DETERMINATION OF THE PLANCK CONSTANT THROUGH THE USE OF LEDS

DETERMINACIÓN DE LA CONSTANTE DE PLANCK MEDIANTE EL USO DE LEDS

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Plancks constant plays a fundamental role in the quantum mechanics theory. The numerical value of this constant can be experimentally obtained by means of the study of a wide variety of physical phenomena. The main goal of the present laboratory exercise is the determination of the numerical value of the Planck constant using LEDs by means of two different theoretical models and an Arduino automatized measurement. To achieve this, four different LEDs spectra were provided and their current-voltage curves were measured. Results from the two proposed methodologies are compared and discussed.

La constante de Planck ocupa un papel fundamental en la mecánica cuántica. La determinación experimental de su valor numérico puede hacerse a través del estudio de diversos fenómenos y dispositivos. Se propone un ejercicio experimental cuyo objetivo es determinar el valor numérico de la constante de Planck de una manera simple y poco costosa, a partir de la construcción de la curva corriente-voltaje y el conocimiento del espectro de emisión característico de un conjunto de diodos emisores de luz (LEDs), en una práctica de laboratorio automatizada con Arduino. Se utilizan dos metodologías diferentes, y se comparan y discuten los resultados de cada una.

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I. INTRODUCTION

Planck constant (denoted by h) is a physical constant of great importance within Quantum Mechanics theory. It is usually defined as the quantum of electromagnetic action. Its numerical value has been fixed as exact by the International Bureau of Weights and Measures (BIPM, after the french name) in 2018 [1] to the number:

$$h = 6.62607015 \cdot 10^{-34} Js. \quad (1)$$

Planck constant owes its name to Max Planck, who introduced it in 1901 within his empirical formula for the Black Body radiation problem. Later in 1905, Einstein used it as the proportionality constant that related the frequency of light quanta (*photons*) and the energy they carried out, in his model for the photoelectric effect. Then, the fundamental relation stands:

$$E = h\nu = \frac{hc}{\lambda}, \quad (2)$$

being E the energy carried by a photon, ν and λ its frequency and wavelength, c the velocity of light and h the Planck constant.

Einstein received the Nobel prize in 1921 "for his services to theoretical physics, and especially for his discovery of the law of the photoelectric effect" [2] and Planck in 1918 "for the services he rendered to the advancement of physics by his discovery of energy quanta" [3].

The numerical value of this constant can be experimentally obtained by means of the study of a wide variety of physical phenomena [4]. A very straightforward and easy to implement way of doing so is by means of Light Emission Diodes (LEDs). LEDs are made of semiconductor materials that emits radiation as a result of a change of energy states, usually of electrons, when the sample is excited by an external energy source that do not vary significantly its temperature [5]. The Laboratory exercise we propose aims the determination of the numerical value of the Planck constant using LEDs by means of two different theoretical models and an Arduino automatized measurement.

Since last quarter of the past century, it is been common to find some studies regarding this topic [6–8]. Even experimental setups were designed and commercialized [9]. In recent years there can be also found similar publications [10, 11, 15, 16], one of them [10] with an automatized experimental proposal. Most of the above mentioned [6, 8, 10, 15, 16] use a rather incorrect approximation, i.e., assume the GAP energy is equal to the energy of the knee voltage, issue noted in [17]. It is discussed later in this paper why this is not correct, even when numerical results seem precise. Some of the above cited publications [7, 11] uses the same methodology we present as first method, with very accurate results. A different methodology can be also found on the internet mainly in students published Laboratory Reports (as an example, two students reports and an exercise program can be found on the links [12–14]). However, the methodology is never successfully explained nor correctly derived the equations of work and with not so accurate results.

Our present proposal has the advantage of combining two different approaches, the first one focusing the student's attention on modeling the effect of light emission, while in the second one the focus is on the functioning of LEDs as electronic devices. Apart from that, a detailed explanation and accurate derivations of the expression for the working equations on both methods are presented. The uncertainty treatment is also discussed for both methodologies. Lastly, it is remarkable that our experimental set up is automatized by using the Arduino platform.

