THE ONSET OF ELECTROHYDRODYNAMIC INSTABILITY IN A COUPLE-STRESS NANOFLUID SATURATING A POROUS MEDIUM: BRINKMAN MODEL

EL UMBRAL DE LA INESTABILIDAD ELECTROHIDRODINÁMICA EN UN NANO-FLUÍDO ACOPLADO POR STRESS QUE SATURA UN MEDIO POROSO: MODELO DE BRINKMAN

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Electrohydrodynamic thermal instability of an elastic-viscous nanofluid saturating a porous in the presence of vertical AC electric field is investigated analytically and numerically. For porous medium, Brinkman model is employed and couple-stress fluid model is used to describe rheological behavior of a nanofluid. On the boundary of fluid layer nanoparticle the flux is assumed to be zero. The problem is solved by applying linear stability analysis based upon perturbation theory and normal mode analysis for isothermal free-free boundaries. The effects of couple-stress parameter, Brinkman-Darcy number, AC electric field, Lewis number, modified diffusivity ratio, nanoparticle Rayleigh number and medium porosity are presented for the case of stationary convection. Under the considered boundary conditions, the oscillatory convection does not exist.

La inestabilidad térmica electro-hidrodinámica de un nanofluido elástico-viscoso que satura un poro en presencia de un campo eléctrico vertical de AC se investiga analíticamente y numéricamente. Para el medio poroso, se emplea el modelo de Brinkman y el modelo de fluido par- estrés se utiliza para describir el comportamiento reológico de un nanofluido. En el límite del flujo de nanopartículas de la capa de fluido el flujo se supone que es cero. El problema se resuelve aplicando un análisis de estabilidad lineal basado en la teoría de perturbaciones y el análisis de modo normal para los límites isotérmicos libre-libre. Los efectos del parámetro de par-estrés, número de Brinkman-Darcy, campo eléctrico en AC, número de Lewis, relación de difusividad modificada, número de Rayleigh de nanopartículas y porosidad media se presentan para el caso de convección estacionaria. Bajo las condiciones de contorno consideradas, la convección oscilatoria no existe.

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I. NOMENCLATURE

a wave number

c specific heat

d thickness of the horizontal layer

D_B diffusion coefficient (m^2/s)

D_T thermophoretic diffusion coefficient

F Couple-stress parameter

g acceleration due to gravity

$\mathbf{g}$ gravitational acceleration vector

$\mathbf{E}$ root mean square value of the electric field

k Thermal conductivity

L_e Lewis number

N_A modified diffusivity ratio

p pressure (Pa)

$\mathbf{q}$ Darcy velocity vector (m/s)

R_a thermal Rayleigh number

R_n concentration Rayleigh number

R_{ea} AC electric Rayleigh number

t time (s)

T temperature (K)

(u, v, w) Darcy velocity components

(x, y, z) space co-ordinates (m)

Greek symbols

α coefficient of thermal expansion

φ noparticles volume fraction

κ thermal diffusivity

κ_m effective thermal diffusivity of porous medium

μ viscosity of the fluid

μ_c Couple-stress viscosity

ρ density of fluid

ρ_p nanoparticle mass density (kg/m3)

ω growth rate of disturbances

∇_H^2 horizontal Laplacian operator

∇ Laplacian operator

ε medium porosity

K dielectric constant

$\tilde{D}_a$ Brinkman-Darcy number

Superscripts

$'$ non-dimensional variables

$''$ perturbed quantity

Subscripts

p particle

f fluid

b basic state

0 lower boundary

1 upper boundary

II. INTRODUCCIÓN

The problem of thermal instability in fluid in a porous medium has many practical applications in physical and industrial processes. It has importance in geophysics, soil sciences, thermal insulation of buildings, winding structures for high power density electric machines, food processing and storage, underground disposal of heavy water ground water hydrology and astrophysics.

Thermal instability of non-Newtonian fluids becomes an important field of research for the last few decades. There are many elastico-viscous fluids which cannot be characterized by linear relationship between stress and rate of strain components. One such type of fluid is couple-stress fluid. The theory of couple-stress fluid was first coined by Stokes [1]. Inertia effect in the squeeze film of a couple-stress fluid in biological bearings was studied by Walicki and Walicka [2] and found that the effects of couple-stress and inertia provide an enhancement in the pressure distribution and in the load-carrying capacity. The application of couple-stress fluid is in the study of the mechanism of lubrication of synovial joints, which has become the main objective of scientific research and found that the synovial fluid in human joints behaves like a couple-stress fluid. The stability of a Boussinesq couple-stress horizontal fluid layer, when the boundaries are subjected to time periodic temperature modulation or in the presence of time periodic body force has been studied by Malashetty and Basavaraja [3] using linear stability theory and found that the effect of couple-stress parameter is to advance the onset of convection whereas Bishnoi et al. [4] studied thermal convection in a couple-stress fluid in the presence of horizontal magnetic field with hall currents and found that couple-stress has destabilizing or stabilizing effect on the thermal convection under certain conditions. Rana [5] has studied thermal convection in couple-stress fluid in hydromagnetics saturating a porous medium and found that couple-stress parameter stabilizes the system.

