

WHAT ARE ELEMENTARY PARTICLES? FROM DARK ENERGY TO QUANTUM FIELD EXCITATIONS

¿QUÉ SON LAS PARTÍCULAS ELEMENTALES? DE LA MATERIA OSCURA A LAS EXCITACIONES DE CAMPOS CUÁNTICOS

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We describe the very nature of the elementary particles, which our (visible) Universe consists of. We underline that they are not point-like, and we depict their ways of interacting. We also address puzzles that occur even in the absence of particles, in the vacuum.

Describimos la naturaleza misma de las partículas elementales, las cuales componen nuestro Universo (visible). Subrayamos que no son puntuales, y representamos sus formas de interactuar. Abordamos también acertijos que ocurren incluso en ausencia de partículas, en el vacío.

PACS: Theory of fields and particles (teoría de campos y partículas), 11.40.-q; supersymmetry (supersimetría) 11.30.Pb; cosmological constant (constante cosmológica), 98.80.Es; dark energy (energía oscura) 95.36.+x.

This article is meant to be qualitative and very simple; slightly technical remarks are added as footnotes and as an appendix.

We do not go through the list of all these known elementary particles (let alone the hypothetical ones); their table can easily be found in many places. Instead we want to address a question, which is seldomly discussed in popular science: *what kind of objects are elementary particles?* Amazingly, even in the physics literature this issue is treated as an orphan: there are numerous textbooks devoted to particle physics, which hardly clarify what these objects actually are.²

I. BASIC BUILDING BLOCKS OF MATTER

If we break up any kind of matter into smaller and smaller pieces, we ultimately reach a point of basic building blocks, which are not divisible anymore: Democritus would have called them “atoms”, but for us these are the *elementary particles*. So far 25 types of elementary particles have been experimentally confirmed; the entire visible Universe consists of them.¹ They are incorporated in the Standard Model of Particle Physics, see *e.g.* Ref. [1]; prominent examples are the *electron* and the *photon* (the particle of light). As if this wasn't enough, the literature of theoretical physics is replete with speculations about additional types of elementary particles.

The common intuitive picture, which is based on our perception of macroscopic objects, views them as “tiny balls”. We are going to point out that this picture is erroneous, and that they are not “point-like objects” either. The latter (mysterious) claim is wide-spread, but that doesn't make it correct.



Figure 1. We can easily decompose a Lego house into its building blocks. If we keep on decomposing matter down to its most fundamental building blocks, we end up with elementary particles.

II. QUANTUM FIELD THEORY

The mathematical formalism that successfully describes elementary particles is called *Quantum Field Theory*. In the course of the 20th century it has replaced Quantum Mechanics.³

In order to symbolically interpret the term *field theory*, we could view the entire Universe as the “meadow-land”, endowed with some kind of “grass blades” everywhere. The latter take an abstract mathematical form: certain variables are permanently attached to each point in space. A “field” is one type of such variables. Each variable, at a given point, can change its value as a function of time, we could say that

¹This includes quarks and gluons, although they cannot be directly detected, as well as leptons, electroweak gauge bosons and the Higgs particle. We refer to the *visible* part of the Universe in order to exclude Dark Matter and Dark Energy; the latter will be addressed in Appendix A. Gravitation belongs to our daily experience, but the particle, which is held responsible for it — the graviton — has not been observed.

²The text is written in terms of “particles”, and the formulae in terms of “fields”, but the question of how these terms are related is by no means as clear as it is supposed to be.

³In contrast to Quantum Mechanics and classical physics, Quantum Field Theory successfully incorporates the concepts of both quantum physics and Special Relativity. (A complete unification of quantum physics with General Relativity has not been achieved.)

it “vibrates” or “oscillates”. In the following we are going to refer to an “oscillator”, a term which can be reasonably well justified, see *e.g.* Ref. [2].



Figure 2. A field in the usual sense. In physical field theories, the meadow-land is converted into space, and the grass blades into abstract mathematical variables, which we denote as “oscillators”.

It is always risky to invoke a simplified picture for illustration purpose, but we do so nevertheless. There is a rough analogy with sound in the air: let us assume the absence of wind in some volume, so the molecules of the air have (essentially) static equilibrium positions, but their vibrations around them represent sound. This bears some similarity with field theory, which we can further strengthen by referring to sound in a crystalline solid, where ions oscillate around their grid sites, with displacements analogous to a field variable. We repeat, however, that actual field variables are abstract mathematical quantities.