The present proposal is organized as follows. The first section presents a quick review on Solid State Physics and Semiconductor Theory, necessary to introduce the mathematical models. The next one introduces the mathematical models we use: we start with a simple model based on Einstein Photoelectric Effect equation, and then also propose a rather more complex based on Shockley diode model. Last two sections present the experimental set up and obtained results. Finally, conclusions are presented and discussed.

II. THEORETICAL BACKGROUND

II.1. Bands Theory

When classical ideas were applied to the study of solid state physics, the classical picture managed to explain the heat capacity in a certain range of temperatures for non conducting materials, and the electrical conduction of metals. However, a wide variety of experimental results in solids remained unexplained. One of them, the existence and behavior of semiconductors. Applying the principles of Quantum Theory to solids derives in the Bands Theory, which successfully explains the behavior of all solids [18].

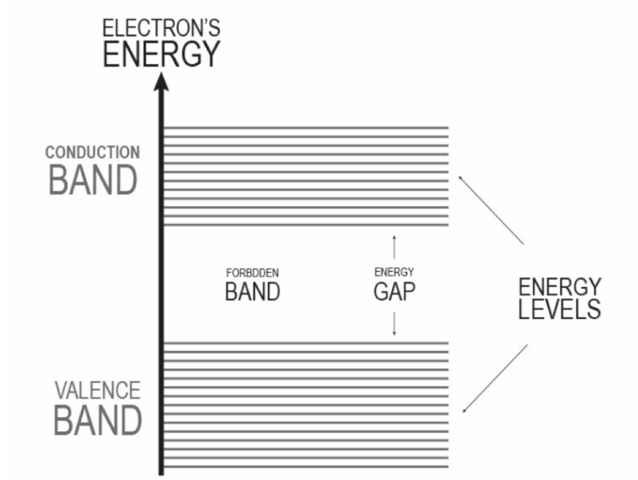

Figure 1. Schematic representation of energy bands within a solid.

Crystalline solids are a periodic arrangement of atoms which constitute a periodic potential for the movement of electrons. The solution of the Schrödinger equation to these periodic systems is a set of energy states clearly separated in bands of allowed and forbidden energies (Figure 1). In semiconductors and insulators electrons are occupying all states of the so called *valence band* and the next energetic region that they could occupy (*conduction band*) is separated by a forbidden region or bandgap (GAP). In general, the GAP of semiconductors is temperature dependent.

As all energy states are occupied in the valence band, the material can not conduct. In order to conduct electricity, the electrons must be able to "gain.energy, i.e., they must be in the conduction band where they have free energy states that they can occupy. Then, the semiconductor can conduct if it is possible to excite electrons from the valence to the conduction band.

II.2. Doping

In order to achieve greater efficiency in semiconductor materials, it is common to dope them with *impurities*. Impurities are any imperfection on the crystaline structure. They can be either the absence of an atom in a place of the structure, a displaced atom, or the substitution of an atom of the solid for another atom from a different chemical species. The latest is the one it is commonly used to dope semiconductors.

A *donor impurity* is an atom that possesses more valence electrons than the solid's constituent atoms and can "donate"these "excess.electrons to the conduction band more easily than the intrinsic atoms. Energetically, it is equivalent to say that the energy level of this "extra.electron is inside the GAP, near the conduction band. When a semiconductor material is doped with donor impurities, it means, is doped with electrons, it is called *n-type*.

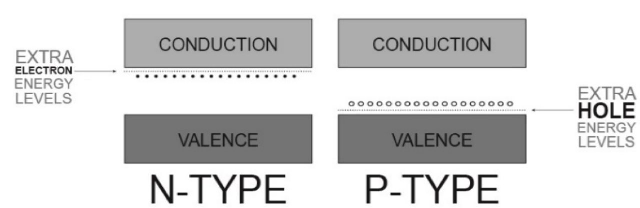

Figure 2. Schematic representation of the energy levels on typical *n-type* and *p-type* semiconductors.

An *acceptor impurity*, on the other hand, is an atom that possesses less electrons than constituent atoms. This kind of impurities creates "holes", vacancies of electrons in places where there should be. These holes are also in energetic states inside the GAP, near the valence band in a way electrons can occupy them easily, leaving mobile holes in the valence band. When a semiconductor material is doped with acceptor impurities, it means, is doped with holes, it is called *p-type*.