In the past few years, thermal convection in a nanofluid saturating a porous medium became an important field of research. Nanofluid is a fluid mixture of nano-sized metallic particles immersed in common fluids such as water, ethanol or engine oils are typically used as base fluids in nanofluids and the nanoparticles may be taken as oxide ceramics such as Al_2O_3 or CuO, nitride ceramics such as AlN or SiN and several metals such as Al or Cu. Nanofluid has various applications in automotive industries, energy saving etc. Further, suspensions of nanoparticles are being developed medical applications including cancer therapy. Different authors [6–14] have studied thermal convection in a layer of nanofluid in a porous medium based upon Buongiorno [15] model. They found that nanofluid has enhancing property of heat transfer.

Recently, considerable interest has been evinced in the study of electrohydrodynamic thermal instability in a viscous and viscoelastic fluid. Takashima [16] discussed the effect of uniform rotation on the onset of convective instability in a dielectric fluid under the simultaneous action of AC electric field. It has various applications in different areas such as EHD enhanced thermal transfer, EHD pumps, EHD in microgravity, micromechanic systems, drug delivery, micro-cooling system, nanotechnology, oil reservoir modeling, petroleum industry, building of thermal insulation, biomechanics, engineering [17–25]. Couple-stress fluid under rotation in electrohydrodynamics has been studied by Shivakumara et al. [26] and found that the couple-stress fluid under rotation has destabilizing effect on the system. The instability of a viscoelastic fluid saturating a porous medium in electrohydrodynamics has been studied by Rana et al. [27–29] whereas Chand et al. [30] studied electrothermo convection in a porous medium saturated by a nanofluid and found that AC electric field has destabilizing effect on the system.

The growing number of applications of electrohydrodynamic thermal instability in an elastico-viscous nanofluid fluid in a porous medium which include several engineering and medical fields, such as automotive industries, energy saving and cancer therapy, motivated the current study. In the present paper, we analyze thermal instability problem in a horizontal layer of an elastico-viscous couple-stress nanofluid in a Darcy-Brinkman porous medium in the presence of AC electric field.

III. FORMULATION OF THE PROBLEM AND MATHEMATICAL MODEL

Here we consider an infinite horizontal porous layer of a couple-stress elastico-viscous nanofluid of thickness d saturating a porous medium, bounded by the planes $z = 0$ and $z = d$ under the action of a uniform vertical AC electric field applied across the layer; the lower surface is grounded and the upper surface is kept at an alternating (60 Hz) potential whose root mean square value is V_1 (see Figure 1). The layer is acted upon by a gravity force $\mathbf{g} = (0, 0, -g)$ aligned in the z direction which is heated from below. The temperature, T, and the volumetric fraction of nanoparticles, φ, at the lower (upper) boundary is assumed to take constant values T_0, and φ_0 (T_1, and φ_0), respectively.

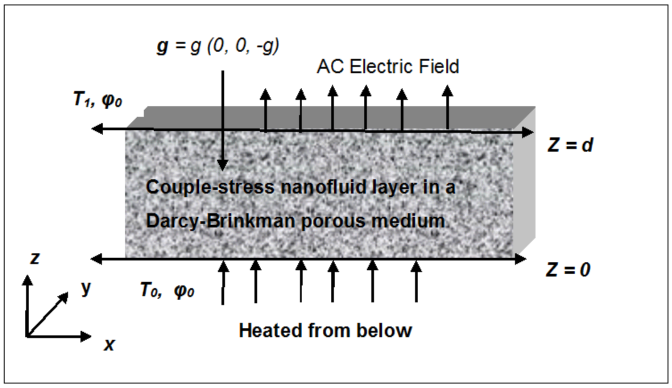

Figure 1. Physical Configuration

III.1. GOVERNING EQUATIONS

Let ρ, μ, μ_c, $\tilde{\mu}$, p, ε, α, φ, ρ_p, k_1, $\mathbf{E}$, K, T and $\mathbf{q}(u, v, w)$, denote, respectively, density, viscosity, couple-stress viscosity, effective viscosity, pressure, medium porosity, coefficient of thermal expansion, volume fraction of nanoparticles, density of nanoparticles, medium permeability, root mean square value of the electric field, temperature and Darcy velocity vector. The equations of mass-balance and momentum-balance for couple-stress nanofluid in the presence of vertical AC electric field saturating a Darcy-Brinkman porous medium by applying oberbeck-Boussines approximation [1,3,4,8,10,11,16] are,

$$\nabla \mathbf{q} = 0, \tag{1}$$

$$0 = -\nabla P - \frac{1}{k_1}(\mu - \mu_c \nabla^2)\mathbf{q} + \tilde{\mu}\nabla^2 \mathbf{q} - -\frac{1}{2}(\mathbf{E}\cdot\mathbf{E}\nabla K + \mathbf{g}(\varphi\rho_p + \rho_0(1-\varphi)(1-\alpha(T-T_0)))), \tag{2}$$

where $P = p - \frac{\rho}{2}\frac{\partial K}{\partial \rho}(\mathbf{E}\cdot\mathbf{E})$ is the modified pressure.

The mass-balance equation for the nanoparticles (Buongiorno [15]) and thermal energy equation for a nanofluid are given by,

$$\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}\cdot\nabla^2\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T, \tag{3}$$

$$(\rho c)_m\left(\frac{\partial T}{\partial t} + \mathbf{q}\cdot\nabla\varphi\right) = k_m\nabla^2 T + \varepsilon(\rho c)_p\left[D_B\nabla\varphi\cdot\nabla T + \frac{D_T}{T_1}\nabla T\cdot\nabla T\right]. \tag{4}$$

where $(\rho c)_m$ is heat capacity of fluid in porous medium, $(\rho c)_p$ is heat capacity of nanoparticles and k_m is thermal conductivity.

