Rev. Cub. Física vol .26 No.2B (2009) p.232-237
ISSN: 0253-9268. Original paper

Revista Cubana de Física
Calle I No. 302 e/ 15 y 17
Vedado, La Habana. CP 10400
www.fisica.uh.cu/biblioteca/revcubfi/index.htm

Spinor field in generalized Krein space quantization

F. Payandeh[a,b†]

a) Physics Dep., Amirkabir University of Technology, Tehran 15914, Iran; payandeh@aut.ac.ir
b) Physics Dep., Payeme Noor University, Tehran 19395-4697, Iran
† Author for correspondence

Received July 01 2008. Final version approval: December 29, 2008

Abstract. In previous papers it has been shown that the presence of negative-norm states or negative- frequency solutions is indispensable for a fully covariant quantization (Krein space quantization) of the minimally coupled free scalar field in de Sitter spacetime. The new method of quantization has been extended to free boson fields (charged scalar field, massive and massless vector field) in Minkowski spacetime. In this paper, quantization of free spinor field is reformulated in generalized Krein space. The presence of unphysical negative-frequency states plays the role of an automatic renormalization tool for the theory.

Sumario. En artículos anteriores se ha demostrado que la presencia de estados de norma negativa o de soluciones de frecuencia negativa es indispensable para una cuantización totalmente covariante (cuantización del espacio de Krein) del campo libre escalar en el espacio-tiempo de de Sitter. El nuevo método de cuantización se ha extendido a los campos de bosones libres (campo escalar cargado, campos vectoriales masivos y sin masa) en el espacio-tiempo de Minkowski. En este artículo la cuantización del campo de spinores libres se reformula en el espacio generalizado de Krein. La presencia de estados de frecuencia negativa no físicos juega un papel en la herramienta automática de renormalización para la teoría.

Key words: QFT, Krein space, negative energy states; 03.70+k, 04.62.+v, 11.10.Cd, 98.80.H

1 Introduction

It has been shown that negative-norm states necessarily appear in a covariant quantization of the free minimally coupled scalar field in de Sitter spacetime.[1,2] Consideration of the negative-norm states was proposed by Dirac in 1942.[3] In 1950 Gupta[4] and Bleuler[5]applied the idea in QED to respect the Lorentz covariance of vector field quantization. The presence of higher derivatives in the Lagrangian also leads to ghosts; states with negative- norms[6]. Mathematically, for the minimally coupled scalar field in de Sitter spacetime[2], which plays an important role in the inflationary model as well as in the linear quantum gravity[7,8], it has been proven that the use of the two sets of solutions (positive and negative frequency states) is unavoidable for preservation of (i) causality (locality), (ii) covariance, (iii) elimination of the infrared divergence. This causal approach was generalized further to the calculation of the graviton propagator in de Sitter spacetime[9], and the one-loop effective action for the scalar field in a general curved spacetime[10]. In this process ultraviolet and infrared divergences have been automatically eliminated[11]. The origin of divergences in standard quantum field theory lies in the singular character of the Green's function at short relative distances (ultraviolet divergence) or in the large relative distances (infrared divergence). The procedures of normal ordering and renormalization have been used for eliminating the divergences of physical quantities. In another word, the problem of divergence in QFT appears when the negative frequency states which are also solutions of the field equation in the classical level, are discarded in the quantum level due to the principle of quantum field theory (positivity condition). This discarding breaks the elegance of the theory (standard QFT) and it causes the appearance of anomaly.

In a previous work it has been shown that the combination of quantum field theory in Krein space together with the consideration of quantum metric fluctuations results in quantum field theory without any divergences[12]. Ignoring the positivity condition (for norm and energy), similar to the Gupta-Bleuler quantization of the electrodynamics in Minkowski spacetime, the free boson field quantization has been performed in Krein space resulting in both positive and negative norms for the unphysical (negative-energy) states[13]. However, in the case of spinor field they are positive-norm states moving forward in time (vise versa of antiparticles). Then, the space of quantization for spinor field is called generalized Krein space[14].

