ABOUT THE UPSTREAM CONTAMINATION

ACERCA DE LA CONTAMINACIÓN A CONTRACORRIENTE

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The effect through which water pollutants initially floating in a lower recipient, can appear at a higher vessel from which the liquid falls down, is investigated. While is effect has been previously studied experimentally for an inclined channel, here we develop a theoretical model for the case of a vertically falling water beam. Two cases are discussed, a simpler one, in which water flows vertically through a cylindrical tube, and a more complex one, in which water falls freely. For the first case, it is derived an expression for the water flux above which the upward flow of particles stops. For the second one, a relation between the water flux, the sizes of the particles and the height along the water beam is obtained which determines whether or not the particles can flow up to the higher recipient. In the case of the free fall it follows that the rising of the particles is only possible if the water surface tension is considered, and only happens below a maximal height difference between the two vessels.

Se investiga el efecto mediante el cual impurezas en agua flotando en un recipiente inferior pueden aparecer en otro situado más arriba y desde el cual el líquido cae hacia el primero. Un trabajo experimental previo reportó este efecto para el caso en que el agua fluye del recipiente superior al inferior a través de un canal abierto. Aquí se complementa la discusión teórica por medio del estudio de chorros de agua cayendo verticalmente. Se discuten dos casos, uno más simple en que el agua fluye a través de un tubo cilíndrico vertical, y otro más complejo en que el agua desciende en caída libre. Para el primer caso, se deriva una expresión para el flujo de agua que es capaz de detener el tránsito de partículas hacia arriba. En el segundo caso se obtiene una relación entre el flujo de agua, el tamaño de las partículas y la altura a lo largo del chorro, que determina si las partículas suben o no hacia el recipiente superior. Para la caída libre se concluye que la subida de las partículas es posible solo si se considera la existencia de la tensión superficial, y además por debajo de una diferencia de altura máxima entre las vasijas.

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I. INTRODUCTION

The present work is devoted to discuss the so called *upstream contamination* effect. This phenomenon was detected when a jet of water fell into a recipient in which with *mate* particles were floating. Then, it was noticed that after a time lapse the particles also appeared at the higher recipient. This effect was observed in the year 2008 by a student at the Physics faculty, University of Havana, and was systematically studied and eventually published few years after in reference [1]. In that work, a water reservoir located at a certain height discharged clean water through an inclined channel on the surface of a second reservoir located approximately 1 cm below. If mate leaves or chalk particles were sprinkled in the surface of the lower container, eventually the particles would reach the upper container moving counter-stream along the channel. At the lower end of the channel there was also a free falling stream of water.

The experimental observations reported in that work, were mainly explained by invoking the so called Marangoni effect [2–7], which consists in the variation of the surface tension of substances when floating contaminants are present. Then, the gradient of the local density of surface contaminants was assumed to create a net force pushing the particles against the stream of water [1].

In this work we will present a complementary discussion of the contamination effect, limited to the simpler case of vertical falling beams of water. The objective is to consider the feasibility of the effect to occur without the additional influence of the Marangoni effect. That is, only the effects of viscosity and surface tension will be under consideration. Specifically, we will determine which conditions among the parameters should be obeyed in order that spherical particles can ascend through the water jet. The discussion will assume that the particles move through the interior of the water beam. The case in which the particle move through the surface will not be analyzed here.

Two different situations will be investigated. In the first one, the water falls down through a cylindrical tube of circular section. In the second case the water is assumed to fall freely under the action of gravity. In both cases the Stokes's frictional force associated to water viscosity plays a central role. However, while for the first case the surface tension plays no role, in the second one it becomes central.

In the first case, we are able to calculate the maximum flux of water above which the particles stop moving upstream. In the second case, we demonstrate that there is a maximum height difference between the recipients above which the particles are unable to flow upstream. This critical value grows with the increasing of the size of the particle and with the reduction of the water flux. It follows that the numerical values predicted for the critical height values (associated to the water fluxes and sizes of the beam of free falling water at the end of the inclined channel) are in the range of the experimental estimates reported in reference [1].