The Maxwell equations are:

$$\nabla \times \mathbf{E} = 0, \tag{5}$$

$$\nabla \cdot (K\mathbf{E}) = 0, \tag{6}$$

Let V be root mean square value of electric potential, the electric potential can be expressed as,

$$\mathbf{E} = -\nabla V \tag{7}$$

The dielectric constant is assumed to be linear function of temperature and is of the form,

$$K = K_0[1 - \gamma(T - T_0)], \tag{8}$$

where γ, is the thermal coefficient of expansion of dielectric constant and is assumed to be small.

We assume that the temperature is constant and nanoparticles flux is zero on the boundaries. The appropriate boundary conditions [3,11] are

$$w = 0,\ T = T_0\ D_B\frac{\partial\varphi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \text{ at } z = 0 \tag{9a}$$

and

$$w = 0,\ T = T_1\ D_B\frac{\partial\varphi}{\partial z} + \frac{D_T}{T_1}\frac{\partial T}{\partial z} = 0 \text{ at } z = d. \tag{9b}$$

Introducing non-dimensional variables as

$$(x', y', z') = \left(\frac{x, y, z}{d}\right),\ (u', v', w') = \left(\frac{u, v, w}{\kappa_d}\right)\ t' = \frac{t\kappa_m}{\sigma d^2},$$

$$P' = \frac{Pk_1}{\mu\kappa_m},\ \varphi' = \frac{\varphi - \varphi_0}{\varphi_0},\ T' = \frac{T}{\Delta T},\ K' = \frac{K}{\gamma E_0 \Delta T d},$$

where $\kappa_m = \frac{k_m}{(\rho c_p)_f}$ is thermal diffusivity of the fluid and $\sigma = \frac{(\rho c_p)_m}{(\rho c_p)_f}$ is the thermal capacity ratio. Eliminating the modified pressure from the momentum-balance equation 8 by operating twice curl and retaining the vertical component, we obtain the following equations in non-dimensional form (after dropping the dashes (′) for convenience) as

$$\nabla \mathbf{q} = 0, \tag{10}$$

$$0 = -(1 - \eta\nabla^2 - \tilde{\mu}\nabla^2)\nabla^2 w + R_a\nabla_H^2 T - Rn\nabla_H^2\varphi + R_{ea}\nabla_H^2\left(T - \frac{\partial K}{\partial z}\right), \tag{11}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}\nabla\varphi = \frac{1}{Le}\nabla^2 T, \tag{12}$$

$$\frac{\partial\varphi}{\partial t} + \mathbf{q}\cdot\nabla T = \nabla^2 T + \frac{N_a}{Le}\nabla\varphi\nabla T + \frac{N_A N_B}{Le}\nabla T\cdot\nabla T, \tag{13}$$

$$\nabla^2 V = \frac{\partial T}{\partial z}. \tag{14}$$

Here $Le = \frac{\kappa_m}{D_B}$ is the thermal Lewis number,

$\eta = \frac{\mu_c}{\mu d^2}$ is the couple-stress parameter,

$R_a = \frac{\rho g \alpha d k_1 (T_0 - T_1)}{\mu \kappa_m}$ is the thermal Rayleigh number,

$R_n = \frac{(\rho_g - \rho)\varphi_0 g k_1 d}{\mu \kappa_m}$ is the nanoparticle Rayleigh number,

$N_A = \frac{D_T (T_0 - T_1)}{D_B T_1 \varphi_0}$ is the modified diffusivity ratio,

$N_B = \frac{\varepsilon (\rho c)_p \varphi_0}{(\rho c)_f}$ is the modified particle-density ratio,

$R_{ea} = \frac{\gamma^2 K E_0^2 d^2 (\Delta T^2)}{\mu \kappa_m}$ is the AC electric Rayleigh number,

$\tilde{D}a = \frac{\tilde{\mu} k_1}{\mu d^2}$, is the Brinkman-Darcy number and ∇_H^2 is the two-dimensional Laplace operator on the horizontal plane, that is $\nabla_H^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$.

The dimensionless boundary conditions are

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial K}{\partial z} = 0,\ T = 1,\ \frac{\partial \varphi}{\partial z} + N_A \frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \quad (15a)$$

and

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial K}{\partial z} = 0,\ T = 0,\ \frac{\partial \varphi}{\partial z} + N_A \frac{\partial T}{\partial z} = 0 \text{ at } z = 1, \quad (15b)$$

III.2. BASIC SOLUTIONS

We assume that the basic state is quiescent [3, 8–11] and is given by

$$u = v = w = 0,\ p = p(z),\ K = K_b(z),\ T = T_b(z),\ \varphi = \varphi_b(z),$$
$$E = E_b(z),\ \psi = \psi_b(z). \quad (16)$$

$$T_b = T_0 - \frac{\Delta T}{dz},\ \varphi_b = \varphi_0 + \left(\frac{D_T \Delta T}{D_b T_1 d}\right) z,$$

$$K_b = K_0 \left(1 + \frac{\gamma \Delta T}{d} z\right) \hat{k},\ E_b = \frac{E_0}{1 + \frac{\gamma \Delta T}{d} z} \hat{k}.$$

Also, we have

$$v_b = -\frac{E_0 d}{\gamma \Delta T} \log\left(1 + \frac{\gamma \Delta T}{d} z\right) \hat{k},$$

where $E_0 = -\frac{V_1 \gamma \Delta T}{d \log(1 + \gamma DeltaT)}$ is the root mean square value of the electric field at $z = 0$.