Here we present the free spinor field, (s = 1/2), quantization in generalized Krein space. In this approach, the auxiliary negative-frequency states have been utilized, the modes of which do not interact with the physical states or the real physical world. Naturally these modes can not be affected by the physical boundary conditions. Following this scheme, the normal ordering procedure is rendered useless because the ultraviolet divergence in the stress tensor disappears and the vacuum energy remains convergent [2]. The most interesting result of this construction is the convergence of the Green's function at large distances, which means that the infrared divergence is gauge dependent.[2,11] Presence of the "unphysical" (negative-energy) states plays the role of an automatic renormalization tool for the theory. The physical interpretation however is not yet clear and any further progress calls for more investigations.[15–18]

It is noteworthy that by the new method of quantization, a natural renormalization of the following problems have been already attained:

• The massive free field in de Sitter spacetime[2].

• The graviton two-point function in de Sitter spacetime[9].

• The one-loop effective action for scalar field in a general curved spacetime[10].

• Tree level scattering amplitude for a scalar field with one graviton exchange in de Sitter spacetime[19].

•The interacting QFT in Minkowski spacetime ($\lambda\varphi^4$ theory).[20]

• Casimir effect in Krein space quantization[21].

• Free boson fields in Krein space quantization [13].

• One-loop approximation of Møller scattering in generalized Krein space quantization[14]. Pursuing the above works and through the same new approach, quantization of free spinor field, its vacuum energy and momentum, and the associated divergence-free two-point function in Minkowski spacetime are worked out in generalized Krein space. Again, it is seen that the presence of unphysical states plays the role of an automatic renormalization tool for QFT.

2 Dirac field quantization in Hilbert space

We briefly recall the spinor field quantization in standard quantum field theory. The Lagrangian density of a classical spinor field $\psi(x)$ with mass m is[22,23]

$$L=\overline{\psi}(i\gamma^{\mu}\partial_{\mu}-m)\psi$$

The spinor field satisfies the Dirac equation

$$(i\partial\!\!\!/-m)\psi(x)=0=(i\eta^{\mu\nu}\gamma_{\mu}\partial_{\nu}-m)\psi(x)$$

$$\eta^{\mu\nu}=diag(1,-1,-1,-1)$$

The two sets of solutions are[23]

$$U^{s}(k,x)=\sqrt{\frac{m}{(2\pi)^{3}\omega_{\vec{k}}}}\,u^{s}(\vec{k})e^{-ik.x}\quad\text{(Positive energy)}$$

$$V^{s}(k,x)=\sqrt{\frac{m}{(2\pi)^{3}\omega_{\vec{k}}}}\,\upsilon^{s}(\vec{k})e^{ik.x}\quad\text{(Negative energy)}$$

with s = 1, 2 and $k^{0}=\omega_{\vec{k}}=(\vec{k}.\vec{k}+m^{2})^{\frac{1}{2}}\geq 0$.

These solutions are lot complex conjugate of each other but they satisfy the following relations[23]

$$u^{s}(\vec{k})=\gamma^{5}\upsilon^{s}(\vec{k})$$
$$\upsilon^{s}(\vec{k})=\gamma^{5}u^{s}(\vec{k})$$
$$\overline{u}^{s}(\vec{k})=\overline{\upsilon}^{s}(\vec{k})\gamma^{5}$$
$$\overline{\upsilon}^{s}(\vec{k})=\overline{u}^{s}(\vec{k})\gamma^{5}$$

where $\overline{u}=u^{\dagger}\gamma^{0}$ is the Dirac adjoint. In order to quantizing the spinor field in the usual way one chooses the positive energy states (standard QFT), and the field operator is given by

$$\psi(x)=\int d^{3}\vec{k}\sum_{s=1,2}\left[b_{\vec{k}s}U^{s}(k,x)+d^{\dagger}_{\vec{k}s}V^{s}(k,x)\right]$$

where $b_{\vec{k}s}$ is the annihilation operator of one-particle state with positive energy and $d^{\dagger}_{\vec{k}s}$ is the creation operator of one-antiparticle state with negative energy ($b_{\vec{k}s}$ and $d^{\dagger}_{\vec{k}s}$ are two independent operators).