So let us denote a field variable at one point as an *oscillator*. It can be in its *ground state*, where its energy E_0 is minimal (in quantum physics we have $E_0 > 0$),⁴ or in an *excited state* with a higher energy $E > E_0$. As we mentioned before, the state of any of these oscillators (which fill the entire space) is time-dependent.

III. VACUUM AND PARTICLES

At this point, we only consider one field, *i.e.* one type of oscillator.

Let us assume all the oscillators in some volume to be in their ground state. We denote this as the *vacuum*, which means that the field takes its state of minimal energy in this volume.⁵ We would say “nothing is there”, although the oscillators are actually there, but none of them vibrates with any excitation

⁴We are referring to the simple case of bosonic fields, like the photon field or the Higgs field. There is another class of particles called *fermions*, which include the electron, and which emerge from fields with $E_0 < 0$, see Appendix A.

⁵For simplicity we refer to a free “neutral scalar field”, where the field is real valued. Other fields involve different types of variables, and when the fields are self-coupled or coupled to other fields, *i.e.* not free, then even the vacuum often takes a complicated structure.

⁶Mathematically this is achieved by applying a so-called creation operator to the vacuum state. It may have a fixed momentum, but it is not restricted to one spatial point.

⁷In mathematical terms, there are field derivatives contributing to the energy, hence a discontinuous spike — or even just a very sharp peak — is not suitable for an excitation with minimal energy.

energy $E > E_0$. It is tempting to interpret the ground state energy throughout the Universe as *Dark Energy*; this leads, however, to a dreadful puzzle, which we address in Appendix A.



Figure 3. A set of classical oscillators; quantum oscillators are hard to depict.

Now let us insert a single particle into this volume, say a particle at rest (with respect to the volume). This requires a massive particle, like the electron, and we denote its mass as $m > 0$.⁶

What does this mean for the field under consideration? It will be excited, such that its total energy inside this volume takes its minimal value above the vacuum energy. In quantum physics, this minimal excitation corresponds to a finite energy gap ΔE ; the energy cannot be increased continuously above the vacuum energy. We also know that this energy gap, *i.e.* the particle’s rest energy, is related to the particle mass by a famous formula, $\Delta E = mc^2$ (where c is the speed of light in vacuum).

If these oscillators were all independent, the obvious way to arrange for a minimal excitation would be to excite just one of them to the first energy level and leave all the rest in their ground state. However, this is not how it works: the oscillators are *coupled* to their nearby neighbours, hence exciting one of them inevitably affects its vicinity (cf. footnote 6).⁷

Instead we obtain a smooth excitation energy profile, which we assume to have a maximum in its centre. It turns out that it decays exponentially with the distance from this centre, where (in the free case) the range of the decay is proportional to the inverse particle mass, $\text{range} \propto 1/m$. This range coincides with the Compton wavelength [3].

For a massless particle, like the photon, this decay is slower: here it only follows some negative power of the distance from the particle centre, but not an exponential decay [3]. In either case, with $m = 0$ or $m > 0$, we see that particles do have

an extent, they are *not* point-like objects.⁸ Such profiles are symbolically illustrated in Figure 4.

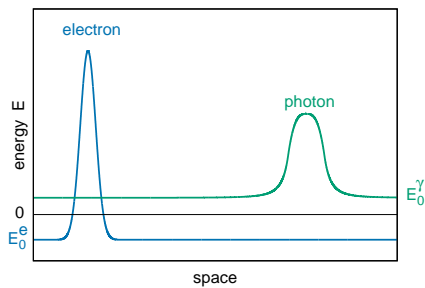


Figure 4. A symbolic illustration of the energy profiles of an electron and a photon, as examples for a massive and a massless elementary particle (E_0^e and E_0^γ are the corresponding ground state energies).