The basic state defined in 16 is substituted into equations 12 and 13, these equations reduce to

$$\frac{d^2 \varphi_b(z)}{dz^2} + N_A \frac{d^2 T_b(z)}{dz^2} = 0, \quad (17)$$

$$\frac{d^2 \varphi_b(z)}{dz^2} + \frac{N_b}{Le} \frac{d\varphi_b(z)}{dz} \frac{dT_b(z)}{dz} + \frac{N_A N_B}{Le} \left(\frac{dT_b(z)}{dz}\right)^2 = 0, \quad (18)$$

Using boundary conditions 15b in equations 17 and 18, on integration equation 17 gives

$$\frac{d\varphi_b(z)}{dz} + N_A \frac{dT_b(z)}{dz} = 0. \quad (19)$$

Using equation 19 in equation 18, we obtain

$$\frac{d^2 T_b(z)}{dz^2} = 0. \quad (20)$$

Applying the boundary conditions 15b, the solution of equation 20 is given by

$$T_b(z) = 1 - z. \quad (21)$$

Integrating equation 19 by applying the boundary conditions 15b, we get

$$\varphi_b(z) = \varphi_0 + N_A z. \quad (22)$$

These results are identical with the results obtained by Sheu [10] and Nield and Kuznetsov [9–11].

III.3. PERTURBATION SOLUTIONS

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, so that

$$\mathbf{q}(w, v, w) = q''(u, v, w),\ T = T_b + T'',\ p = p_b + p'', \quad (9)$$
$$E = E_b + E'',\ V = V_b + V''. \quad (23)$$

Introducing equation 9 along with equations 21 and 22 into equations 10 – 14, linearizing the resulting equations by neglecting nonlinear terms that are product of prime quantities and dropping the primes ('') for convenience, we obtain the following equations

$$\nabla \cdot \mathbf{q} = 0, \quad (24)$$

$$0 = -\nabla P - \frac{1}{k_1}(\mu - \mu_c \nabla^2) + \tilde{\mu} \nabla^2) \nabla^2 w + Ra \nabla_H^2 T$$
$$- R \nabla_H^2 \varphi + R_{ea} \nabla_H^2 \left(T - \frac{\partial K}{\partial z}\right), \quad (25)$$

$$\frac{1}{\sigma} \frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon} w = \frac{1}{Le} \nabla^2 \varphi + \frac{N_A}{Le} \nabla^2 T, \quad (26)$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{Le}\left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z}\right) - \frac{2 N_A N_B}{Le} \frac{\partial T}{\partial z}, \quad (27)$$

$$\nabla^2 V = \frac{\partial T}{\partial z}, \quad (28)$$

Boundary conditions for equations 24-28 are

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial K}{\partial z} = 0,\ T = 0,\ \frac{\partial \varphi}{\partial z} + N_A \frac{\partial T}{\partial z} = 0 \quad (29)$$

at $z = 0$ and at $z = 1$.

IV. NORMAL MODE ANALYSIS

We express the disturbances into normal modes of the form

$$[w, T, \varphi, V] = [W(z), \Theta(z), \Phi(z), \Psi(z)]exp[i(lx + my + \omega t)] \quad (30)$$

where l, m are the wave numbers in the x and y direction, respectively, and ω is the growth rate of the disturbances.

Substituting equation 30 into equations 24-30, we obtain the following eigenvalue problem

$$(1 - \eta(D^2 - a^2) - \tilde{D}a(D^2 - a^2))(D^2 - a^2)W + \\ + a^2 Ra\Theta - a^2 Rn\Phi + a^2 R_{ea}(\Theta - D\Psi) = 0 \quad (31)$$

$$W + \left(D^2 + \frac{N_A}{Le}D - \frac{2N_A N_B}{Le}D - d^2 - \omega\right)\Theta - \frac{N_B}{Le}D\Phi = 0 \quad (32)$$

$$\frac{1}{\varepsilon}W - \frac{N_A}{Le}(D^2 - a^2)\Theta - \left(\frac{1}{Le}(D^2 - a^2) - \frac{\omega}{\sigma}\right) = 0, \quad (33)$$

$$(D^2 - a^2)\Psi = D\Theta, \quad (34)$$

where $D = d/dz$ and $a^2 = l^2 + m^2$ is the dimensionless resultant wave number.

The boundary conditions of the problem in view of normal mode analysis are

$$W = 0,\ D^2W = 0,\ \Theta = 0,\ D\Psi = 0,\ D\Phi + N_A D\Theta = 0 \quad (35)$$

at $z = 0$ and $z = 1$.

V. LINEAR STABILITY ANALYSIS AND DISPERSION RELATION

The case of two free boundaries, though a little artificial is the most appropriate for stellar atmospheres. Using the boundary conditions 35, the proper solution of equations 31–34 is given by,

$$W = W_0 \sin \pi z, \quad (36a)$$

$$\Theta = \Theta_0 \sin \pi z, \quad (36b)$$

$$\Phi = \Phi_0 \cos \pi z, \quad (36c)$$

$$\Psi = \Psi_0 \cos \pi z, \quad (36d)$$

where W_0, Θ_0, Φ_0 and Ψ_0 are constants.

