENERGY SENSITIVITY OF THE LOW-ENERGY PARAMETERS OF NEUTRON-PROTON SCATTERING FOR VARIOUS NUCLEON-NUCLEON POTENTIALS

SENSIBILIDAD A LA ENERGÍA DE PARÁMETROS DE BAJA ENERGÍA EN EL SCATTERING NEUTRÓN-PROTÓN PARA VARIOS POTENCIALES NUCLEÓN-NUCLEÓN

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The triplet and singlet low-energy parameters in the effective-range expansion for neutron–proton scattering were determined with the aid of the most popular modern realistic nucleon–nucleon potentials (Nijm I), (Nijm II) and (Reid93). We compared our results with the latest partial wave analysis experimental data from the SAID nucleon–nucleon database and newest values a_t, a_s, r_t and r_s parameters were presented. Our calculations based on these three potentials at incident neutron energy less than 60 KeV show that there are some discrepancies with experimental data. In order to decrease the discrepancy between our results and the experimental ones at very low energy, we suggest to include coupling terms $^3S_1$ + $^3D_1$ for constructing various realistic nuclear-force models. Furthermore, the calculated effective range expansions in this work are not very accurate for very low energy unless considering into account many terms v_n in the expansions.

Se determinaron los parámetros de singlete y triplete de baja energía en la expansión de rango efectivo para el scattering neutrón-protón, con la ayuda de los potenciales nucleón-nucleón realistas más populares (Nijm I), (Nijm II) y (Reid93). Comparamos nuestros resultados con la data experimental más avanzada basada en análisis parcial de ondas de la base de datos SAID nucleón-nucleón y se rpresentaron los valores más actuales de los parámetros a_t, a_s, r_t y r_s. Nuestros cálculos basados en estos tres potenciales con energía de neutrones incidentes menores de 60 kEV muestran que hay algunas discrepancias con la data experimental. Para disminuir la discrepancia a muy bajas energías, sugerimos incluir los términos de acoplamiento $^3S_1$ + $^3D_1$ para construir varios modelos de fuerza nuclear realistas. Las expansiones de rango calculadas en este trabajo no son muy precisas oara muy bajas energías, excepto si se considera muchos términos vn en las expansiones.

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I. INTRODUCTION

In the past years, researchers were focused on the subtleties and various extensions of the nuclear force leading to setting up more sophisticated two and few-nucleon potentials. Therefore, various high-quality models and forms for Nucleon-Nucleon interaction has been presented nowadays [1–7]. One way to study the nuclear two-body interactions is to use a two-nucleon system namely, the deuteron that has two nucleons ($n + p$).

The primary goals of this article is to use a model-independent analysis, to extract the best possible values of the effective range theory (ERT) parameters for np elastic scattering, the spin-triplet and spin-singlet scattering lengths a_r and a_s, and their effective ranges r_t and r_s, and the zero-energy free neutron cross section. The secondary goal is to check the accuracy of effective range expansion method for various modern realistic nucleon–nucleon potentials, namely, Nijm I, Nijm II and Reid93 [8, 9] at very low energy of incident neutron.

Along with the deuteron parameters, the low-energy parameters in the effective-range expansion for neutron–proton scattering were given by:

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \dots, \quad (1)$$

where $k^2 = 2\mu E/\hbar^2$ and δ-the phase shift.

The effective-range expansion, including the scattering length a, the effective range r, the shape parameter v_2, and the higher order parameters v_n for neutron–proton scattering are fundamental quantities that play a key role in studying strong nucleon–nucleon interaction [10]. These parameters are of great importance not only to construct various realistic nuclear-force models, but also to form a basis for studying the structure of nuclei and various nuclear processes. The theoretical value of these parameters greatly depends on the used nuclear-force model. As we go over from one model to another, it follows that the shape parameter is very sensitive to nucleon–nucleon interaction.

We would like to note that the shape parameter v_2 depends not only on the form of interaction, but depends on the scattering length a, and the effective range r. In particular, a change of only a few tenths of a percent in the scattering length may lead to a several fold change in the shape parameter.