Defining the canonical conjugate field to $\psi(x)$ by (dot stands for time-derivative)

$$\pi(x)=\frac{\partial L}{\partial\dot\psi}=i\psi^{\dagger}(x)$$

one obtains the nonzero (equal-time) anticommutation relation

$$\left\{\psi_\alpha(x),\psi^{\dagger}_\beta(x')\right\}_{x^0=x^{0\prime}}=\delta_{\alpha\beta}\,\delta^3(\vec x-\vec x')$$

$$(\alpha,\beta=1,2,3,4)$$

Creation and annihilation operators are constrained to obey the following anticommutation rules

$$\left\{b_{\vec ks},b^{\dagger}_{\vec k's'}\right\}=\left\{d_{\vec ks},d^{\dagger}_{\vec k's'}\right\}=\delta_{ss'}\,\delta^3(\vec k-\vec k')$$

$$\left\{b_{\vec ks},b_{\vec k's'}\right\}=\left\{b^{\dagger}_{\vec ks},b^{\dagger}_{\vec k's'}\right\}=\left\{d_{\vec ks},d_{\vec k's'}\right\}=$$

$$\left\{d^{\dagger}_{\vec ks},d^{\dagger}_{\vec k's'}\right\}=0$$

$$\left\{b_{\vec ks},d_{\vec k's'}\right\}=\left\{b^{\dagger}_{\vec ks},d^{\dagger}_{\vec k's'}\right\}=\left\{b^{\dagger}_{\vec ks},d_{\vec k's'}\right\}=$$

$$\left\{b_{\vec ks},d^{\dagger}_{\vec k's'}\right\}=0$$

The vacuum state $|0>$ is defiled as a state that is destroyed by all annihilation operators,

$$b_{\vec ks}\big|0>=0\qquad,\qquad d_{\vec ks}\big|0>=0,\qquad\forall\vec k$$

One can obtain the one-physical particle/antiparticle state by letting creation operator act on the vacuum state

$$b^{\dagger}_{\vec ks}\big|0>=\big|1^b_{\vec k,s}>,\quad d^{\dagger}_{\vec ks}\big|0>=\big|1^d_{\vec k,s}>\ \forall\vec k$$

Where $\left|1^b_{\vec ks}\right\rangle$ $\left(\left|1^d_{\vec ks}\right\rangle\right)$ is called a one-physical particle (antiparticle) state. The anticommutation relations together with the normalization of the vacuum, $<0|0>=1$, lead to positive norms for these physical parts

$$\left\langle 1^b_{\vec k',s}\,\middle|\,1^b_{\vec k,s}\right\rangle=+\delta^3(\vec k-\vec k'),$$

$$\left\langle 1^d_{\vec k',s}\,\middle|\,1^d_{\vec k,s}\right\rangle=+\delta^3(\vec k-\vec k')$$

The Hamiltonian and momentum operators of Dirac field are defiled as

$$H=\int d^3\vec x\,(\pi\dot\psi-L)=\int d^3\vec x\,\overline{\psi}(i\gamma^0)\dot\psi$$

$$\vec P=-i\int d^3\vec x\,\psi\,\vec\nabla\psi$$

Calculating the energy and momentum operators in terms of Fourier modes gives

$$H=\int d^3\vec k\ \omega_{\vec k}\sum_s(b^{\dagger}_{\vec ks}b_{\vec ks}-d_{\vec ks}d^{\dagger}_{\vec ks})$$

$$\vec P=\int d^3\vec k\ \vec k\sum_s(b^{\dagger}_{\vec ks}b_{\vec ks}-d_{\vec ks}d^{\dagger}_{\vec ks})$$

In this case one constructs a covariant quantization but there appears an ultraviolet divergence in the vacuum energy. The imposition of normal ordering prescription is required to lead $<0|:H:|0>=0$.