The presentation of the work will be as follows. In Section 2 the relations among the parameters for the water to be stopped to contaminate the higher recipient for the case of water falling through a vertical tube, will be discussed. In Section 3, we will consider the derivation of the similar relation for the case of the free falling water beam. The determination of an approximate stationary form of the beam is also presented. Finally, the results are reviewed at the Summary.

II. WATER FALLING THROUGH A CYLINDRICAL PIPE

Let us consider that the reference frame for coordinates will be at the bottom of the upper recipient. The positive axis for the z coordinate will point downwards and along the symmetry axis of the water flow. Under this assumptions the Bernoulli theorem will be applied between two points a and b laying at different heights. Both points will be assumed to lay on a curve being tangent to the velocity field. Then, it is possible to write

$$\frac{1}{2}\rho\, v_a^2 - \rho\, g\, z_a + P_a = \frac{1}{2}\rho\, {v_b}^2 - \rho\, g\, z_b + P_b. \tag{1}$$

In the following this relation will be employed to discuss the two types of water flows under consideration. Assuming that the index $a = 0$ and the b is the z-coordinate, the Bernoulli Law can be written as

$$\frac{1}{2}\rho v_0^2 + P_0 = \frac{1}{2}\rho\, v(z)^2 - \rho g\, z + P(z),$$
$$\frac{1}{2}\rho v_0^2 = \frac{1}{2}\rho\, v(z)^2 - \rho\, g\, z + (P(z) - P_0),$$

where v_0, v_z are the flow velocities at the two points and P_0, P_z the corresponding pressures.

But, for the case under consideration in this section, the transversal area of the cylinder is constant at different heights. Therefore, the incompressibility of water determines that the velocity $v(z)$ is in fact not changing with z, i.e.

$$v(z) = v_0 \equiv v. \tag{2}$$

Thus, it also follows that the distribution of pressures along the vertical inside the tube is identical to the one in static water, thus

$$P(z) - P_0 = \rho\, g\, z. \tag{3}$$

Now, let us write the Newton equation of motion for a small spherical particle of radius R and density ρ_m located inside the water beam, as

$$\rho_m\, V\, \frac{dv_m(t)}{dt} = \rho_m V\, g + f_e - k\,(v_m(t) - v), \tag{4}$$
$$m = \rho_m\, V, \tag{5}$$
$$V = \frac{4}{3}\pi R^3, \tag{6}$$

which expresses that the acceleration is determined by the vector addition of the weight of the body, the floating forces produced by the pressure and the viscosity force defined by the Stoke's law. But, Archimedes Law turns out to be exactly valid for the floating force, due to the linear behavior of the pressure coinciding with the functional dependence of pressure with vertical distance for static water. Thus the floating force is given by

$$f_e = -V\frac{dP(z)}{dz} = -\rho\, g\, V, \tag{7}$$

where the negative sign is because the force tend to move the body in the negative direction of the height coordinate z. Then, the Newton equation can be rewritten in the form

$$\frac{dv_m(t)}{dt} = g\,(1 - \frac{\rho}{\rho_m}) - \frac{k}{m}(v_m(t) - v). \tag{8}$$

This equation includes the frictional Stokes force acting on a moving sphere in a fluid when the liquid movement is laminar

$$f_S = -\,k\,(v_m(t) - v), \tag{9}$$

where the constant k is given in terms of the viscosity constant μ and the radius of the sphere as

$$k = 6\,\pi\mu\, R. \tag{10}$$

This is a simple first order equation for the velocity of the particle in the observer's reference frame. Expressing it in the integral form, we can write

$$\int_0^{v(t)} \frac{dv_m}{(v_m(t) - v) + \frac{m\,g}{k}\,(\frac{\rho}{\rho_m} - 1)} = -\int_0^t \frac{k}{m}dt,$$
$$\log\left[\frac{v_m(t) - v + \frac{m\,g}{k}\,(\frac{\rho}{\rho_m} - 1)}{-v + \frac{m\,g}{k}\,(\frac{\rho}{\rho_m} - 1)}\right] = -\frac{k}{m}t. \tag{11}$$

