EXPLORING THE REASONS BEHIND THE CIRCULAR SHAPE OF DRUMS

EXPLORANDO LAS RAZONES DETRÁS DE LA FORMA CIRCULAR DE LOS TAMBORES

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Many of the drums played all over the world are circular in shape. There are many practical reasons for the instrument makers to choose a circular shape for the drums. Apart from practical side, there are some scientific bases for the circular shape of drums. This paper investigates the physics behind the circular shape of the drums.

La mayoría de los tambores utilizados alrededor del mundo tienen forma circular. Hay muchas razones prácticas para seleccionar esta forma. Además the razones prácticas, hay razones científicas de la forma circular de los tambores. Este artículo investiga la física detrás de esta forma circular.

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I. INTRODUCTION

Musical instruments are an integral part of any visual or audio performance. Among these instruments, drums are used for producing either music or rhythm [1, 2]. Most of the drums are made with wood and animal skin. It is seen that almost all drum heads made with animal skin are circular in shape. The Fig. 1 shows an ensemble of chenda, a temple musical instrument played in Kerala (popularly known as God's own country), a small state in the southern part of India [3].

Figure 1. An ensemble of chenda played in Kerala, India.

The reason for the circular shape of drums is an interesting research problem which has to be addressed by the tools of Physics. With common reasoning ability one simple reason is that the shaping of the wood and skin could be easily done with minimum labor for making a circular form. Another possible argument regarding the practical side of instrument making is that, to stretch membrane uniformly on drum head and to adjust uniform tension on the membrane, the circular shape is the best one. The question of identification of shape of drums from their eigenvalue spectrum was put forward by Kac [4]. Kac and others studied the problem mathematically by considering two drums with same set of eigenfrequency spectrum and tried to prove that the drums have the same shape [5, 6]. Later investigations [7] found that same set of eigenvalues can happen for drums with different complex shapes also. But, for simple shaped drums like circular, many information of the geometry of the drum head can be identified from its eigenfrequency spectrum [8,9]. Both western and Indian drums have circular shapes and our paper evaluates some of the physics behind the circular shape.

II. EIGENFREQUENCY SPECTRUM OF SOME DIFFERENT SHAPED DRUMS

To differentiate and identify the characteristics of membranes, we consider the modes of vibrations of rectangular, equilateral triangular and circular membranes. Let us represent the frequency of vibration of different modes as f_{nm} where n, m are the number of half waves in normal modes of vibration of membranes in x and y direction respectively. The frequencies of different modes of vibration of drums may be different but their frequency ratio remains the same. Hence the frequency ratios of all three membranes are found for first 10 modes by dividing frequency of each mode by the frequency of the first or fundamental mode.

Rectangular membrane: For a rectangular membrane, the frequencies of vibration of different modes f_{nm} are given by [10]:

$$f_{nm} = \frac{v}{2}\sqrt{\frac{n^2}{a^2} + \frac{m^2}{b^2}}, \tag{1}$$

where a, b are sides of the membrane and v is the velocity of sound through the membrane . We calculated the frequencies and the frequency ratio of modes of rectangular membrane and the results obtained are given in Table 1. For the calculation, the sides are chosen to be $a = 0.08$m and $b = 0.1$m. Since v is taken as constant for all membranes and we are more interested in the frequency ratio, the value of f_{nm}/v is tabulated.

Table 1. The frequency ratio of the rectangular membrane.

Mode of vibration	Frequency/v (f_{nm}/v)	Frequency ratio
1.1	8.0039	1
1.2	11.7924	1.4733
2.1	13.4629	1.6820
2.2	16.0078	2
1.3	16.2500	2.0302
3.1	19.4052	2.4244
2.3	19.5256	2.4395
1.4	20.9538	2.6179
3.2	21.2500	2.6549
2.4	23.5849	2.9466

Equilateral membrane: The frequency of vibration of an equilateral triangular membrane is given by [11]:

$$f_{nm} = \frac{v}{a}\sqrt{n^2 + m^2 + nm}. \tag{2}$$

For an equilateral triangular membrane all sides are equal. For calculations the sides are assigned the value $a = 0.08$m. The frequency ratio is given in Table 2.

Table 2. The frequency ratio of the equilateral membrane.