In the framework of this qualitative note, we stay with the observation that an elementary particles is not a point particle, only marginally touching upon the somewhat distracting question what its size amounts to. We could, for instance, define the size of a free electron with the exponential profile decay that we just mentioned, or we could refer to the so-called *form factor*, which is observed in electron scattering experiments. Mathematically, the electron form factor is the spatial Fourier transform of its electric charge distribution.⁹

Finally one might be tempted to refer to the resolution of a particle detector, for instance to the size of a pixel in a raster image. However, no matter how small the pixels are, a single photon will always be detected in only one of them, so we cannot determine a photon size in this way. This does not imply that the photon has zero extent: its profile “collapses” into just one pixel at the moment of the detection.¹⁰

IV. PARTICLES IN MOTION

We have seen that elementary particles can be understood as small regions, or zones, where a (quantum) field is excited. These zones, *i.e.* the particles, can *move* (say, relatively to each other). A descriptive picture is that the excited oscillators lose energy, and eventually drop down to the ground state, transferring their energy to nearby other oscillators, and so on. In this way, the particle centre moves, and with it all the excitation zone.

This picture is reminiscent of an ordinary (though Lorentz invariant) wave,¹¹ but one issue must be stressed: if the

⁸This is correctly emphasised *e.g.* in Refs. [4,5].

⁹Ref. [6] explains how to compute this radius in Quantum Electrodynamics.

¹⁰We don't know how exactly this happens, but it does happen. It is analogous to the notorious “collapse of a wave function” in Quantum Mechanics.

¹¹Frank Wilczek occasionally denotes a quantised field excitation as a “*wavicle*” [7], which makes sense, but the established term is “particle”.

¹²Even the “empty” cosmic space is packed with about 411 photons and 366 neutrinos per cm^3 , see *e.g.* Ref. [8]. These are the most abundant types of particles.

¹³Capturing the dynamics of particle creation and annihilation is an essential achievement of Quantum Field Theory, in contrast to traditional Quantum Mechanics. This property is intimately related to the statement in footnote 3, see *e.g.* Ref. [9].

¹⁴For further reading about particle scattering and decay, we recommend Ref. [14].

¹⁵1 GeV = 10^9 eV (electronvolt) is a unit of energy, $1 \text{ eV} \approx 1.6 \cdot 10^{-19} \text{ J}$. For comparison, the electron and the proton have a masses of $0.000511 \text{ GeV}/c^2$ and $0.938 \text{ GeV}/c^2$, respectively.

¹⁶The question of how forces really emerge in Quantum Field Theory is another subject, which is not well covered in the literature, despite its importance. A sound pedagogical explanation is given in Ref. [15].

terms “particle” and “wave” are understood from a classical (not quantum) perspective, *i.e.* as concepts, which match our macroscopic perception, then neither of these two terms describes a quantum particle appropriately. For the lack of quantum terminology in colloquial language, we are using those terms, which have led to never-ending confusion.

V. MULTIPLE FIELDS AND PARTICLE INTERACTIONS

We now go beyond the consideration of just one particle type. From Section 1 we know that there are at least 25 different fields, *i.e.* at least 25 types of “oscillators” being present at each point of the Universe, at any time. Some of these fields are self-coupled or *coupled to each other*, this is specified in the Standard Model. Each field has its zones of excitations, corresponding to a huge number particles in the Universe,¹² which may all be in motion.

If such excitation zones of coupled fields *overlap*, particles can directly *interact* and we talk about *particle collision*. The rule that instantaneous interactions do not happen over a distance is known as *locality*. When a collision takes place, energy can be transferred from one field to another. This dynamics can also affect fields, that have not been excited before in this zone, so additional types of particles can be generated.¹³

Obviously the creation of a heavy particle requires a lot of energy. Therefore some laboratories, like CERN, accelerate massive particles almost to the speed of light and arrange for collisions at extremely high energy, in order to investigate whether this creates heavy particles, which have not yet been observed — possibly one of the hypothetical particles that theoretical physicists speculate about. The famous Higgs particle, which had been predicted since 1964 [10], was finally observed in this manner at CERN in 2011/12 [11,12] (a popular science description is given in Ref. [13]).

Once a rather heavy particle is generated, it tends to decay very rapidly (unless there is a conservation law preventing this). Then it transfers its energy to other fields, thus creating several lighter particles, which corresponds to a process of energy diffusion.¹⁴ For instance the Higgs particle, with a mass of $125 \text{ GeV}/c^2$ — the second-most heavy elementary particle that we know¹⁵ — has a lifetime is only about 10^{-22} sec.