This relation allows to write the following explicit solution for the velocity of the particle

$$v_m(t) = [v - \frac{m\,g}{k}\,(\frac{\rho}{\rho_m} - 1)](1 - \exp(-\frac{k}{m}t)), \tag{12}$$

which after being evaluated for large times, yields the following expression for the limit velocity of the particle with respect to the observer's frame

$$v_m(\infty) = [v - \frac{m\,g}{k}\,(\frac{\rho}{\rho_m} - 1)] \tag{13}$$
$$= v - \frac{2\pi R^2 g\,\rho_m}{9\,\pi\mu}\,(\frac{\rho}{\rho_m} - 1).$$

In the above relations it has been substituted

$$\frac{m}{k} = \frac{2\pi R^2\rho_m}{9\,\pi\mu}. \tag{14}$$

Therefore, the single condition for the particles not to be allowed to climb to the upper reservoir is

$$v - \frac{2\pi R^2 g}{9\ \pi\mu}\,(\rho - \rho_m) > 0.$$

But, the velocity is defined in terms of the volume flux of water Q and the area A of the vertical cylinder as $v = \frac{Q}{A}$. Thus, the inequality becomes

$$Q > \frac{2\pi R^2 g A}{9\ \pi\mu}\,(\rho\ - \rho_m). \tag{15}$$

This relation indicates that if Q stops all the particles regardless the value of their density ρ_m, the following modified relation should be satisfied

$$Q > \frac{2\pi R^2 g A \rho}{9\ \pi\mu}\ . \tag{16}$$

Let us now assume that the fluid is water and that the experience is performed under normal gravity conditions. Then, the following set of parameters can be substituted $\rho = 1000\ \ \mathrm{Kg/m^3}$, $g = 9.8\ \ \mathrm{m/s^2}$, $\mu = 8.9 \times 10^{-4}\ \mathrm{Ns/m^2}$.

We now illustrate the resulting regions of values of the flux of water Q , the radius R of the spherical particles and the area A of the water conducting cylinder, for which the particles are not allowed (or allowed) to climb upstream along the cylinder. These regions are shown in figure 1. The surface shown defines the critical boundary in the space of parameters at which the particles are in the limit of being allowed and not being allowed to contaminate the upper recipient. The colored zone over the surface is the set of values of triplets of coordinates (Q, R, A) for which the particles can move against the flow up to the higher recipient. On the contrary, the white region below the surface represent the triplets of parameters for which the particles remain trapped in the lower vessel.

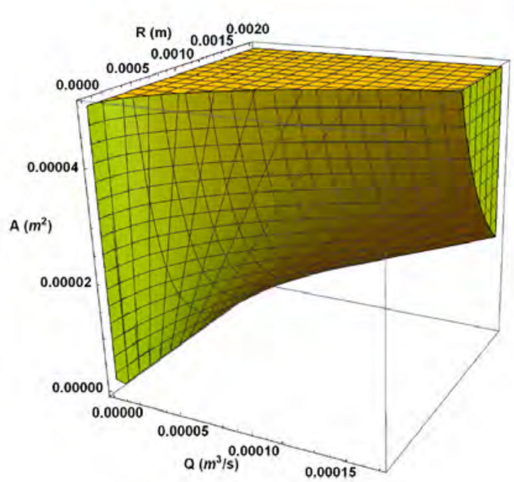

Figure 1. The plotted surface contains the set of points satisfying the equality in relation (16) as a function of R, A and Q. That is, at such points the velocity of the particles vanishes. Below the surface the particles have positive velocities and then do not contaminate the upper recipient. Above it, in the colored region, the particles tend to ascend against the flow.