3 Dirac field quantization in generalized krein space

In the new method of quantization both sets of solutions

$$U^s(k,x)=\sqrt{\frac{m}{(2\pi)^3\omega_{\vec k}}}\,u^s(\vec k)e^{-ik.x}\ \text{(Positive energy)}$$

$$V^s(k,x)=\sqrt{\frac{m}{(2\pi)^3\omega_{\vec k}}}\,\upsilon^s(\vec k)e^{ik.x}\ \text{(Negative energy)}$$

are needed for obtaining a naturally renormalized theory. These modes are orthogonal and normalized. It is worthwhile to note that in the standard quantization of Dirac field, the creation and annihilation of positive frequency modes (i.e. Positive energy states) are considered in the field operator expansion. However, in the new approach the creation and annihilation of both positive and negative frequency modes (i.e. Positive and negative energy states) are applied for the expansion of field operator. Then, the new field operator is defined (subscript K shall, hereafter, always stands for quantities in generalized Krein space)

$$\psi_K(x)=\frac{1}{\sqrt2}\left[\psi_p(x)+\psi_n(x)\right]\quad\text{where}$$

$$\psi_p(x)=\int d^3\vec k\sum_{s=1,2}\left[b_{\vec ks}U^s(k,x)+d^{\dagger}_{\vec ks}V^s(k,x)\right]$$

$(positive\ energy\ solution)$

$$\psi_p(x)=\int d^3\vec k\sum_{s=1,2}\left[a_{\vec ks}V^s(k,x)+c^{\dagger}_{\vec ks}U^s(k,x)\right]$$

$(negative\ energy\ solution)$

and hence

$$\psi_K(x)=\frac{1}{\sqrt2}\int d^3\vec k\sum_{s=1,2}\begin{bmatrix}(b_{\vec ks}+c^{\dagger}_{\vec ks})\,U^s(k,x)\ +\\ (d^{\dagger}_{\vec ks}+a_{\vec ks})\,V^s(k,x)\end{bmatrix}$$

Similar to the standard QFT one can obtain the nonzero (equal-time) anticommutation relation

$$\left\{\psi_\alpha(x)\ ,\ \psi^{\dagger}_\beta(x')\right\}_{x^0=x^{0\prime}}=\delta_{\alpha\beta}\,\delta^3(\vec x-\vec x'),$$

$$(\alpha,\beta)=(1,2,3,4)$$

Creation and annihilation operators are also constrained to obey the following anticommutation rules

$$\{b_{\vec ks},\,b^{\dagger}_{\vec k's'}\}=\{d_{\vec ks},\,d^{\dagger}_{\vec k's'}\}=\ \delta_{ss'}\,\delta^3(\vec k-\vec k')$$

$$\{a_{\vec ks},\,a^{\dagger}_{\vec k's'}\}=\{c_{\vec ks},\,c^{\dagger}_{\vec k's'}\}=\delta_{ss'}\ \delta^3(\vec k-\vec k')$$