If the system has both kind of impurities, the material can decrease its energy by means of a recombination, for example when a free electron falls from the conduction band to a free hole in the valence band. Recombination process can be accompanied by photon emission. This is the kind of processes that produces luminescence within a LED.

II.3. *p-n junction*

The most effective way to inject charges into a semiconductor, in order to produce light, is by means of a *p-n junction* [19]. As it is stated by the name, on one side of the junction there is an *n-type* semiconductor while on the other side there is a *p-type*, i.e., in one side of the semiconductor there are excess electrons near the conduction band while on the other side holes predominate near the valence band. If the system is excited (with room temperature, for example), the excess electrons in the *n-type* zone jump to the conduction band, and the holes of the *p-type* zone are filled due to electrons that jump from the valence band leaving free holes behind. Therefore, there is an excess of electrons within the conduction band in one side and an excess of mobile holes within the valence band in the other side.

At the moment of making the junction, the system tries to reach equilibrium between electrons and holes in both bands, and electrons start moving from the n to the p zone by diffusion. This process creates a negative net charge in the p zone and a positive net charge in the n zone, because the diffusion process has effectively created a charge unbalance in both sides of the junction. An electric field is then established and it equilibrates the diffusion movement. The region with unbalanced charge is called *depletion zone* or *space charge region*.

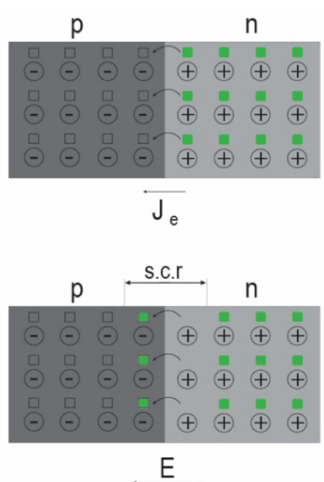

Figure 3. Schematic representation of a p-n junction and the creation of the *depletion zone*.

If the semiconductor is supposed to conduct electric current (or illuminate, in the case of LEDs), it must be forward-biased. It means it must be connected in a way external voltage is opposed to the internal voltage in the depletion zone (this is, the p zone should be wired to positive). Then, if the voltage applied is greater than the internal one, electrons will keep moving through the depletion zone from the n to the p zone, conducting electricity. The so called knee voltage is the minimum voltage needed for the LED to conduct. Therefore, the energy of knee voltage equals the energy of this electric field, which is not the same and can not be identified with the GAP energy, although they are indeed related.

If recombination of an electron and a hole occurs within the depletion zone, then energy is emitted. This energy loss can be radiative, in which case a photon would be emitted, or non radiative, in which case phonos would be produced. The radiated energy (if the first occurs) and therefore the frequency of the produced light, dependes on the GAP, it means, on the material.

III. MODELS

III.1. *Method I*

To make a numerical analysis of the light production phenomena within LEDs, we can start by using a model similar to the one used by Einstein for the Photoelectric Effect (FE) [20, 21]. Although these are two different phenomena, the logic followed by Einstein for the FE was based on the Conservation of Energy, which is a universal law. Then, the equation for the light emission within diodes would be:

$$K_{min} = E - W_0. \tag{3}$$

In our problem, we speak of a minimum kinetic energy, i.e., the minimum energy amount that is necessary for an electron to penetrate the depletion zone and that is provided by the external voltage. This can be expressed as:

$$K_{min} = eV_0, \tag{4}$$

where V_0 is the so called knee point voltage, i.e., the voltage for which current starts flowing trough the diode. The latest can be obtained by simple inspection on the IV curve, although there exits more refined methods, as the one that fits the lineal part of the curve to a line and takes the intersection with the abscissa as knee voltage.

This energy carried by electrons can be turned into radiation energy (E) as it is expressed by Eq. 3 where is also taken into account possible energy loss due to a wide variety of non radiative processes that can occur within the depletion zone (W_0). For simplicity, we asume this lost energy is the same for every LED. This term corrects the invalid assumption widely found in literature that $eV_0 = E$.