VI. THE INTERWOVEN UNIVERSE

Another manifestation of particle interactions are attractive or repulsive *forces*.¹⁶ In contrast to Newton's formulation of

gravity, such forces do not act instantaneously at a distance — field theory is consistent with the principle of locality, as we pointed out before. For instance, the Coulomb force between two electrons is indirect: each electron affects at its location the photon field (they are coupled by the electric charge of the electron). When the electrons move closer, the energy of the coupled field system is enhanced, which implies a repulsive force¹⁷ (in jargon, this is due to the “exchange of virtual photons”).

We know that the electromagnetic force can also be attractive, for instance between the electron and its anti-particle, the positron, which carries positive electric charge.¹⁸ Other types of intermediate fields (so-called “gluon fields”) give rise to the “strong interaction”, and in particular to strong attraction, which (in suitable circumstances) outweighs electromagnetic repulsion. Due to such forces, some elementary particles form *bound states*, which are composite particles. The best known examples are the proton and the neutron, which build the atomic nuclei. Together with the electrons we obtain atoms, which can be further clustered to molecules. Following this sequence of composition, we reconstruct the larger structures of matter, which we have decomposed in the very beginning of this article.

However, from a fundamental perspective, such composite objects, and the entire visible Universe, ultimately consist of the elementary particles that we have described before. This is the particle physicist’s view of the world: the interactions among these particles imply a very complicated, interwoven dynamics, following probabilistic rules, which we investigate.

At last, returning to the simplistic analogy with sound, we could — figuratively speaking — call this interwoven dynamics the “cosmic symphony”, or “cosmic salsa concert”, whatever you prefer.¹⁹

A. THE MYSTERY OF DARK ENERGY

The consideration in the second paragraph in Section 3 suggests the presence of a non-zero energy density ρ_{E_0} throughout the Universe. Actually ρ_{E_0} even seems to diverge: for a given field there is not just one oscillator at each space-point, but there is an oscillator for each possible frequency.

For the free, neutral, massless scalar field, one oscillator

¹⁷As a simple classical picture: if static electrons are next to each other, an electric field $\vec{E}(\vec{x})$ emerges, which is almost doubled compared to a single electron. Thus the field energy $\propto \int d^3x \vec{E}(\vec{x})^2$ decreases when the electrons move apart.

¹⁸When they are close, they form a small, electrically neutral compound, which hardly affects the photon field, hence their proximity is energetically favoured.

¹⁹Symbolically, this seems to bear some similarity with the concept of the “harmony of spheres and numbers” or “musica universalis”, a philosophical concept which was supposedly advocated by the Pythagoreans over 2500 years ago [16]. Unlike them, however, we do not focus on the motion of celestial bodies, and we discard mystical interpretations.

²⁰Considering well-defined differences of physical quantities, while putting aside, or isolating, a divergent additive constant, is the basic idea of *renormalisation*.

²¹Albert Einstein introduced such a constant in his formulation of General Relativity, in order to construct a static Universe [17]. Once Edwin Hubble and others convinced him that the Universe is rapidly expanding, and it turned out that his static solution would be unstable, he dismissed this constant and accepted the expanding solutions to his theory by Alexander Friedmann and Georges Lemaître [18–20] (see Ref. [21] for a historic account). Then, for almost seven decades, the Cosmological Constant was assumed to vanish, which implies a decelerated expansion. Einstein’s original value was positive, just at the (unstable) transition between a decelerated and an accelerated expansion. Today a somewhat larger Cosmological Constant is appreciated as the most obvious explanation for the observed accelerated expansion. If we understand General Relativity as a low-energy effective theory, then the presence of this constant is natural.

contributes the ground state energy $E_0 = \frac{1}{2}\hbar\omega$, where $\omega = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2}$, ω_i being the angular frequencies in different directions (in 3 spatial dimensions), and \hbar is Planck’s constant. If we compute ρ_{E_0} for the photon field by integrating $\int d^3\omega \hbar\omega$, we obviously run into a divergence (a factor 2 accounts for the two photon polarisation states).