$$\{b_{\vec{k}s}, b_{\vec{k}'s'}\} = \{b^{\dagger}_{\vec{k}s}, b^{\dagger}_{\vec{k}'s'}\} = \{d_{\vec{k}s}, d_{\vec{k}'s'}\} = \{d^{\dagger}_{\vec{k}s}, d^{\dagger}_{\vec{k}'s'}\} = 0$$
$$\{a_{\vec{k}s}, a_{\vec{k}'s'}\} = \{a^{\dagger}_{\vec{k}s}, a^{\dagger}_{\vec{k}'s'}\} = \{c_{\vec{k}s}, c_{\vec{k}'s'}\} = \{c^{\dagger}_{\vec{k}s}, c^{\dagger}_{\vec{k}'s'}\} = 0$$
$$\{a_{\vec{k}s}, b_{\vec{k}'s'}\} = \{a^{\dagger}_{\vec{k}s}, b^{\dagger}_{\vec{k}'s'}\} = \{a^{\dagger}_{\vec{k}s}, b_{\vec{k}'s'}\} = \{a_{\vec{k}s}, b^{\dagger}_{\vec{k}'s'}\} = 0$$
$$\{a_{\vec{k}s}, c_{\vec{k}'s'}\} = \{a^{\dagger}_{\vec{k}s}, c^{\dagger}_{\vec{k}'s'}\} = \{a^{\dagger}_{\vec{k}s}, c_{\vec{k}'s'}\} = \{a_{\vec{k}s}, c^{\dagger}_{\vec{k}'s'}\} = 0$$
$$\{a_{\vec{k}s}, d_{\vec{k}'s'}\} = \{a^{\dagger}_{\vec{k}s}, d^{\dagger}_{\vec{k}'s'}\} = \{b_{\vec{k}s}, c_{\vec{k}'s'}\} = \{b^{\dagger}_{\vec{k}s}, c^{\dagger}_{\vec{k}'s'}\} = 0$$
$$\{b_{\vec{k}s}, c_{\vec{k}'s'}\} = \{b^{\dagger}_{\vec{k}s}, c^{\dagger}_{\vec{k}'s'}\} = \{b^{\dagger}_{\vec{k}s}, c_{\vec{k}'s'}\} = \{b_{\vec{k}s}, c^{\dagger}_{\vec{k}'s'}\} = 0$$
$$\{b_{\vec{k}s}, d_{\vec{k}'s'}\} = \{b^{\dagger}_{\vec{k}s}, d^{\dagger}_{\vec{k}'s'}\} = \{b^{\dagger}_{\vec{k}s}, d_{\vec{k}'s'}\} = \{b_{\vec{k}s}, d^{\dagger}_{\vec{k}'s'}\} = 0$$

The vacuum state $|0>$ is defined as a state that is destroyed by all annihilation operators

$$b_{\vec{k}s}\left|0>=0,\ d_{\vec{k}s}\right|0>=0 \qquad \forall \vec{k}$$
$$a_{\vec{k}s}\left|0>=0,\ c_{\vec{k}s}\right|0>=0 \qquad \forall \vec{k}$$

One "physical" and one "unphysical" particle states are obtained by acting the creation operators on vacuum state

$$b^{\dagger}_{\vec{k}s}\left|0>=\right|1^{b}_{\vec{k}s}>=\left|one\ physical\ particle\ state\right\rangle\ \forall\vec{k}$$
$$c^{\dagger}_{\vec{k}s}\left|0>=\right|\overline{1}^{c}_{\vec{k}s}>=\left|one\ unphysical\ particle\ state\right\rangle\ \forall\vec{k}$$
$$d^{\dagger}_{\vec{k}s}\left|0>=\right|1^{d}_{\vec{k}s}>=\left|one\ physical\ antiparticle\ state\right\rangle\ \forall\vec{k}$$
$$a^{\dagger}_{\vec{k}s}\left|0>=\right|\overline{1}^{a}_{\vec{k}s}>=\left|one\ unphysical\ antiparticle\ state\right\rangle\ \forall\vec{k}$$

Comparing the four statements $b^{\dagger}_{\vec{k}s}$, $d^{\dagger}_{\vec{k}s}$ are called the creation operators of physical one-particle and one-antiparticle states with positive energy, respectively, running forward in time.

$c^{\dagger}_{\vec{k}s}$, $a^{\dagger}_{\vec{k}s}$ are also called the creation operators of unphysical one particle and one-antiparticle

with negative energy, respectively, running backward in time.

The commutation relations together with the normalization of the vacuum, $<0|0>=1$, lead to positive norms for both physical and unphysical parts

$$<1^{b}_{\vec{k}s}\left|1^{d}_{\vec{k}'s}\right> = \delta_{bd}\ \delta^{3}(\vec{k}-\vec{k}')$$
$$<\overline{1}^{a}_{\vec{k}s}\left|\overline{1}^{c}_{\vec{k}s}\right> = \delta_{ac}\ \delta^{3}(\vec{k}-\vec{k}')$$

It is seen that the unphysical (negative-energy) states for spinor field vise versa of the boson fields' (which have both positive and negative norms and therefore are defined in Krein space), are positive-norm states moving forward in time (vise versa of antiparticles[14]). Hence, the space of quantization for spinor fields is called generalized Krein space.