Starting with Schrodinger equation:

$$\left[\frac{d}{dr^2} + k^2 - U(r)\right] u_k(r) = 0, \tag{2}$$

where $U(r) = (2\mu/\hbar^2)V(r)$. The boundary conditions of $u_k(r)$ are given in terms of phase shift δ:

$$u_k(0) = 0; \qquad u_k(r \to \infty) \to \frac{\sin(kr+\delta)}{\sin\delta}.$$

Whereas, the wave function $u_k(0)$ was taken when the incident energy is zero, $k = 0$. The equation 2 for this case is given as follows:

$$\left[\frac{d}{dr^2} - U(r)\right] u_0(r) = 0. \tag{3}$$

The above equation holds even when $U(r) = 0$. Thus, we can have the same discussion with the following Schrodinger equation with different wave function:

$$\left[\frac{d}{dr^2} + k^2\right] \omega_k(r) = 0. \tag{4}$$

Along with the boundary condition, $\omega_k(r \to \infty) \to u_k(r \to \infty)$.

$$\omega_0 \frac{d^2\omega_k}{dr^2} - \omega_k \frac{d^2\omega_0}{dr^2} = -k^2 \omega_0 \omega_k \tag{5}$$

According to the boundary conditions, $[u_0 u_k' - u_k u_0']_0^\infty$ will cancel out and only when $r = 0$, we can have the following:

$$[\omega_0 \omega_k' - \omega_k v_0']_{r=0} = k^2 \int_0^\infty (\omega_0 \omega_k - u_0 u_k) dr. \tag{6}$$

Where

$$\omega_k = \cos kr + \cot\delta \sin kr. \tag{7}$$

From this, we can derive $\omega_0(r)$ which is the asymptotic function when $k \to 0$,

$$\omega_0 \equiv \lim_{k\to 0} \omega_k(r) = 1 + r \lim_{k\to 0} k \cot\delta. \tag{8}$$

We also perform an expansion with k^2; then, we have for the scattering length a and the effective range r

$$k\cot\delta = -\frac{1}{a} + \frac{1}{2} r k^2 + O(k^4). \tag{9}$$

Where the parameter r is the usual effective range at zero energy, which is a good approximation in studying nucleon–nucleon (NN) scattering at low energies.

II. RESULTS AND DISCUSSIONS

The total scattering cross-section of two nucleonic systems for different singlet and triplet states is written as:

$$\sigma = \frac{3}{4}\sigma_t + \frac{1}{4}\sigma_s. \tag{10}$$

Where σ_t and σ_s are the cross-sections for scattering in the triplet and singlet states, respectively. The scattering length a is defined in such a way the low-energy cross-section is equal to $4\pi a^2$, where

$$\lim_{k\to 0} \sigma = 4\pi a^2, \tag{11}$$

with

$$a = \pm \lim_{k\to 0} \frac{\sin\delta_0}{k}. \tag{12}$$

Where δ_0 is the zero-energy phase shift.

In order to determine the triplet and singlet scattering lengths (a_t and a_s, respectively), we employ equations that relate to the above quantities with the total cross-section for zero-energy scattering of neutrons by protons,

$$\sigma_0 = \pi(3a_t^2 + a_s^2). \tag{13}$$

The coherent scattering length then is defined as:

$$f = \frac{1}{2}(3a_t + a_s). \tag{14}$$

The values of the triplet effective range r_t were determined primarily in an approximation that does not depend on the form of interaction that is,

$$r_t \equiv \delta(-\varepsilon_d, 0) = 2R(1 - \frac{R}{a_t}). \tag{15}$$

Where $\delta(-\varepsilon_d, 0)$ is the mixed effective radius of the deuteron.

$$R = \frac{1}{\alpha}, \tag{16}$$

R is a parameter that characterizes the spatial dimensions of the deuteron; and α is the deuteron wave number, which is directly related to the deuteron binding energy ε_d by the equation $\varepsilon_d = (\hbar^2\alpha^2)/m_N$.

The singlet effective range r_s is usually determined on the basis of the analysis of the total cross-section for neutron–proton scattering σ_0 in the low-energy region at fixed values of the parameters a_t, a_s, and r_t. To determine the other parameters, we fitted the total cross section for various realistic nucleon-nucleon potentials, namely the most popular modern realistic nucleon–nucleon potentials (Nijm I, Nijm II and Reid93) using the equation 13 for zero-cross section σ_0 and the equation 14 for the coherent scattering length f. We chose the values of experimental data for the total cross section σ_0 from SAID nucleon–nucleon database [11]. Here we determined the values of the triplet effective range r_t using the equations 15 and 16, then we compared our results for all low energy parameters a_t, a_s, r_t and r_s by using the value of coherent scattering length $f = -3.756$ fm from previous experimental data [12–14]. The chosen experimental data along with our results for various realistic potential (Nijm I, Nijm II and Reid93) were summarized in Table .