If we substitute E as in Eq. 2 and K_{min} as in Eq. 4, we obtain the linear model:

$$V_0 = \frac{hc}{e\lambda} - \frac{W_0}{e}, \tag{5}$$

in which, from the slope of a lineal fit to a set of values for the knee voltage and the wavelength of different LEDs, the numerical value of the Planck's constant can be computed.

III.2. *Method II*

A more detailed model can be constructed looking into the mechanisms of the diode itself. As they are complex devices, there are various mathematical models that tries to approximate the behavior of their IV curves. One of the most commonly used is the Shockley diode model [19]. This relates the current and voltage flowing through the diode as:

$$I(V) = I_S\left(e^{\frac{eV}{\eta k_B T}} - 1\right), \tag{6}$$

where

I_S is the reverse saturation current,

e is the electronic charge,

η is the so called non-ideality factor, that is set to 1 if an ideal diode is under consideration, but for real diodes it is often a number higher than 1,

k_B is the Boltzmann constant,

T is the temperature, and

V, I are the voltage and current flowing through the diode.

However, even at rather low voltages and room temperature it is common to assume $V \gg \frac{k_B T}{e}$, where $\frac{k_B T}{e}$ is commonly named as thermal voltage. This approximation should not affect the value of the Planck constant, due to the very small value of the fraction (on the order of $10^{-2}V$ for room temperature) compared to the measured voltages (on the order of Volts). An inspection to Eq. 6 suggests that due to the presence of the non-ideality factor η within the exponential, this assumption should be done carefully, depending on the order of this parameter. However in worst case scenario (for our measurements, one of them is slightly higher than 10) the measured voltage remains at least an order higher than the thermal voltage. This clearly affects the expected result for the constant, even when only one of the LEDs presents the problem, because there are no enough LEDs to statistically corrects the deviation this one will cause.

If this above approximation is done, the previous relationship becomes:

$$I(V) = I_S e^{\frac{eV}{\eta k_B T}}. \tag{7}$$

The reverse saturation current is sometimes expressed in terms of the energy band gap as:

$$I_S \propto e^{-\frac{E}{\eta k_B T}}, \tag{8}$$

where E is the GAP energy. It is remarkable that there are lots of approximation for the saturation current as a function of the GAP energy and this is the simplest one. Some more accurate descriptions also include a temperature dependence factor instead of a simple proportionality factor. The substitution of expression 8 into Eq. 7 finally gives:

$$I(V) = Ae^{\frac{eV-E}{\eta k_B T}}, \tag{9}$$

with A being a proportionality constant.

If we take logarithm in both members, we obtain a lineal relation:

$$\ln\{I(V)\} = mV + b, \tag{10}$$

where

m is the slope of a line and is equal to $\frac{e}{\eta k_B T}$, and

b is the curve intercept with the vertical axis and is equal to $\ln\{A\} - \frac{E}{\eta k_B T}$.

In Equation 10 should be noted that the logarithm has been taken to the values of $I(A)$. There seems to be a dimensional error in this action, but this is not the case, because the term $\ln\{A\}$ in the intercept corrects the dimensionality of the equation.

A careful inspection on the expressions for the slope and intercept unravel their inner relation:

$$-\frac{be}{m} = h\nu - \ln\{A\}\eta k_B T, \tag{11}$$

where we have written $E = h\nu$.

Then, a lineal fit to measurements of $\ln\{I\}$ vs. V leads us to a new characterization of the LEDs, i.e, the values of m and b (Eq. 10). It must be taken care that measurements are done within the validity region of Eq. 6, the exponential grow part. Then if the characteristic m, b and ν for each LED are again plotted, the slope of a lineal fit to this set of values, as in Eq. 11, is exactly the Planck constant.

IV. EXPERIMENTAL SETUP

The spectra of the different LEDs were obtained in a Laboratory of the Physics Faculty using traditional methods. They are given to the students for them to find the wavelength and uncertainty of LEDs.