Substituting equation 36a into the eigen-value problem given by equations 31–34 we obtain,

$$\begin{bmatrix} (\pi^2 + a^2)(1 + \eta(\pi^2 + a^2)) + \tilde{D}a(\pi^2 + a^2) & -a^2(Ra + R_{ea}) & -a^2 N_A Rn & -a\pi R_{ea} \\ 1 & -(\pi^2 + a^2 + \omega) & 0 & 0 \\ \frac{1}{\varepsilon} & \frac{1}{Le}(\pi^2 + a^2) & -\left(\frac{1}{Le}(\pi^2 + a^2) + \frac{\omega}{\sigma}\right) & 0 \\ 0 & -\pi & 0 & -(\pi^2 + a^2) \end{bmatrix} \begin{bmatrix} W \\ \Theta \\ \Phi \\ \Psi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (37)$$

The above system of matrix equations has non-trivial solution if

$$\begin{vmatrix} (\pi^2 + a^2)(1 + \eta(\pi^2 + a^2)) + \tilde{D}a(\pi^2 + a^2) & -a^2(Ra + R_{ea}) & -a^2 N_A Rn & -a\pi R_{ea} \\ 1 & -(\pi^2 + a^2 + \omega) & 0 & 0 \\ \frac{1}{\varepsilon} & \frac{1}{Le}(\pi^2 + a^2) & -\left(\frac{1}{Le}(\pi^2 + a^2) + \frac{\omega}{\sigma}\right) & 0 \\ 0 & -\pi & 0 & -(\pi^2 + a^2) \end{vmatrix} = 0. \quad (38)$$

which gives an expression for Rayleigh number Ra as

$$Ra = \frac{(1 + \eta(\pi^2 + a^2) + \tilde{D}a(\pi^2 + a^2))(\pi^2 + a^2)(\pi^2 + a^2 + \omega^2)}{a^2} - \frac{a^2}{\pi^2 + a^2}R_{ea} - \frac{\varepsilon N_A(\pi^2 + a^2) + Le(\pi^2 + a^2 + \omega^2)}{(\pi^2 + a^2)\sigma + \omega Le}\frac{\sigma}{\varepsilon}Rn \quad (39)$$

Equation 39 is the required dispersion relation accounting for the effect of Lewis number, couple-stress parameter, AC electric Rayleigh number, nanoparticle Rayleigh number, modified diffusivity ratio and medium porosity on thermal instability in a layer of couple-stress elastico-viscous nanofluid saturating a porous medium under the influence of vertical AC electric field.

STATIONARY CONVECTION

The growth rate ω is, in general, a complex quantity such that $\omega = \omega_r + i\omega_i$. The system with $\omega_r < 0$ is always stable, for $\omega_r > 0$, it will become unstable (neutral stability $\omega_r = 0$).

For the case of steady state (i. e., principle of exchange of stability), we put $\omega = 0$ in equation 39, we obtain

$$Ra = \frac{(1 + \eta(\pi^2 + a^2) + \tilde{D}a(\pi^2 + a^2))(\pi^2 + a^2)^2}{a^2} \\ - \frac{a^2}{\pi^2 + a^2}R_{ea} - \left(N_A + \frac{Le}{\varepsilon}\right)Rn \quad (40)$$

Equation 40 expresses the Rayleigh number as a function of the dimensionless resultant wave number a, the couple-stress parameters η, the Brinkman-Darcy number the AC electric Rayleigh number R_{ea}, the medium porosity ε, the nanoparticle Rayleigh number Rn, the Lewis number Le, modified-diffusivity ratio N_A. Equation 40 is identical to that obtained by Kuznetsov and Nield [8], and Rana et al. [27–29] in the absence of the couple-stress parameters η, the Brinkman-Darcy number and the AC electric Rayleigh number R_{ea}. Since equation 51 does not contain the particle increment parameter N_B but contains the diffusivity ratio parameter N_A in corporation with the nanoparticle Rayleigh number Rn. This shows that the nanofluid cross-diffusion terms approach to be dominated by the regular cross-diffusion term.

In the absence of Brinkman-Darcy number $\tilde{D}a$, equation 40 becomes,

$$Ra = \frac{(1+\eta(\pi^2+a^2))(\pi^2+a^2)^2}{a^2} - \frac{a^2}{\pi^2+a^2}R_{ea} - \left(N_A + \frac{Le}{\varepsilon}\right)Rn \tag{41}$$

In the absence of AC electric field R_{ea}, equation 41 reduces to

$$Ra = \frac{(1+\eta(\pi^2+a^2)+\tilde{D}a(\pi^2+a^2))(\pi^2+a^2)^2}{a^2} - \left(N_A + \frac{Le}{\varepsilon}\right)Rn, \tag{42}$$

which is identical with the result derived by Kuznetsov and Nield [8], Rana et al. [27–29].

In the absence of couple-stress parameter, equation (42) reduces to

$$Ra = \frac{(\pi^2+a^2)^2}{a^2} - \left(N_A + \frac{Le}{\varepsilon}\right)Rn, \tag{43}$$

which is identical with result derived by Kuznetsov and Nield [8].

Thus, the presence of nanoparticles and vertical AC electric field lower the value of the critical Rayleigh number by usually by substantial amount. Also parameter N_B does not appear in the Eq. 40, thus instability is purely phenomena due to buoyancy coupled with conservation of nanoparticles. Thus average contribution of nanoparticles flux in the thermal energy equation is zero with one-term Galerkin approximation.