Table 1. Low-energy parameters of neutron–proton scattering from various experimental studies compared with our results based on potentials Nijm I, Nijm II and Ried 93. References were provided next to each experimental study.

Model	$a_t(fm)$	$a_s(fm)$	$r_t(fm)$	$r_s(fm)$
(Exp.data) Dilg [12]	5.423(21)	-23.749(54)	1.740(28)	2.772(11)
(Exp.data) Houlk. [13]	5.405(11)	-23.728(28)	1.738(12)	2.56(9)
(Exp.data) Noyes [14]	5.396(4)	-23.678(13)	1.727(4)	2.51(10)
Nijm I (Our Calculations)	5.08236	-22.7371	1.315	2.445
Nijm II. (Our Calculations)	5.08343	-22.7603	1.316	2.455
Ried 93. (Our Calculations)	5.08388	-22.7616	1.317	2.51
Babenko and Petrov (theory) [15]. < 150 keV .	5.411(27)	-23.7155 (8)	1.7601(27)	2.706(21)

As we can see from Table our calculated results using Nijm I, Nijm II and Ried 93 potentials for parameters a_t, a_s, r_t and r_s, are in a good agreement with each other. However, some discrepancies were observed between the obtained data and the experimental data. In particular for the effective range in the triplet state r_t, the discrepancies are about 25 %. In addition we also compared our obtained results with other theoretical calculations at energies below 150 keV, (Babenko and Petrov [15]) for the total cross sections of neutron–proton scattering at zero energy.n We noticed that Babenko and Petrov results below 150 keV also were at odds with Dilg's experimental cross section as ours. According to Babenko and Petrov, the discrepancies with Dilg's experimental cross section is indicates that Dilg's cross section is in a glaring contradiction with experimental data in the energy region of several keV units and is likely to be erroneous.

This fact deserves special attention because all modern realistic nucleon–nucleon potentials (Nijm-I, Nijm-II,Reid93); Argonne [16]; and CD-Bonn [17, 18] are based on a fit to the Nijmegen nucleon–nucleon database, which includes Dilg's cross section as an input parameter, and therefore lead to an insufficiently accurate description of the present-day experimental data at low energies.

Table 2. A comparison between experimental data of total cross-section for neutron scattering on a proton (σ_0) at low energies ($l = 0$) [11] with the obtained data of total cross-section for the three potentials (Nijm I, Nijm II and Reid93).

Energy (MeV)	Experimental data (σ_0), barn	Nijm I potential (σ_0), barn	Nijm II potential (σ_0), barn	Reid93 Potential (σ_0), barn
00.000132	20.491∓ 0.014 [39]	0.040852467(13)	0.038483489(22)	0.038528427(43)
00.000300	20.436∓0.023 [42]	0.097287109(27)	0.097382615(44)	0.097382615(33)
00.001970	20.130∓0.030 [43]	02.04260796(34)	02.04330996(19)	02.04392938(28)
00.060000	15.400∓0.462 [44]	14.90949419(11)	14.91276065(43)	14.90874451(37)
00.075000	14.200∓0.426 [44]	14.01762298(19)	14.02030094(11)	14.01455952(41)
00.090000	13.000∓0.390 [44]	13.24459177(22)	13.24681517(14)	13.23983230(39)
00.120000	12.050∓0.121 [44]	11.99111572(29)	11.99270806(13)	11.98420184(33)
00.143000	11.210∓0.030 [43]	11.20928574(28)	11.21055428(23)	11.20141927(26)
00.492600	06.202∓0.011 [45]	06.20429833(38)	06.20397122(22)	06.19607508(27)
00.555000	06.041∓0.103 [46]	05.82498798(19)	05.82531337(44)	05.81289620(19)
00.600780	05.557∓0.088 [46]	05.58979939(45)	05.59012464(46)	05.58292632(44)
00.702690	05.173∓0.052 [46]	05.14674364(47)	05.14629833(41)	05.13965183(41)
00.803430	04.817∓0.043 [46]	04.79434200(38)	04.79404296(29)	04.78785422(17)
00.902520	04.472∓0.036 [46]	04.50706514(11)	04.50687126(39)	04.50106316(11)
01.053000	04.274∓0.001 [47]	04.15012476(33)	04.15046381(33)	04.14512978(10)
02.082000	02.819∓0.003 [47]	02.85372251(45)	02.85398756(28)	02.85021791(22)
03.069000	02.246∓0.004 [47]	02.26502099(29)	02.26517930(12)	02.26194264(41)
03.986000	01.889∓0.004 [47]	01.91441046(11)	01.91447913(11)	01.91150386(19)
05.115000	01.608∓0.005 [47]	01.61142047(14)	01.61139859(10)	01.60863412(11)
06.032000	01.411∓0.004 [47]	01.42759440(19)	01.42751368(16)	01.42488243(18)
07.222000	01.219∓0.004 [47]	01.24191462(23)	01.24177352(17)	01.23929313(27)
08.363000	01.092∓0.005 [47]	01.10230937(18)	01.10212380(18)	01.09977674(22)
09.281000	01.000∓0.007 [47]	01.00955693(21)	01.00934309(38)	01.00709898(18)
10.360000	00.903∓0.008 [47]	00.91734345(19)	00.91710324(33)	00.91497690(13)
10.972000	00.861∓0.009 [47]	00.87154787(27)	00.87129543(26)	00.86923464(19)