Let us use the above derived relation to estimate the critical value of the flux for particles with size of the chalk powder used in reference [1]. Since the authors estimate it as of the order of hundreds of microns, we will assume a radius of the spherical particles of $R = 100\ \mu m = 0.0001$ m. The flux of water falling from the upper to lower recipients was chosen as $Q = 16\ \mathrm{cm^3/s} = 0.000016\ \ \mathrm{m^3/s}$. For these values of the size of the particle and the flux of water, the point laying on the critical surface has a value of the sectional area of the cylinder given by $A = 0.000163469\ \mathrm{m^2} = 1.63\ \ \mathrm{cm^2}$.

This result indicates that for the flux of water employed in the experiment in reference [1] and a size of the chalk particle powder of the estimated one in that reference, the area of the cylinder section considered in our model is of the order of a few $\mathrm{cm^2}$. But, the sectional area of the waterfall at the end of the inclined channel employed in the experiments is expected to be of the same order. Therefore, it can be concluded that floating effects alone, without the consideration of the Marangoni effect can, partially justify the contamination against current effect through the waterfall at the end of the channel, at least for particles which are able to float.

We would like here to remark on a question that looks of interest in connection with the experiments reported in reference [1]. It should be underlined that the viscous character of water is expected to establish a zero velocity of the water relative to the channel when they meet. This situation suggests that the contact boundary of water with the walls of the channel should be expected to be a preferred path for the moving of the particles upwards. Therefore, it seem necessary to compare the dimensions of the particles with the sizes of the boundary layer in which the velocity varies from zero at the wall up to its maximum value at the center of the channel.

III. FREE FALLING WATER FLOW

Let us now consider the second situation in which the water free falls in a beam to the lower recipient. Firstly, we will determine the pressure difference between two points: one in the air just over the water surface, and one just underneath the surface. Then, we will consider the balance of momentum of the surface element illustrated in figure 2. The picture illustrates the two principal curvature radii R_{in} and R_{out} of the chosen symmetric surface element, and also the surface tension forces which are mainly exerted on the two arcs of circle corresponding to the respective two circumferences defining the curvature radii. The surface tension parameter is represented by γ. Therefore, considering that the two arc elements are infinitesimal with equal length dl, the momentum balance for the surface element gives

$$\begin{aligned}(P_{in} - P_{out})\ dl^{\,2} &= \gamma\ dl\ (d\theta_{in} - d\theta_{out}).\\ &= \gamma\ dl^{\,2}(\frac{1}{R_{in}} - \frac{1}{R_{out}}),\end{aligned} \tag{17}$$

$$d\theta_{in} = \frac{dl}{R_{in}}, \quad d\theta_{out} = \frac{dl}{R_{out}}. \tag{18}$$

Now, in order to simplify the discussion, let us assume that water beam shows curvature radii satisfying

$$R_{out} >> R_{in}. \tag{19}$$

This relation indicates that the tangent vector to the water surface contained in a plane including the axis of the beam, becomes close in direction with the beam axis. But, this property in turns implies that the small radius is approximately given by the radius of the beam taken at the fixed height value defined by the z coordinate. This radius will be defined by $r(z)$. Thus,

$$P_{in} - P_{out} \; = \frac{\gamma}{r(z)}. \tag{20}$$

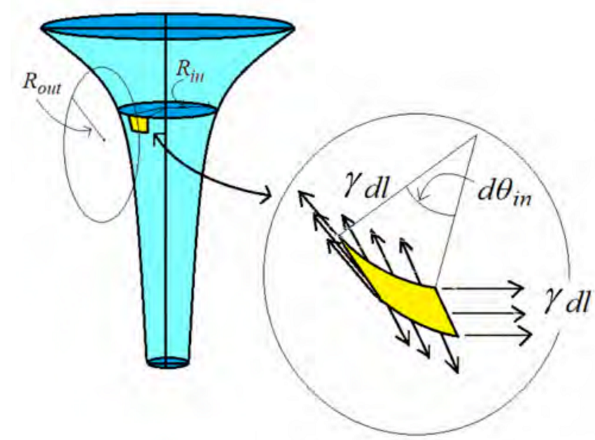

Figure 2. A model of the water beam that includes a surface element for which the momentum balance is worked out. The internal and external principal curvature radii are indicated. The surface element is colored in yellow and the surface tension forces acting on it are shown in the figure depicted at right.