VI. OSCILLATORY CONVECTION

For the marginally oscillatory state, we put $\omega = i\omega$ in equation 39, we obtain

$$Ra = \Delta_1 + i\omega\Delta_2, \tag{44}$$

where

$$\Delta_1 = \frac{(1+\eta(\pi^2+a^2)+\tilde{D}a(\pi^2+a^2))(\pi^2+a^2)^2}{a^2} - \frac{a^2}{\pi^2+a^2}R_{ea} - \frac{(\pi^2+a^2)^2\left(N_A+\frac{Le}{\varepsilon}\right)+\frac{\omega^2}{\sigma\varepsilon}}{(\pi^2+a^2)^2+\left(\frac{\omega Le}{\sigma}\right)^2}Rn, \tag{45}$$

and

$$\Delta_2 = \frac{(1+\eta(\pi^2+a^2)+\tilde{D}a(\pi^2+a^2))(\pi^2+a^2)^2}{a^2} + \frac{(\pi^2+a^2)\left(\frac{Le}{\sigma}\left(N_A+\frac{Le}{\varepsilon}\right)-\frac{Le}{\varepsilon}\right)+\frac{\omega^2}{\sigma\varepsilon}}{(\pi^2+a^2)^2+\left(\frac{\omega Le}{\sigma}\right)^2}Rn, \tag{46}$$

Since Ra is a physical quantity, so it must be real. Thus, it follows from the Eq. 44 that either $\omega = 0$(exchange of stability, steady state) or $\Delta 2 = 0$ ($\omega \neq 0$ overstability or oscillatory onset) which gives an expression for the frequency of oscillations in the form ω^2. But $\omega^2 < 0$ when $10^2 \leq Le \leq 10^4$ (Lewis number), $-1 \leq Rn \leq 10$ (nanoparticles Rayleigh number), $0.1 \leq \varepsilon 1$ (porosity parameter) (Rana et al. [27–29]) and $10 \leq R_{ea} \leq 10^4$ (AC electric Rayleigh number) (Shivakumara et al. [26]). Hence, oscillatory convection does not exist under the considered boundary conditions.

VII. RESULTS AND DISCUSSIONS

To study the effect of couple-stress parameter, AC electric Rayleigh number, Lewis number, nanoparticle Rayleigh number, modified diffusivity ration nanoparticle Rayleigh number, medium porosity and Brinkman-Darcy number, we examine the behavior of $\frac{\partial Ra}{\partial \eta}$, $\frac{\partial Ra}{\partial R_{ea}}$, $\frac{\partial Ra}{\partial Le}$, $\frac{\partial Ra}{\partial N_A}$, $\frac{\partial Ra}{\partial N_A}$, $\frac{\partial Ra}{\partial Rn}$, $\frac{\partial Ra}{\partial \varepsilon}$ and $\frac{\partial Ra}{\partial \tilde{D}a}$ analytically.

From equation (40), we obtain

$$\frac{\partial Ra}{\partial \eta} = \frac{(\pi^2+a^2)^3}{a^2}, \tag{47}$$

$$\frac{\partial Ra}{\partial R_{ea}} = -\frac{a^2}{\pi^2+a^2}, \tag{48}$$

$$\frac{\partial Ra}{\partial Le} = -\frac{Rn}{\varepsilon}, \tag{49}$$

$$\frac{\partial Ra}{\partial N_A} = -Rn, \tag{50}$$

$$\frac{\partial Ra}{\partial Rn} = -N_A + \frac{Le}{\varepsilon}, \tag{51}$$

$$\frac{\partial Ra}{\partial \varepsilon} = \frac{LeRn}{\varepsilon^2}, \tag{52}$$

$$\frac{\partial Ra}{\partial \tilde{D}a} = \frac{(\pi^2+a^2)^3}{a^2}, \tag{53}$$

From equations 47, 52 and 53, we see that the partial derivative of Rayleigh number Ra with respect to couple-stress parameter η, medium porosity ε and Brinkman-Darcy number $\tilde{D}a$ is positive implying implying thereby couple-stress, medium porosity and Brinkman-Darcy number stimulate the stationary convection. Thus, couple-stress parameter, medium porosity and Brinkman-Darcy number has stabilizing effect on the which is in an agreement with the result derived by Shivakumara [25], Rana et al. [26–28] and Chand et al. [29].

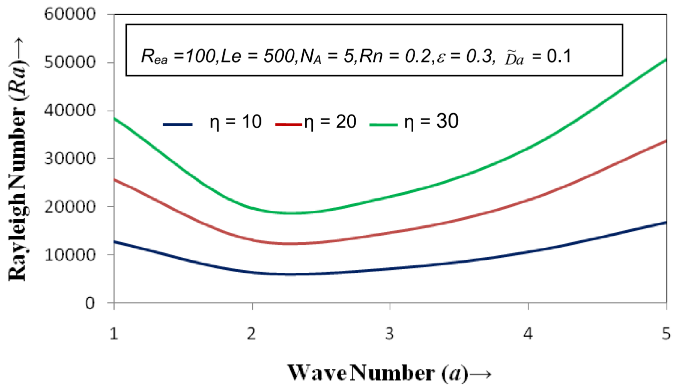

Figure 2. The variations of thermal Rayleigh number Ra with the wave number a for different values of the couple-stress parameter $\eta = 10$, $\eta = 20$ and $\eta = 30$.

The right hand sides of equations 48-51 are negative implying thereby the AC electric Rayleigh number R_{ea}, Lewis number Le, modified diffusivity ratio NA and nanoparticle Rayleigh number Rn inhibit the stationary convection. Thus, the AC electric Rayleigh number, Lewis number, modified diffusivity ratio and nanoparticle Rayleigh number have destabilizing effects on the system which is in an agreement with the results derived by Sheu [10], Nield and Kuznetsov [16], Takashima [22] and Shivakumara [25], Rana et al. [26–28] and Chand et al. [29].