Mode of vibration	Frequency/v (f_{nm}/v)	Frequency ratio
1.1	21.6506	1
1.2	33.0718	1.5275
2.1	33.0718	1.5275
2.2	43.3012	2
1.3	45.0693	2.0816
3.1	45.0693	2.0816
2.3	54.4862	2.5166
3.2	54.4862	2.5166
1.4	57.2821	2.6457
4.1	57.2821	2.6457

Circular membrane: The frequency of vibration of a circular membrane is given by [12]:

$$f_{nm} = \frac{x_{nm} v}{2\pi a}. \tag{3}$$

Here x_{nm} are the roots of Bessel function of order n. Here n and m represents the number of half waves of modes of vibration in θ and r direction since polar coordinate is used for circular membrane problem. The radius of the membrane is chosen as $a = 0.08$m for calculation. The obtained values are given in Table 3.

Table 3. The frequency ratio of the circular membrane.

Mode of vibration	Frequency/v (f_{nm}/v)	Frequency ratio
0.1	4.7866	1
1.1	7.6267	1.5933
2.1	10.2221	2.1355
0.2	10.9874	2.2954
3.1	12.6994	2.6531
1.2	13.9641	2.9173
4.1	15.1041	3.1554
2.2	16.7539	3.5001
0.3	17.2247	3.5985
3.2	19.4287	4.0589

From the Table 1, 2 and 3 the ratio of frequencies of different membrane gives some important insights. They are:

- It is found that the tenth mode of rectangular membrane produces 2.9466 times fundamental frequency and for equilateral triangular membrane same mode produces a frequency 2.6457 times the fundamental frequency. But for a circular membrane, the tenth mode produces 4.0589 times the fundamental frequency. This shows that with same number of modes circular membrane can produce wide range of frequency than other membranes. So a player must have to excite less number of modes to obtain higher frequencies compared other shaped membranes. This reduces the strain of the player if one uses the circular shaped drum.
- It is also seen that the frequency set of all membranes is different. The pitch, tone color and amplitude are interrelated and all depends on fundamental frequency, intensity of sound and overtone structure [13]. Hence the tone color of the sound produced by the drums of these membranes will be heard differently.

III. ISOPERIMETRIC THEOREM AND ITS EFFECTS

In two dimension, out of many shapes with same perimeter, circle has the largest area and this is called isoperimetric theorem [14]. For a drum, if the shape of the membrane is circular then the area of vibration will be more than other shapes. This increase the sound intensity or amplitude. Isoperimetric theorem has deeper effects on the sound produced by drum which was found by Lord Rayleigh [15]. He studied about membranes of different shapes such as rectangle, equilateral triangle, circle and many more of same area and found that the pitch or fundamental frequency of the deepest tone is smallest for circle. In their paper Z. Lu and J. M. Rowlett [16] found mathematically that a listener could identify the corners of a drum . This indicates that the sound produced by a circular membrane and other shaped membranes with different number of corners such as rectangle or triangle will be heard differently. The isoperimetric theorem and other works [4, 17] suggest following ideas. Circular shaped drums:

- Produce more sound compared to other drums
- Produce low pitched sounds or bass sound
- Can maintain the particular tone quality or timbre.

IV. SYMMETRY AND SHAPE OF THE DRUM

In our daily life we find many natural objects with beautiful symmetry such as flowers, leafs and the physics behind the symmetry of objects is a vast and promising field of research [18]. If some transformation such as rotation or reflection is performed on an object with any shape and if the shape remains unaffected, then the object with that particular shape is said to be symmetric. For a 2D shape, a line of symmetry is a line passing through the centre which divides it into identical halves. As the symmetry of the object increases the number of lines will also increase. For the rectangle there are two lines of symmetry, for the equilateral triangle it is three and for the square the number is four and so on. For circular shape there is infinite number of lines of symmetry and hence it is the most symmetrical shape in 2D [19]. From Group theory, the group formed by circular shape have infinite number of rotation and reflection symmetry [20]. The symmetry and degeneracy are interrelated. The circular drum has large number of degenerate modes represented by sine and cosine solutions of the circular membrane problem. In real circular drums, the tampering of the circular symmetry of the rim, application of varying tension on the membrane and the change in the thickness of the membrane creates a shift in frequency of the degenerate modes and the beats produced can be removed by the player [21]. So the circular symmetry creates the following effect:

- The same sound is produced by the drum played from any side of the circular head.

V. CONCLUSIONS

The paper discussed some features of physics behind the circular shape for the drums. The practical easiness in construction is one aspect to choose the circular shape for the drums. The circular shape gives the drum most low pitch or bass sound compared with other shapes. The particular symmetry helps in tuning of the drum and even distribution of the tension on the membrane.

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