It seems natural to introduce an *ultraviolet cutoff*, say at the Planck scale $E_{\text{Planck}} = \sqrt{\hbar c^5/G} \approx 1.2 \cdot 10^{28}$ eV, where G is Newton’s gravitational constant. This restricts the integral to $4\pi \int_0^{E_{\text{Planck}}} d\omega \hbar\omega^3$. Taking into account the Fourier normalisation factor $1/(2\pi)^3$, we obtain — due to the ground state energy of the free photon field — the energy density

$$\rho_{E_0} \approx \frac{E_{\text{Planck}}^4}{8\pi^2(\hbar c)^3}.$$

In usual studies of quantum physics, such an additive constant in the energy is irrelevant, since we are only concerned with energy *differences*.²⁰ If we add a constant term to the potential of some system, then this does neither affect the (field) equations of motion, nor the expectation values in Quantum Field Theory. However, this changes when we include *gravity*: note that a constant energy density ρ cannot be added to the potential in the simple space-time integrated form $\propto \int dt d^3x \rho$ — that term is not covariant. Instead the space-time metric must be involved, which is therefore affected by the quantity ρ (in General Relativity even the metric is dynamical).

As a consequence, such a constant leads to a prominent physical effect, namely (if it is sufficiently large) the *accelerated expansion of the Universe*. The driving energy density is denoted as *Dark Energy*, which can be interpreted (up to a constant factor) as Einstein’s *Cosmological Constant*.²¹ It corresponds to a negative pressure, which is occasionally denoted as “gravitational repulsion”.

So at the *qualitative* level, it seems that we have found a neat explanation for this accelerated expansion, which was discovered at the very end of the 20th century [22, 23]. The 2011 Physics Nobel Prize was awarded for this observation.

However, our enthusiasm comes to an abrupt end when we proceed to the *quantitative* level: the observation of Refs. [22, 23], which is based on the distance and redshift of a set of type Ia supernovae, corresponds to a vacuum energy density

of about $\rho_{\text{obs}} \approx (0.002 \text{ eV})^4/(\hbar c)^3$. Now we are stunned by a tremendous discrepancy from the theoretical estimate ρ_{E_0} ,

$$\frac{\rho_{E_0}}{\rho_{\text{obs}}} \approx 10^{121}.$$

This is perhaps the worst discrepancy between a theoretical prediction and an observed value in the history of science.

E_{Planck} is the most natural energy cutoff, but a conceivable alternative might be $E_{\text{GUT}} \approx 10^{25} \text{ eV}$, the energy where the three gauge couplings of the Standard Model are predicted to converge to the same strength. That reduces the above discrepancy to $\rho_{E_0, \text{GUT}}/\rho_{\text{obs}} \approx 10^{109}$, which is no salvation. If we really wanted to maintain the previous derivation, we had to lower the energy cutoff to $\approx 0.006 \text{ eV}$ instead of E_{Planck} , but such a ridiculously low cutoff does not make any sense whatsoever: even the rest energy of an electron is almost 10^8 times higher.²²

To make it even worse, there is in addition the *coincidence problem*: by default, the Cosmological Constant, and therefore the Dark Energy density, is assumed to be really *constant* during the evolution of the Universe, whereas the matter density decreases due to its expansion; it has decreased by many orders of magnitude since the time of the Early Universe, see Figure 5. At that time matter density²³ dominated over vacuum energy, and in the far future it will be the other way around. It so happens that just in our time the matter density — which is dominated to about 85% by Dark Matter (which does not interact with the photon field) — is of the same magnitude as the Dark Energy density (they only differ by about a factor of 3). Is it by accident that we just have the privilege to witness this transition, or does this “coincidence” require an explanation?²⁴ (Some people try to argue with the “anthropic principle”, which seems like an act of desperation.)

So far we have considered the example of a *bosonic* field, in particular the photon. As we anticipated in footnote 4, there are other types of particles called *fermions* (the electron is an example), where such a huge vacuum energy density emerges with a *negative* sign, so one might hope for a large amount of cancellation.

If we were living in a perfectly *supersymmetric* world, the bosons and fermions would appear in pairs with the same mass, and indeed the vacuum energy would exactly cancel. However, even if supersymmetry exists, it has to be *broken*, in the low-energy regime where we are living: for instance the bosonic partner of the electron, the “selectron”, must be much heavier than the electron — if it exists at all — otherwise it would have been observed. The extent of supersymmetry breaking, which is required to avoid contradictions with observations, would allow for a strong reduction of the ratio $\rho_{E_0}/\rho_{\text{obs}}$. It still has to be huge, though, estimates suggest at least $\approx 10^{60}$ [31] (even before knowing the LHC results),

²²At this point, the question arises whether also the field theoretic vacuum energy has been observed, since the Casimir force is now confirmed experimentally [25, 26]. Then this discrepancy would be even more puzzling. However, this conclusion is not compelling, since the Casimir effect can also be derived without referring to the vacuum energy of the photon field [27–29].