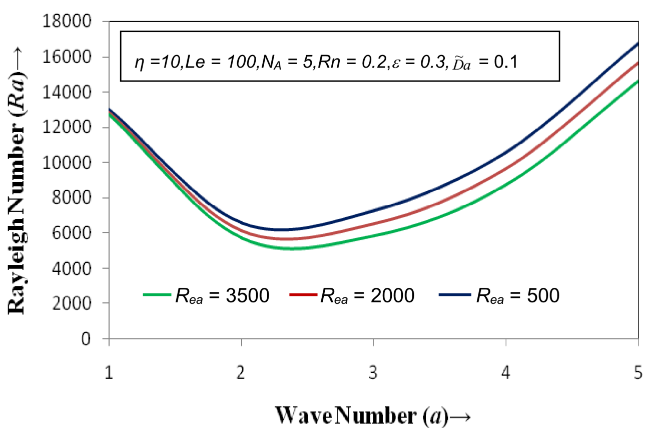

Figure 3. The variations of thermal Rayleigh number Ra with the wave number a for different values of the AC electric Rayleigh number $R_{ea} = 500$, $R_{ea} = 2000$ and $R_{ea} = 3500$.

The dispersion relation 40 is also analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters to depict the stability characteristics, e. g., Lewis number ($10^2 \leq Le \leq 10^4$), nanoparticles Rayleigh number ($-1 \leq Rn \leq 10$), porosity parameter ($0.1 \leq \varepsilon 1$) (Rana et al. [27–29]) and AC electric Rayleigh number ($10 \leq Re \leq 10^4$) (Shivakumara et al. [26]). Stability curves for couple-stress parameter η, AC electric Rayleigh number R_{ea}, Lewis number Le, nanoparticles Rayleigh number Rn, modified diffusivity ratio N_A and porosity parameter ε are shown in figures 2-8.

The variations of thermal Rayleigh number Ra with the wave number a for three different values of couple-stress parameter, namely, $\eta = 10, 20, 30$ is plotted in Fig. 2 and it is observed that the thermal Rayleigh number increases with the increase in couple-stress parameter which shows that couple-stress parameter stabilizes the system.

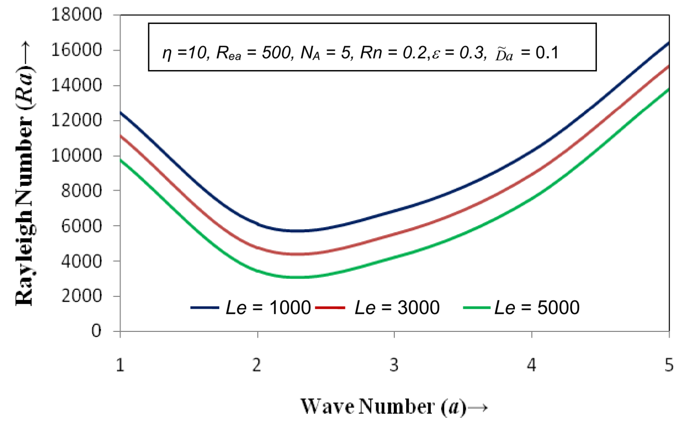

Figure 4. The variations of thermal Rayleigh number Ra with the wave number a for different values of the Lewis number $Le = 500$, $Le = 2000$, $Le = 3500$.

In Fig. 3, the variations of thermal Rayleigh number Ra with the wave number a for three different values the AC electric Rayleigh number, namely, $R_{ea} = 500, 2000$ and 3500 is plotted and it is observed that the thermal Rayleigh number decreases with the increase in AC electric Rayleigh number implying there by AC electric Rayleigh number destabilizes the system.

In Fig. 4, the variations of thermal Rayleigh number Ra with the wave number a for three different values of the Lewis number, namely, $Le = 1000$, 3000 and 5000 which shows that thermal Rayleigh number increases with the increase in Lewis number. Thus, Lewis number has stabilizing effect on the system.

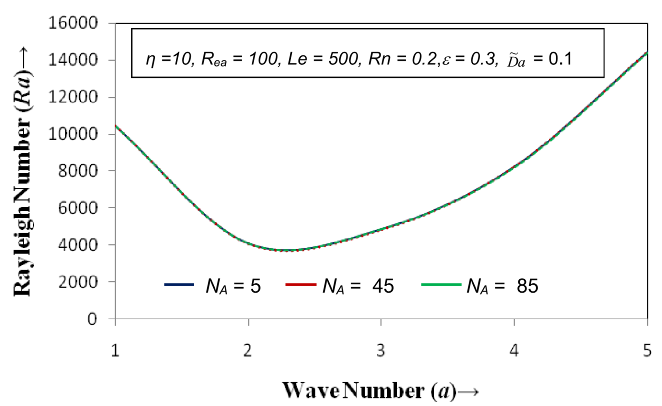

Figure 5. The variations of thermal Rayleigh number Ra with the wave number a for different values of the modified diffusivity ratio $N_A = 5$, $N_A = 45$ and $N_A = 85$.

The variations of thermal Rayleigh number Ra with the wave number a for three different values of the modified diffusivity ratio, namely, $NA = 5, 45, 85$ is plotted in Fig. 5 and it is found that thermal Rayleigh number decreases very slightly with the increase in modified diffusivity ratio implying thereby modified diffusivity ratio has slightly destabilizing effect on the system.