But, the external pressure on the beam is given by the atmospheric one P_0, which implies the following relation between the internal pressure at any height z the formula

$$P(z) = P_0 + \frac{\gamma}{r(z)}. \tag{21}$$

Consider now at the origin of the height coordinates $z = 0$, the following initial conditions: the speed of the fluid when emerging form the upper recipient is $v(0) = v_0$ and the radius of the beam is $r(0) = a$. Then, substituting in the Bernoulli's equation, it follows

$$\frac{1}{2}\rho v_0^2 + P_0 + \frac{\gamma}{a} = \frac{1}{2}\rho\, v(z)^2 - \rho g\, z + P_0 + \frac{\gamma}{r(z)}, \tag{22}$$

$$\frac{1}{2}\rho v_0^2 + \frac{\gamma}{a} = \frac{1}{2}\rho\, v(z)^2 - \rho\, g\, z + \frac{\gamma}{r(z)}, \tag{23}$$

from which the velocity can be expressed in the form

$$v(z)^2 = v_0^2 + 2\, g\, z + \frac{2\,\gamma}{\rho\, a} - \frac{2\,\gamma}{\rho\, r(z)}.$$

Let us consider now a special origin for measuring the height coordinate z. Note first that if we assume the velocity v_0 of the flux is tending to zero in (23), the constancy of the total flux of water $Q = \pi a^2 v_0$ implies that the radius of the beam a should tend to infinity. Therefore, if we consider this point as the origin of coordinates $z = 0$, relation (23) becomes

$$\rho\, g\, z = \frac{1}{2}\frac{\rho\, Q^2}{\pi\, r(z)^4} + \frac{\gamma}{r(z)}. \tag{24}$$

But, the unique real and positive solution of this equation gives for $r(Q, z)$ (after fixing the density, surface tension and viscosity associated to water) is plotted in figure (3) as a function of the flux and the height z (as measured from the point at which the velocity of water vanishes). Note that this chosen origin of values of z is an unphysical point of the curve $r(z)$, since the section of a real beam never tends to infinity.

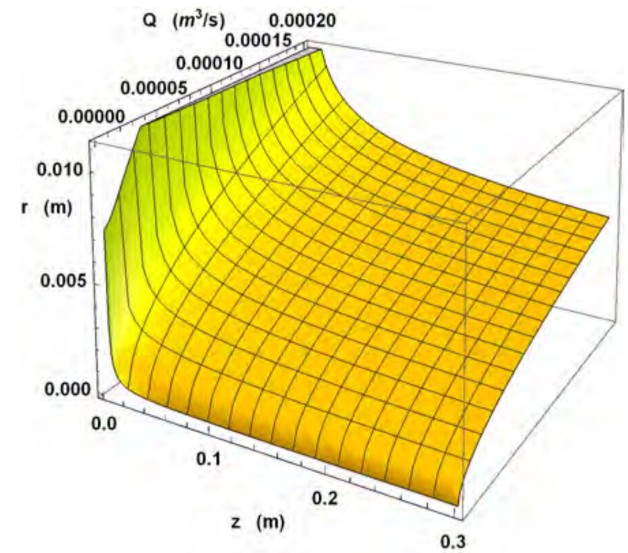

Figure 3. Radius of the free falling water beam as a function of the height coordinate z and the water flux Q passing through the beam.

The above discussion in this section, describing the flow of water regime in the falling beam, mainly follows the one given in reference [8]. It was presented for completeness.