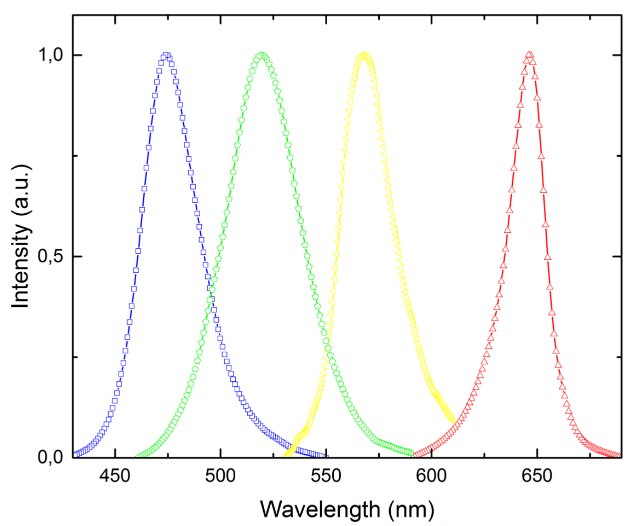

Figure 4. Measured LEDs spectra. The colors on the curves corresponds to the colors of each LED.

The wavelength of the LEDs is taken from the maximum of the spectra. The band width of the spectra was taken as the associated uncertainty for the wavelength of each LED. Table IV shows the different wavelengths obtained for our four LEDs with their uncertainties and Figure 4 shows the measured spectra.

Table 1. Wavelength value for each one of the LEDs in our experiment.

LED	Wavelength (nm)
Yellow	$567,80 \pm 24,14$
Green	$520,27 \pm 36,27$
Red	$646,40 \pm 18,61$
Blue	$474,29 \pm 27,33$

For the experimental setup the following elements are needed:

- LEDs of different colors (green, red, yellow and blue)
- Arduino Genuino UNO
- various 2 kΩ resistances
- a potentiometer
- an LCD 16×2 screen
- a protoboard
- conection cables
- remote control

IV.1. *Experimental determination of the IV curve*

For the measurement of the IV curve, a circuit like the one represented in the Figure 5 should be assembled. A program for the control of the measurement system has been implemented on Arduino. With a remote control it is possible to select the LED whose IV curve is going to be measured. With a lineal potentiometer the voltage applied to the terminals of the LED is varied. The values of voltage and current are shown in the LCD screen. It is important to remark once more that LEDs should be forward biased. The procedure is repeated 20 times for every LED.

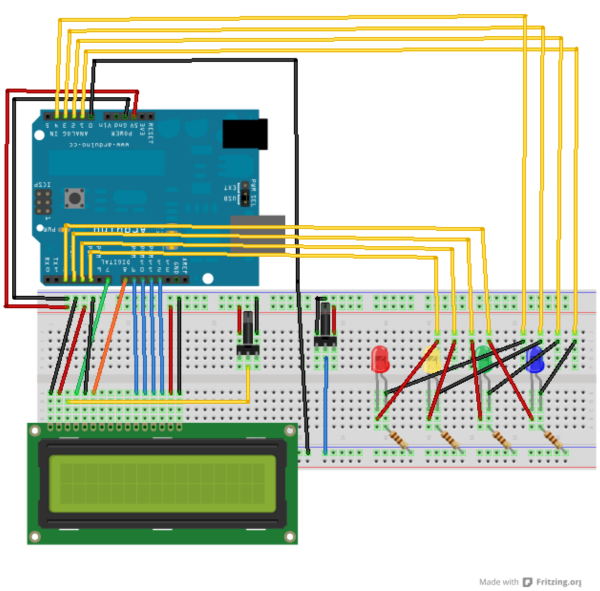

Figure 5. Circuit for the IV characterization of LEDs.

IV.2. *A remark on uncertainties sources*

The proposed experimental setup use an analog digital converter (ADC) from an Arduino Genuino Uno [22]. This is a successive approximation ADC, and those kind of converters are usually very precise. The least variation that the converter is able to detect is determined by the relation:

$$V_{min} = \frac{V}{2^N}, \tag{12}$$

where $V = 5V$ is the supply voltage and $N = 10$ *bits* is the resolution of the ADC. This implies that the minimum voltage variation that could be detected by the ADC is of $V_{min} = 4.88 \cdot 10^{-3} V$.