In the present work to explain the discrepancies of our obtained results for low energy parameters with the experimental data, we collected a set of experimental values of the total cross-section of neutron-proton scattering at specific low energies from the SAID nucleon–nucleon database, then compared the calculated values of the total np scattering cross-section based on the three potentials (Nijm I, Nijm II and Reid93) at the same energies range [0.0002 – 10 MeV]. In Table we summarized the obtained results based in our studied potentials with experimental total cross section from the SAID nucleon–nucleon database at the interval of energies [0.0002 – 10 MeV].

As we can see from Table , there is a huge discrepancy between the values of total cross-section of neutron-proton scattering σ_0 for three potentials Nijm I, Nijm II and Reid93 with experimental data at energy of incident neutron less than 0.06 MeV. However, as the incident neutron goes above the value 0.06 MeV, a good agreement with experimental data was observed.

Table 3. The contributions of expansion terms v_n (v_2, v_3 and v_4) in the low-energy scattering potentials Nijm I, Nijm II and Reid93 in energy interval [0.0002 – 10 MeV].

Potentials	v_2	v_3	v_4
NijmI	0.046	0.675	–3.97
NijmII	0.044	0.672	–3.95
Reid93	0.045	0.676	–3.90

Figures 1, 2 and 3 show the total cross section at zero energy (σ_0) vs. energy of incident neutron for all studied potentials (Nijm I, Nijm II and Reid93), compared with experimental data. These data were taken from the SAID nucleon–nucleon database. Figures show a good agreement between our calculations and experimental data above the incident neutron energy of 0.06 Mev; as energy decreases we see the discrepancies with experimental data increases.

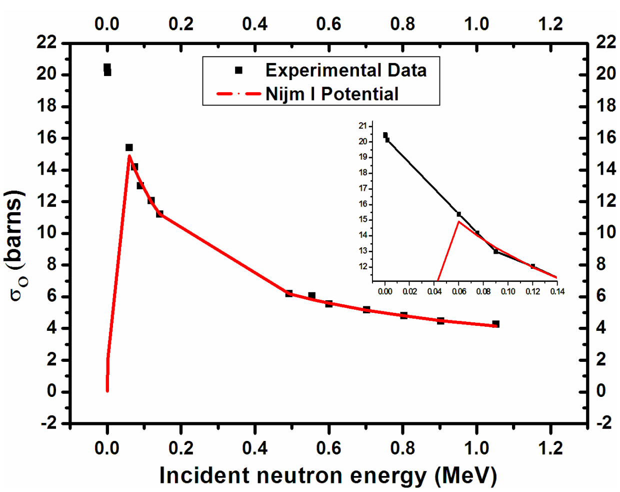

Figure 1. The total cross-section for neutron scattering by a proton for the potential Nijm I as function incident neutron energy, the experimental data were obtained from [11]. The inset represents a zoom for the cross section at low energy from (0.0–0.06) Mev.

The insets of Figs. 1, 2 and 3 represent a zoom for the total cross section at interval energy from (0.0 – 0.06). This observation leads us to an important conclusion: the three potentials have an obvious defect when the incident neutron energy is less than 0.06 MeV. In Table we presented the contributions of the higher order expansion terms v_2, v_3 and v_4 in the effective range expansions at very low energies for all studied potentials (Nijm I, Nijm II and Reid93). The higher order terms have an important contribution in the total cross section, constituting about 10-15 %; it follows that the shape parameter is very sensitive to nucleon–nucleon interaction at very low energies. Moreover, in Table we presented some of the effective ranges for various potentials and the contributions of terms v_n in the expansions of effective range theory in energy interval [0.0002 – 10] MeV. In a future paper we intend to show the contributions of all expansion terms v_2, v_3 and v_4 separately in our calculations for the cross section at very low incident energies.