Let us now consider how a particle situated in the water beam moves. The forces acting on this particle are determined by the variation of the pressure with the height, the weight of the particles and the viscosity. The viscosity force is proportional to the relative speed between the liquid and the particles (Stokes Law). Then, the Newton equation of motion takes the following form:

$$\rho_m\, V\, \frac{d^2 z}{dt^2} \; = \; \rho_m V\, g - k\, (\frac{dz}{dt} - v(z)) - V\frac{dP(z)}{dz}, \tag{25}$$

$$m = \rho_m\, V, \tag{26}$$

which is very similar to relation (4) in the previous section with the only difference that the expression for the pressure as a function of the height z is different here. In this equation $\frac{dz}{dt} = v_m(t)$ is the speed of the particle relative to the observer and ρ, V (as in the previous subsection) the density and the volume of the floating particle, respectively. But, the derivative of the pressure in (21) can be expressed in terms of $\frac{d\,r(z)}{dz}$, which in turns can be written as a function of $r(z)$ by taking the derivative of the relation (24). Using the result of this evaluation, the floating force (equal to minus the derivative of the pressure respect to the height z times the volume) can be calculated as

$$\begin{aligned} f_e &= -V\frac{dP(z)}{dz} \\ &= -m\,g\frac{\rho}{\rho_m}\frac{\gamma\, r(z)^3}{\frac{2\rho\, Q^2}{\pi}+\gamma\, r(z)^3}. \end{aligned} \tag{27}$$

This result for the floating force deserves a comment. Note that by discarding the viscosity, the only force pointing in the -z (that is pushing the particle to the upper recipient) is this floating force that completely disappears if the surface tension vanishes. This allows to conclude that, in the here considered free falling case, the surface tension is a central element for the possibility of contamination of the upper recipient by particles coming from the lower one.

Then, by dividing the Newton equation by the mass of the particle $m = \rho_m\, V$, it is possible to write

$$\begin{aligned} \frac{d^2z}{dt^2} = (g - \frac{9\pi\mu}{2\pi\rho_m R^2}(\frac{dz}{dt}-v(z)) - \\ g\frac{\rho}{\rho_m}\frac{\gamma\, r(z)^3}{\frac{2\rho\, Q^2}{\pi}+\gamma\, r(z)^3}), \end{aligned} \tag{28}$$

in which all the entering parameters are already well defined. We will consider now the restrictions implied by this equation on the values these parameters for allowing or not the transportation of particles from the lower recipient to the upper one. A drastic simplification for the derivation of these conditions follows after noting that the term in the equation pushing the particles down is a constant equal to g. Thus, when density ρ_m tends to zero we have the situation in which there is a maximal tendency to move up. That is the limit in which the particle is an empty bubble.

In this limit the equation reduces to the simpler form

$$0 = \frac{9\pi\mu}{2\pi R^2}(\frac{dz}{dt}-v(z)) + \ g\rho\frac{\gamma\, r(z)^3}{\frac{2\rho\, Q^2}{\pi}+\gamma\, r(z)^3}, \tag{29}$$

from which the velocity of the particle at any point can be expressed in terms of the already known magnitudes of the problem as

$$\frac{dz}{dt} = v(z) - \frac{2R^2 g\rho}{9\mu}\frac{\gamma\, r(z)^3}{\frac{2\rho\, Q^2}{\pi}+\gamma\, r(z)^3}. \tag{30}$$

Therefore, the condition for the particle to move up at a height value z can be written as

$$\begin{aligned} \frac{dz}{dt} &= v(z) - \frac{2R^2 g\rho}{9\mu}\frac{\gamma\, r(z)^3}{\frac{2\rho\, Q^2}{\pi}+\gamma\, r(z)^3} \\ &= \frac{Q}{\pi r(z)^2} - \frac{2R^2 g\rho}{9\mu}\frac{\gamma\, r(z)^3}{\frac{2\rho\, Q^2}{\pi}+\gamma\, r(z)^3} < 0. \end{aligned} \tag{31}$$