On the other hand, the LCD screen has a resolution of $V_{min}^{screen} = 1 \cdot 10^{-2} V$. This means that the screen is the main source of uncertainties in the measurement of voltage.

For the measurement of current, the voltage through a resistance of known value is measured. The value of the resistance is taken as exact. Therefore, it does not contributes to the uncertainties.

Taking the above considerations into account, we can establish that the uncertainties due to the measurement instrumenta is constant throughout the experiment and with an extremely low value, not comparable with the uncertainties that arise from the fitting process, and therefore will not be taken into account.

As it was mentioned before, the uncertainties related to the wavelength of the different LEDs were taken as the bandwidth of the emission spectra. This way the uncertainty is somehow overestimated, but it is important to take into account the fact that LEDs are not entirely monochromatic devices. With this, the uncertainty on the frequency is simply calculated by standard propagation of uncertainties.

For the knee voltage, the estimation of the uncertainties is rather complex. This is a factor that is highly dependent on the choices of the student, when inspecting the IV curve. However, if a line is fitted to the lineal part of the curve and the intercept is taken as the knee voltage, then the uncertainty of the intercept can be used to approximate the uncertainty of the knee voltage.

For the first method, the uncertainty on the value of the constant is the uncertainty associated with the slope of the fitted line, which was computed taking into account the measured uncertainties for the frequencies and the knee voltages.

As for the second method, the uncertainties needs to be propagated through all the steps of the methodology. The linearization of the IV curve gives the values of m and b for every LED, as in Eq. 10, with the associated uncertainties from the fitting process. Then the combination $-be/m$ is calculated and uncertainty propagated from the ones of b and m. That way, the uncertainties of the $-be/m$ factors as well as the frequencies, are used in the fitting process to compute the slope and its uncertainty, according to Eq. 11.

V. RESULTS

Figure 6 shows the experimental values for knee voltage and wavelengths as well as the lineal fit, with its parameters and related uncertainties. As it is stated by Eq. 5, conveniently selecting the quantities to plot (V_0 vs. $\frac{c}{e\lambda}$), the slope of the line is simply the Planck constant. Then,

$$h = (6.41 \pm 0.16) \cdot 10^{-34} Js, \qquad (13)$$

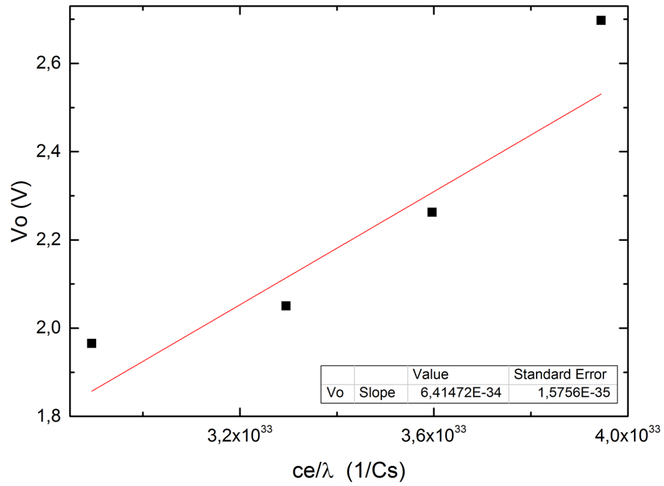

Figure 6. Lineal fit of knee voltage vs. $\frac{c}{e\lambda}$.

As for the second methodology proposed, Figure 8 shows the lineal fits to the IV curves for each of the measured LEDs. Table V shows the parameters m and b corresponding to the LEDs as well as the value of the combination $-be/m$ with its associated uncertainty.