²³Here we include radiation, unlike the terminology of Ref. [30], but it doesn’t matter for the statements in this paragraph.

²⁴Meanwhile a number of cosmologists speculate that the Cosmological “Constant” might have changed in the course of the evolution of the Universe [24] (this is reminiscent of the coupling “constants” in Quantum Field Theory, which actually depend on the energy scale; in jargon, they are “running”).

so supersymmetry does not overcome this problem either. Also the string community tried to solve the Cosmic Constant problem, without arriving at any key clue [31, 32].

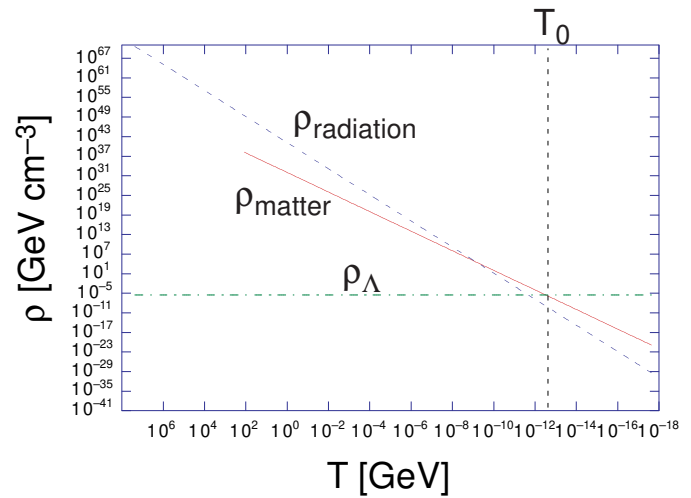


Figure 5. Illustration of the coincidence problem, adapted from Ref. [30]. That work (and many others) distinguishes “radiation” (fast moving, *i.e.* relativistic particles, mostly photons and neutrinos) from “matter”: in the former (latter), most energy is kinetic (contained in the rest mass). In these terms, the first ≈ 47000 years after the Big Bang were radiation-dominated, followed by the matter-dominated era, which lasted until $\approx 9.8 \cdot 10^9$ years after the Big Bang. Today, the age of the Universe is $\approx 13.8 \cdot 10^9$ years, the cosmic microwave background has the temperature $T_0 \approx 2.7 \text{ K}$ (indicated in the figure), and our era is dominated by Dark Energy (ρ_Λ), to about 70%. (For the Hubble constant, Ref. [30] inserted $H_0 = 65 \text{ km}/(\text{s Mpc})$.)

It is outrageous that we do not have any convincing solution to this stunning puzzle, so this appendix finishes without a happy ending. For reviews of this outstanding issue we refer to Refs. [24, 31].

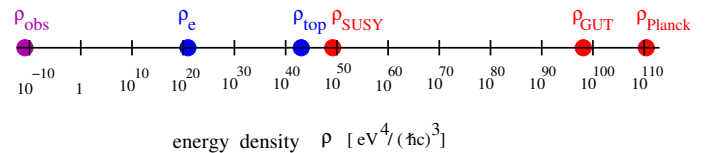


Figure 6. The energy density as observed in the Universe, ρ_{obs} , and as obtained from the vacuum energy in Quantum Field Theory, if we insert a cutoff at the rest energy of the electron, ρ_e , or of top-quark (with a mass of $173 \text{ GeV}/c^2$ the heaviest known elementary particle), ρ_{top} . These cutoffs are not motivated, they are included just for comparison. Scenarios that could be considered as somehow motivated are referring to broken supersymmetry (ρ_{SUSY} corresponds to the lower bound for its breaking), a cutoff at the Grand Unification scale, ρ_{GUT} , or at Planck scale, ρ_{Planck} . In all these cases, the theoretical energy density exceeds ρ_{obs} by *many* orders of magnitude.

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Jaime Nieto-Castellanos and Lilian Prado for reading the manuscript, as well as the authors of Ref. [30] and the American Physical Society for the permission to reproduce Figure 5. This work was supported by UNAM-DGAPA-PAPIIT, grant number IG100219.

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