Assuming that the fluid under consideration is water, the above conditions are equivalent to

$$C(Q,R,z) = Q - \frac{2\pi R^2 g\rho}{9\mu}\frac{\gamma\, r(z)^5}{\frac{2\rho\, Q^2}{\pi}+\gamma\, r(z)^3} < 0. \tag{32}$$

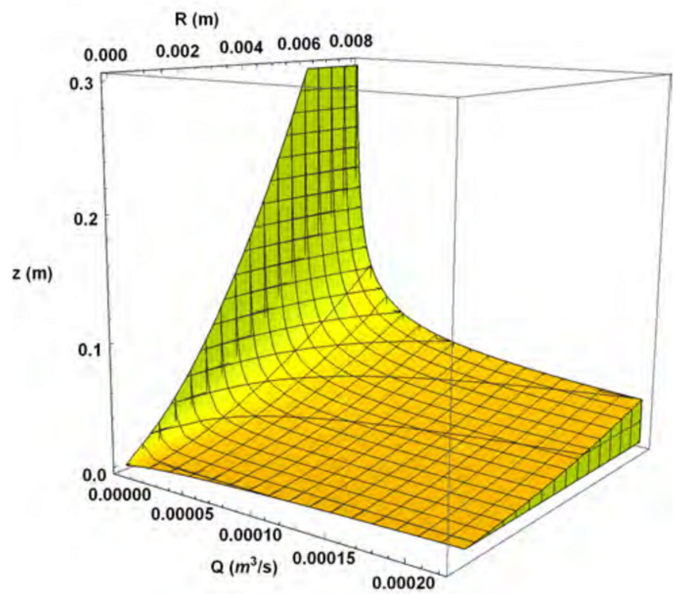

Figure 4. The plotted surfa9ce is formed by the set of points satisfying the equality in relation (32). That is, at such points the velocity of the bubble particles vanishes when the triplet of parameters take the plotted values. Above the surface the particles have positive velocities and then do not contaminate the upper recipient. Below it, in the colored region, the particles have negative velocities and then, tend to ascend against the flow.

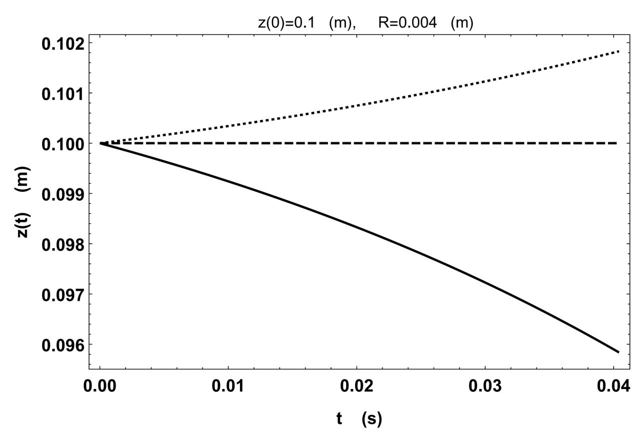

Figure 5. The plotted curves illustrate that when the flux is smaller, and such that the triplet of parameters (Q,z,R) is below the critical surface, the motion of the particle is towards negative values of z. That is, the particle ascends to the upper reservoir. For higher values of flux (for which the triplet of parameters is over the critical surface) the particle moves in the direction of the positive values of z. Thus, in this case the particle moves down and does not tend to contaminate the upper container.

The satisfaction of the relation (32) in the space of the three still free parameters: the flux Q, the radius of the particles R and the height coordinate z (measured from the point of zero velocity) is illustrated in figure 4. The shown surface describes the triplets of values of the parameters (Q, R, z) at which the velocity of the particle becomes equal to zero. For all the points being over this surface the particle shows a positive value of its velocity and then it is not able to climb to the upper reservoir (remember that the increments of the height z are defined as positive ones if the point moves down). Correspondingly, the points being below the surface correspond to particles that will contaminate the upper vessel. That is, having a negative velocity value. In this way, given the parameters of the system, the conditions for the particle to appear in the upper recipient have been identified also for this free falling water situation. In the next subsection we will check how the direct solution of the exact system of equations reproduces the same conclusions extracted from the simplified analysis for a spherical bubble of the same radius as a particle with density ρ_m.