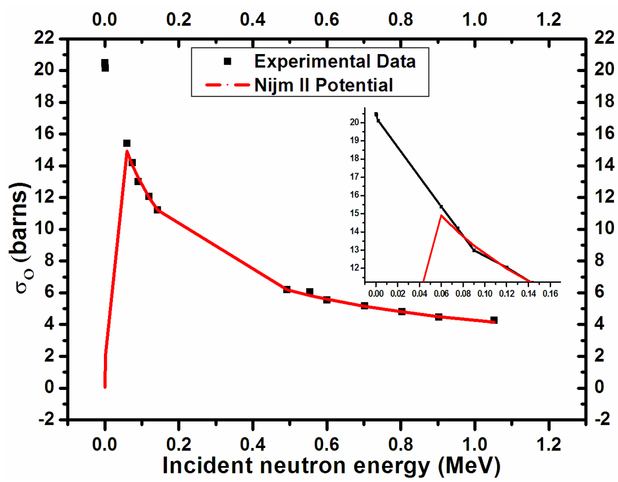

Figure 2. The total cross-section for neutron scattering by a proton for the potential Nijm II as a function incident neutron energy, the experimental data were obtained from [11] the inset represent a zoom for the cross section at low energy from (0.0–0.06) Mev.

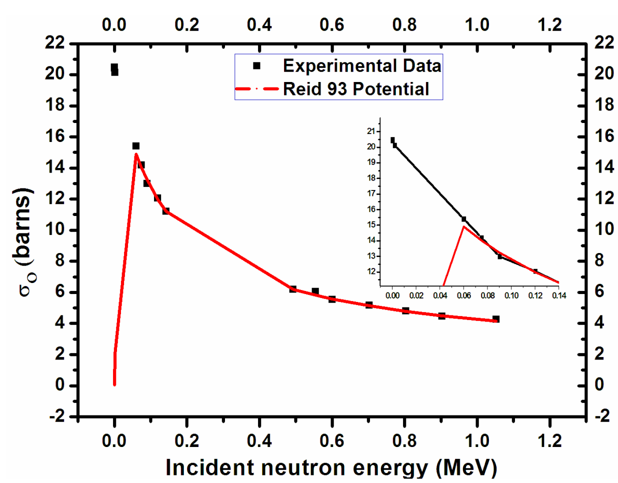

Figure 3. The total cross-section for neutron scattering by a proton for the potential Reid93 as function incident neutron energy, the experimental data were obtained from [11]. The inset represents a zoom for the cross section at low energy from (0.0–0.06) Mev.

So we may conclude that the obtained results for various nucleon- nucleon potentials such Nijm I, Nijm II and Reid93 show that the construction of these potentials are incomplete at zero-energy specially at incident energy less than 60 keV. To eliminate the discrepancy between our results and the experimental one at very low energy, we may suggest including coupling terms in addition to the term $^1S_0$ for constructing various realistic nuclear-force models. Furthermore, to achieve a good agreement for the low energy parameters, one should take into account many terms v_n in the expansions of effective range theory.

III. CONCLUSIONS

We determined the four low energy parameters: singlet scattering length a_s, triplet scattering length a_t, singlet effective range r_s and triplet effective range r_t of low-energy neutron-proton scattering using three neutron-neutron potentials: Nijm I, Nijm II and Reid93. Compared to experimental data, some discrepancies were observed. We mentioned here that the calculated effective range expansions in this work are not very accurate for very low energy unless one takes into account many terms v_n in the expansions. Besides, our calculations lead us to an important conclusion that the three potentials, Nijm I, Nijm II and Reid93 have obvious defect to calculate the values of total cross-section of neutron-proton scattering when the incident neutron energy is less than 60 KeV. As the energy of incident neutrons increases, the calculated results become in a good agreement with the experimental data for all three potentials. This is a good indication that the methodology adopted for constructing these potentials follows the same logic. To improve the consistency between our results and the experimental results at very low energy we suggest to include coupling terms $^3S_1$ + $^3D_1$ in addition to the term $^1S_0$ for constructing various realistic nuclear-force models. Finally, we found that a reliable experimental determination of the total cross section for neutron–proton scattering at zero energy, σ_0, and of the coherent scattering length, f, is now quite a pressing problem. Precise values of these quantities would make it possible to determine unambiguously the triplet and singlet scattering lengths and to solve the problem of choosing a correct set of the low-energy parameters and phase shifts among currently recommended experimental values.

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