Since the Dirac equation is one degree less than the Klein-Gordon equation, therefore the norm of spinor field in generalized Krein space is positive. However, the unphysical positive- norm states of spinor field are different from physical antiparticles. Physical antiparticles (e.g. positron), although having negative energy, move backward in time, and therefore they are observable particles. But, unphysical negative energy states of spinor field move forward in time and are not observable.

It is noteworthy that the two sets of solutions of the scalar and vector (boson) fields are complex conjugate of each other. However, in the case of spinor field there is no such relation between them. It is due to the Dirac equation which is not real.

The calculation of the Hamiltonian and momentum operator of Dirac spinor field in terms of the new Fourier modes defined in generalized Krein space leads to

$$H=\int d^{3}\vec{k}\,\omega_{\vec{k}}\sum_{s=1,2}\begin{bmatrix} b^{\dagger}_{\vec{k}s}b_{\vec{k}s}+b^{\dagger}_{\vec{k}s}c^{\dagger}_{\vec{k}s}+c_{\vec{k}s}b_{\vec{k}s}+c_{\vec{k}s}c^{\dagger}_{\vec{k}s}- \\ d_{\vec{k}s}d^{\dagger}_{\vec{k}s}-d_{\vec{k}s}a_{\vec{k}s}-a^{\dagger}_{\vec{k}s}d^{\dagger}_{\vec{k}s}-a^{\dagger}_{\vec{k}s}a_{\vec{k}s}\end{bmatrix}$$

$$\vec{P}=\int d^{3}\vec{k}\,\vec{k}\sum_{s=1,2}\begin{bmatrix} b^{\dagger}_{\vec{k}s}b_{\vec{k}s}+b^{\dagger}_{\vec{k}s}c^{\dagger}_{\vec{k}s}+c_{\vec{k}s}b_{\vec{k}s}+c_{\vec{k}s}c^{\dagger}_{\vec{k}s}- \\ d_{\vec{k}s}d^{\dagger}_{\vec{k}s}-d_{\vec{k}s}a_{\vec{k}s}-a^{\dagger}_{\vec{k}s}d^{\dagger}_{\vec{k}s}-a^{\dagger}_{\vec{k}s}a_{\vec{k}s}\end{bmatrix}$$

It is immediately seen that the energy and momentum of the vacuum state are automatically zero, $<0|\ H\ |0>=0$, $<0|\ P\ |\ 0>=0$, and the imposition of normal ordering prescription is not required. (This prescription is vital for the vacuum energy in the Hilbert space quantization.)

The Feynman propagator of spinor field in standard quantum field theory is defined as the time-ordered product of fields

$$i\,S^{p}_{F}(x,x') = \left\langle 0\left|T\,\overline{\psi}(x)\,\psi(x')\right|0\right\rangle$$

giving

$$S^{F}_{P}(x,x') = (i\partial\!\!\!/+m)\,G^{F}_{P}(x,x') =$$
$$\int \frac{d^{4}k}{(2\pi)^{4}}\,\frac{k\!\!\!/+m}{k^{2}-m^{2}+i\in}\,e^{-ik.(x-x')}$$

which suffers from an ultraviolet divergence. The time-ordered propagator of spinor field in generalized Krein space is defined

$$iS_{T}(x,x') =<0\left|T\left[\overline{\psi}_{K}(x)\ \psi_{K}(x')\right]\right|0>$$

yielding

$$S_{T}(x,x') = (i\partial\!\!\!/+m)\frac{1}{2}\left[G^{p}_{F}(x,x')+G^{p*}_{F}(x,x')\right] =$$
$$(i\partial\!\!\!/+m)G_{T}(x,x') \qquad (1)$$

or

$$S_{T}(x,x') = \frac{1}{2}\left(S^{p}_{F}(x,x') + \gamma^{5}\gamma^{0}\,S^{p\dagger}_{F}(x,x')\,\gamma^{0}\gamma^{5}\right)$$