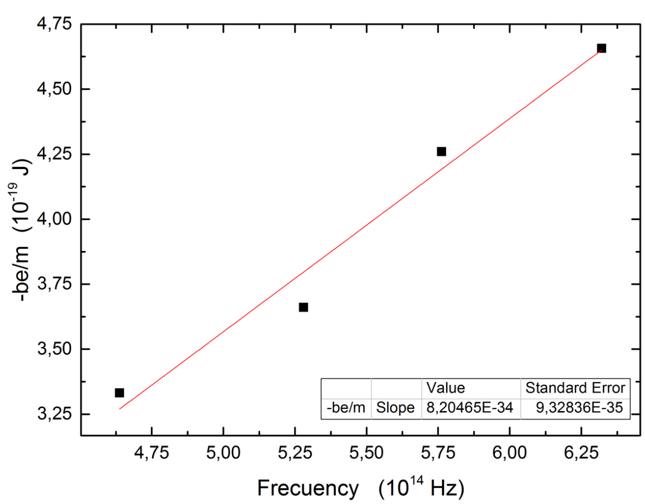

Figure 7. Lineal fit of $-\frac{be}{m}$ vs. ν according to Eq. 11.

Table 2. Intercept and slope values for the linear fit to the logarithmic IV curve, with the combination $-be/m$ and its associated uncertainty.

LED	b	m	$-be/m(10^{-19})$	$\Delta(10^{-20})$
Yellow	$-18,18$	$7,95$	3.66	$2,52$
Green	$-31,70$	11.92	4.26	$1,98$
Red	$-29,70$	$14,28$	3.33	1.39
Blue	$-34,87$	11.99	4.65	2.14

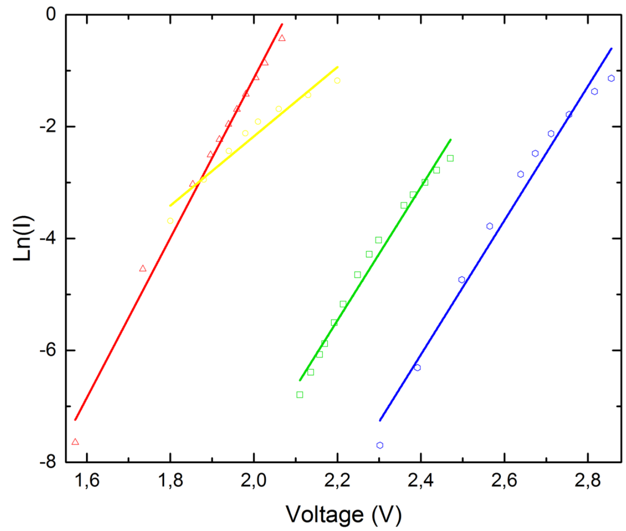

Figure 8. Lineal fits to the IV curves for each of the measured LEDs.

It is easily observed that the behaviour of the Yellow LED is different from the others, specially for the slope of the line. The only variation for the LEDs is the η parameter (temperature is considered constant through the experiment), which means the value of this parameter is considerably higher (an order of magnitude) with respect to the other three. However, as it was discussed previously, the measured voltages remains an order higher than the thermal voltage, so the Eq. 7 stands.

Figure 7 shows the experimental values and the lineal fit following Eq. 11, with the slope and related uncertainty. Then,

$$h = (8,20 \pm 0.93) \cdot 10^{-34} Js, \qquad (14)$$

With none of the two methods is possible to obtain the value of the constant within the experimental uncertainty, however the results are close to it. It is remarkable that with the first method, despite its simplicity, it is possible to obtain a very good approximation to the numerical value of the Planck constant and with a reasonably low uncertainty, being not only a simple but also a precise method. The second one is less precise, in the sense that uncertainties are greater, which was an expected result due to the propagation that needs to be done, and also in the sense that the value obtained for the constant is further from the one reported in literature than the value obtained by the first method, also expected due to the influence that might be caused by the high η parameter on the Yellow LED.

VI. CONCLUSIONS

Two methods have been proposed that allow the student to compute the numerical value of the Planck constant with a reasonable good approximation to the one reported in literature. With the first method, widely found in literature, the student is able to compute the value of a universal constant by means of a simple mathematical expression and general physical considerations on the rather complex phenomena that takes place within a LED. With the second one, not so commonly found in literature and rather successfully explained, the student deals with a more detailed overview of the functioning of LEDs as electronic devices. In a simple way the student is also put in contact with the Arduino platform, widely used in experimental measurements in the scientific research. The use of the Arduino also implies that the Laboratory Exercise can be implemented in any Physics Laboratory and it is an almost costless Exercise.

VII. ACKNOWLEDGMENTS

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