III.1. Direct solution of the full Newton equation around the critical surface

Let us consider the Newton equation

$$\frac{d^2z}{dt^2} = g - \frac{9\pi\mu}{2\pi\rho_m R^2}\left(\frac{dz}{dt} - v(z)\right) - g\frac{\rho}{\rho_m}\frac{\gamma\, r(z)^3}{\frac{2\rho\, Q^2}{\pi} + \gamma\, r(z)^3}, \tag{33}$$

for values of the triplet of parameters (Q, R, z) being close to the critical surface in figure 4. For concreteness, it will be assumed the following specific values for the height position and the radius of the particle

$$z(0) = 0.1 \ \text{m}, \qquad R = 0.004 \ \text{m}, \tag{34}$$

by also selecting two values of the flux

$$Q_1 = 1.4\times 10^{-6}\ \text{m}^3/s \qquad Q_2 = 1.6\times 10^{-6}\ \text{m}^3/s, \tag{35}$$

being close to an specific value $Q^* = 1,53\times 10^{-6}$. This value Q^* is chosen for making sure that the triplet $(Q^*, R, z(0))$ is exactly on the critical surface. Then, the two triplets associated to the fluxes Q_1 and Q_2 are situated, one of them over and the other below, the critical surface.

Now, it will be considered the solution of the equations (33) by assuming that the density of the particle is very small, by example satisfying

$$\frac{\rho_m}{\rho} = 10^{-9}.$$

In addition, we will assume two independent boundary condition for the velocity of the particles as coinciding at the initial time $t = 0$ with the velocities of the water flow in the form

$$\frac{d}{dt}z(t)_{t=0} = \dot{z}(0) = \frac{Q}{\pi r(z(0))^2},$$

as calculated for the two specified values of the flux: one of them associated to a higher value and defining a point over the critical surface and the other one with a smaller flux, which is linked with a point being below the critical surface. The specific values of the two fluxes were defined in (35).

The solutions of the equation of motion for the height coordinate z of the particle are shown in figure 5. The curves clearly show that when the flux is the smaller one, and such that the tripelet of parameters is below the critical surface, the movement of the particle is tending to the negative values of z. That is, the motion is ascending to the upper reservoir. However, for the higher flux, when the triplet of parameters is over the critical surface, the particle moves in the direction of the positive values of z. Thus, in this case it does not tend to contaminate the upper vessel.

IV. SUMMARY

We have theoretically modeled the effect associated to the rising of particles from a lower recipient to a higher one, through a water flux falling from the higher one [1]. Conditions for the occurrence of the effect for two types of mechanisms for the water falling were established. It follows that for water going down through a cylindrical tube of constant cross section, there is a critical value of the flux were above which the particles are not allowed to rise up to the upper recipient. For the case in which the water is free falling, it firstly became clear that the possibility of the particles to climb up exists only under the presence of surface tension in water. If the surface forces are assumed to be absent, the particles can not flow up, at least for the discussed case of particle motion through the volume. In this case, it also follows that there is a critical value of the difference of height between the vessels, above which the particles are not allowed to climb the water flow. These critical values rise with the increasing size of the particles and decrease as the amount of water flux through the falling beam is reduced. The discussion results in a simple criterion for the occurrence of the effect which is based on a simplified equation of motion for empty bubbles of the same sizes as the particles. The almost vanishing of the density of the bubbles permits to reduce the equation of motion to a first order in the time derivative one, in the limit of zero density. The satisfaction of the criterion for the occurrence of the effect obtained for bubbles is checked by solving the full Newton equations of motion for particles near the critical surface, under simplified conditions.

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