In the momentum space for the new propagator we obtain[23,24]

$$\tilde{S}_T(k) = \tilde{S}_F^P(k) - \gamma^0 \tilde{S}_F^{P\dagger}(k)\gamma^0$$

$$\frac{1}{\not k - m + i\varepsilon} - \frac{1}{\not k - m - i\varepsilon} =$$

$$-i2\pi(\not k + m)\,\delta(k^2 - m^2) = PP\frac{1}{\not k - m}$$

where P P is the principal part symbol. It has been shown that the time-ordered propagator of the scalar field in Krein space is[12,20]

$$G_T(x,x') = \frac{1}{2}[G_p^F(x,x') + G_p^{F*}(x,x')] =$$

$$\int \frac{d^4k}{(2\pi)^4} e^{-ik.(x-x')} PP\frac{1}{k^2 - m^2}$$

giving

$$G_T(x,x') = \mathrm{Re}\,G_F^p(x,x') =$$

$$-\frac{1}{8\pi}\delta(\sigma_0) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}, \qquad (2)$$

$$\sigma_0 \geq 0$$

where $2\sigma_0 = \eta_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu)$. This function is singular only on the light cone. However it has been shown that the incorporation of quantum metric fluctuations removes the singularities of Green's functions on the light cone[25]. In a previous work it has been established that the combination of QFT in Krein space together with the consideration of quantum metric fluctuations results in QFT without any divergence[12]:

$$\langle G_T(x-x')\rangle = -\frac{1}{8\pi}\sqrt{\frac{\pi}{2<\sigma_1^2>}} \times$$

$$\exp\left(-\frac{\sigma_0^2}{2<\sigma_1^2>}\right) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}$$

where $<\sigma_1^2>$ is related to $_1$ the density of gravitons. When $\sigma_0 = 0$, due to the metric quantum fluctuation $<\sigma_1^2> \neq 0$, and we have

$$\langle G_T(0)\rangle = -\frac{1}{8\pi}\sqrt{\frac{\pi}{2<\sigma_1^2>}} + \frac{m^2}{16\pi}$$

Inserting (2) into (1) the time-ordered propagator of Dirac spinor field in generalized Krein space is obtained

$$S_T(x,x') = \frac{1}{8\pi} i\gamma^\mu(x_\mu - x'_\mu) \times$$

$$\left\{ \begin{array}{l} \sqrt{\frac{\pi}{2<\sigma_1^2>}} e^{-\frac{\sigma_0^2}{2<\sigma_1^2>}} \left[\frac{\sigma_0}{<\sigma_1^2>} + m^2 \frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}\right] + \\ \frac{m}{2\sqrt{2}}\theta(\sigma_0)\left[\sqrt{2m^2\sigma_0}\; J_0(\sqrt{2m^2\sigma_0}) - 2J_1(\sqrt{2m^2\sigma_0})\right] \end{array} \right\}$$

$$+\frac{m}{8\pi}\left[-\sqrt{\frac{\pi}{2<\sigma_1^2>}} e^{-\frac{\sigma_0^2}{2<\sigma_1^2>}} + m^2\theta(\sigma_0)\frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}\right]$$

which is free of any divergence. It should be noted that the auxiliary negative-frequency states can not propagate in the physical world, and they only play the role of an automatic renormalization tool for the theory.

4 Conclusions

In standard quantum field theory, to eliminate the divergences that appear in the physical quantities, the normal ordering (renormalization) procedure has been adopted for free (interacting) fields. However, the divergences seem to disappear once the requirement of the positivity of norm and energy is relaxed. The addition of the new unphysical states, thus, leads us to the Krein/generalized Krein space quantization for boson/spinor fields. In this paper the quantization of free spinor field is reformulated in generalized Krein space. Once again it is found that the theory is automatically renormalized. The new method has also been applied successfully to reformulate QED in Krein space[14].

The physical interpretation of the unphysical negative energy states is not yet clear. How- ever, in the case of spinor field one can interpret as the unphysical particle and antiparticle running in the inverse time direction! This case will need a careful consideration, which will be discussed in the coming papers. But the most important question is that: Does this quantization only regularize the theory without changing the physical content of the theory?

References

1. De Bievre S., Renaud J., "The Massless Quantum Field on the 1+1 Dimensional de Sitter Space" , Phys. Rev. D, 1998, 57 : 6230-6241.