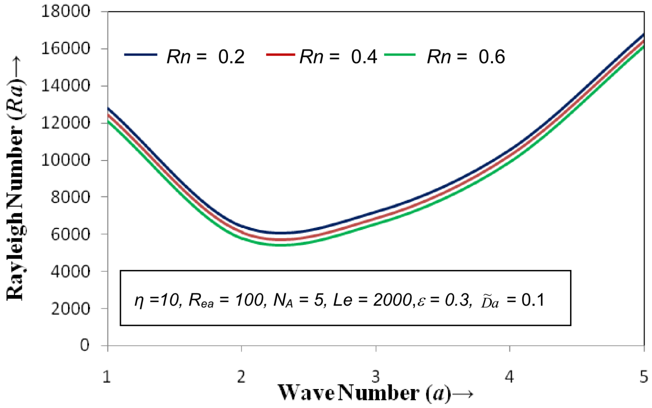

Figure 6. The variations of thermal Rayleigh number Ra with the wave number a for different values of the nanoparticle Rayleigh number $Rn = -0.2$, $Rn = 0.4$ and $Rn = 0.6$.

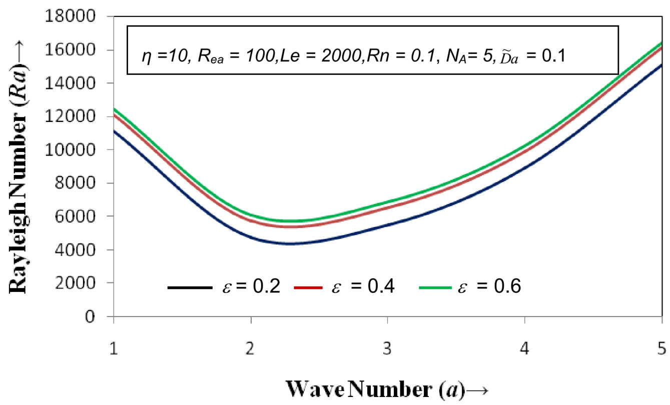

Figure 7. The variations of thermal Rayleigh number Ra with the wave number a for different values of the medium porosity $\epsilon = 0.2$, $\epsilon = 0.4$ and $\epsilon = 0.6$.

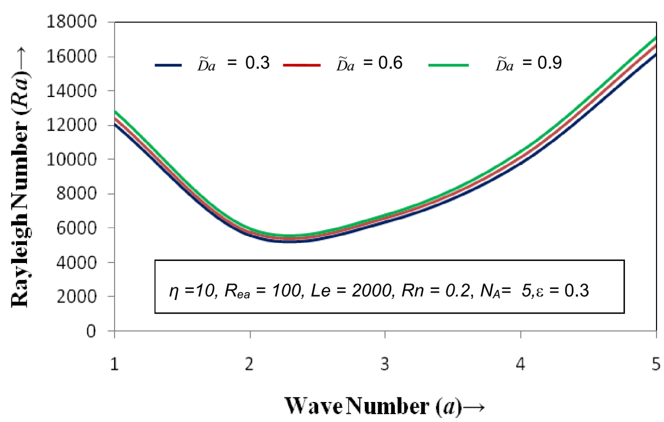

Figure 8. The variations of thermal Rayleigh number Ra with the wave number a for different values of the Brinkman-Darcy number $\tilde{D}a = 0.3$, $\tilde{D}a = 0.6$ and $\tilde{D}a = 0.9$.

In Fig. 6, the variations of thermal Rayleigh number Ra with the wave number a for three different values of the nanoparticle Rayleigh number, namely $Rn = 0.2$, 0.4, 0.6 which shows that thermal Rayleigh number increases with the decrease in nanoparticle Rayleigh number. Thus, nanoparticle Rayleigh number has destabilizing effect on the system. The variations of thermal Rayleigh number Ra with the wave number a for three different values of medium porosity, namely $\varepsilon = 0.2$, 0.4 and 0.6 is plotted in Fig. 7 and it is found that thermal Rayleigh number increases with the increase in medium porosity implying thereby medium porosity has stabilizing effect on the onset of stationary convection in a layer of couple-stress elastico-viscous nanofluid saturating a porous medium. In Fig. 8, the variations of thermal Rayleigh number Ra with the wave number a for three different values of the Brinkman-Darcy number $\tilde{D}a = 0.3$, $\tilde{D}a = 0.6$ and $\tilde{D}a = 0.9$ which shows that thermal Rayleigh number increases with the increase in nanoparticle Rayleigh number. Thus, Brinkman-Darcy number has stabilizing effect on the system.

VIII. CONCLUSIONS

The onset of electrohydrodynamic thermal instability in an elastico-viscous couple-stress nanofluid saturating a Darcy-Brinkman porous medium has been investigated analytically and numerically for free-free boundaries by applying a linear stability analysis. Dispersion relation for Rayleigh number accounting for the effect of Lewis number, couple-stress parameter, AC electric Rayleigh number, nanoparticle Rayleigh number, modified diffusivity ratio, medium porosity and Brinkman-Darcy number has been derived. Analytical expression for the occurrence of steady and oscillatory convection has been obtained for isothermal free-free boundaries. For the case of stationary convection, it is found that couple-stress parameter, médium porosity and Brinkman-Darcy number have stabilizing effect whereas AC electric Raleigh number, Lewis number and modified diffusivity ratio and nanoparticle Rayleigh number have destabilizing effect on the system. Oscillatory convection has been ruled out under the considered boundary conditions (see, Nield and Kuznetsov [11]).

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