2. Gazeau J. P., Renaud J., Takook M. V., "Gupta-Bleuler Quantization for Minimally Coupled Scalar Field in de Sitter space", Class Quantum Gravity, 2000, 17 : 1415-1434.

3. Dirac P. A. M., "A Theory of Electrons and Protons", Proc. Roy. Soc. A, 1942, 180 : 1-40.

4. Gupta S. N., "Theory of Longitudinal Photons in Quantum Electrodynamics", Proc. Phys. Soc. A, 1950, 63 : 681-691.

5. Bleuler K., "A New Method of Treatment of the Longi-

tudinal and Scalar Photons", Helv. Phys. Acta, 1950, 23: 567-586.

6. Hawking S. W., Hertog T., "Living with Ghosts", Phys. Rev. D, 2002, 65 : 103515-103527.

7. Takook M.V., Théorie quantique des champs pour des syst`emes élémentaires "massifs" et de "masse nulle" sur l'espace-temps de de Sitter (Th`ese de l'universit´e Paris VI, 1997).

8. Garidi T., Gazeau J. P., Takook M. V., "Massive Spin-2 Field in de Sitter Space", J. Math. Phys., 2003, 44 : 3838-3862.

9. Takook M.V., "Covariant Two-point Function for Linear Gravity in de Sitter Space", Proceedings of the 6th International Wigner Symposium (WIGSYM6), Istanbul, Turkey, 16-22 August 1999. 10. Takook M. V., Int. J. Mod. Phys. E 14, 219 (2005).

10. Takook M. V., "A Natural Renormalization of the One-loop Effective Action for Scalar Field in Curved Space-time", Int. J. Mod. Phys. E, 2005, 14 : 219-224.

11. Takook M. V., "Covariant Two Point Function for Minimally Coupled Scalar Field in de Sitter Space-time", Mod. Phys. Lett. A, 2001, 16 : 1691-1698.

12. Rouhani S., Takook M. V., "A Naturally Renormalized Quantum Field Theory", 2006, gr-qc/0607027

13. Payandeh F., Mehrafarin M., Takook M.V., "Field quantization in Krein space", "13th International Symposium on Particles, Strings, and Cosmology" (London, 2-7 July 2007), AIP conference proceedings, 2007, 957 : 249-252.

14. Payandeh F., Mehrafarin M., Takook M. V., "One-loop Approximation of Møller Scattering in Generalized Krein Space Quantization", to be published in "Science in China Series G: Physics, Mechanics and Astronomy, issue 2".

15. Ramirez A., Mielinik B., "The Challenge of Non-hermitian Structures in Physics ", Rev. Mex. Fis., 2003, 49S2: 130-133.

16. Helfer A. D., "The Physics of Negative Energy Densities", Mod. Phys. Lett. A, 1998, 13: 1637.

17. Visser M., Barcelo C., "Energy conditions and their cosmo´ logical implications", Class. Quantum Grav.,2000, 17: 3843.

18. Blinnikov S., "Cosmic Gamma-ray Bursts" , Surv. High Energy. Phys. 2000, 15: 37.

19. Rouhani S., Takook M. V., "Tree-level Scattering Amplitude in de Sitter Space", Europhys. Lett., 2004, 68 : 15-20.

20. Takook M. V., "Negative Norm States in de Sitter Space and Quantum Field Theory without Renormalization Procedure", Int. J. Mod. Phys. E, 2002, 11 : 509-518.

21. Khosravi H., Naseri M., Rouhani S., Takook M. V., "Casimir Effect in Krein Space Quantization",

22. Greiner W., Reinhardt J., "*Field quantization"* (Springer-Verlag, Berlin, 1996).

23. Itzykson C., Zuber J. B., "*Quantum Field Theory*", (McGraw-Hill, 1980).

24. Takook M. V., " Negative Norm States in de Sitter Space ", 2002, gr-qc/0006019.

25. Ford H. L., "Quantum Field Theory in Curved Space-time", 1997, gr-qc/9707062.