THE 2021 PHYSICS NOBEL PRIZE: CLIMATE AND DISORDER

EL PREMIO NOBEL DE FÍSICA DE 2021: CLIMA Y DESORDEN

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The 2021 Nobel Prize in Physics was jointly awarded to Syukuro Manabe, Klaus Hasselmann and Giorgio Parisi, *"for groundbreaking contributions to our understanding of complex physical systemas"*. The Prize was divided in two parts, Manabe and Hasselman were recognized *for their modelling of Earth's climate* and Giorgio Parisi *for the discovery of the interplay between disorder and fluctuations in physical systems*. Here we review the more important aspects of these contributions and try to put them within a unique conceptual framework.

El premio Nobel en Física del 2021 fue otorgado a Syukuro Manabe, Klaus Hasselmann y Giorgio Parisi, por el impacto de sus contribuciones a la comprensión de los sistemas complejos. El premio se dividió en dos partes, Manabe y Hasselmann fueron reconocidos *por sus trabajos de modelación del clima de la Tierra* y Giorgio Parisi *por haber descubierto la conexión entre el desorden y las fluctuaciones en los sistemas físicos*. Aqui resumimos los aspectos más importantes de estas contribuciones tratando de colocarlas dentro de un mismo marco conceptual.

PACS: Complex systems (sistemas complejos), 89.75.-k; modeling of global climate (modelación del clima global), 92.70.Np

I. INTRODUCTION: FLUCTUATIONS AND DISORDER

When physicists talk about fluctuations they usually formalize the idea using a Langevin-like equation [1] that in its simplest version takes the following form:

$$\frac{dx}{dt} = -\frac{\partial V(x)}{\partial x} + \eta(t) \tag{1}$$

where x represents the system's coordinate, $V(x)$ is a potential and η encodes the information about the fluctuations in the system. The latter term is a "noise"function defined by its statistical properties, for example, the mean and the variance. The task is to understand, given $V(x)$ and the statistical properties of η how x evolves in time. The difficulty comes from the fact that given the statistical nature of η there is a possible ensemble of $x(t)$ consistent with equation (1).

This ensemble of trajectories can be characterized by a probability distribution that, under very general conditions, and in the long time limit, can be written as [2]:

$$P(x) = \frac{e^{-\beta V(x)}}{Z} \tag{2}$$

where β is defined by the fluctuations in η, and Z is a partition function.

Things become much more difficult when x is not a coordinate, but a bunch of them, for example a high-dimensional vector $\vec{x}$, or more generally when it is a field ϕ, with both spatial and temporal coordinates $\phi(\vec{r}, t)$. It is even harder when $V(\vec{r})$ also is defined statistically. For example, when the interaction between two particles in the system depends specifically on the two particles under consideration. In this later case we often say that the system is *disordered*.

In these general situations equation (1) may adopt the form of a "Langevin-like equation", i.e.:

$$\frac{d\phi(\vec{r}, t)}{dt} = F(\vec{r}, \phi(\vec{r})) + \eta(\vec{r}, t) = -\frac{\partial V(\vec{r}, \phi(\vec{r}))}{\partial \phi(\vec{r})} + \eta(\vec{r}, t) \tag{3}$$

where ϕ is a vector field, $F(\vec{r}, \phi(\vec{r, t}))$ is a general function that sometimes can be written as the derivative of a potential V that may depend on the coordinate $\vec{r}$ and the field. As before, η plays the role of the noise. If the noise is absent we are in the presence of a deterministic equation for the field ϕ.

In what follows we explain how the work of Manabe, Hasselmann and Parisi can be cast within the above described framework and which were their main contributions to their respective fields of research.

II. CLIMATE MODELS OF THE EARTH

One half of the Nobel Prize in Physics of 2021 was awarded to Pr. Syukuro Manabe and Pr. Klaus Hasselmann. Their work is at the basis of our current knowledge of the Earth's climate and the influence of humans on it.

II.1. Syukuro Manabe: Carbon Dioxide and Radiation Balance

Probably the *older* and at the same time *modern* model for climate dynamics was proposed by S. Arrhenius already in 1896 [3].

In short, the Earth receives energy from the Sun, and radiates energy as a black body. The atmosphere is an intermediate layer that mitigates the energy arriving from the Sun, and radiates to the empty space, and back to the Earth, the energy received from the Earth's radiation.

This can be considered as the starting point for every model of Earth's climate. For example, the effects of CO_2 in the temperature of the Earth can be taken into account by changing the radiative properties of the layer representing the atmosphere. This was the approach followed by Manabe and Wetherald [4], in what is now considered for many as the most important paper in the history of climatology [5].

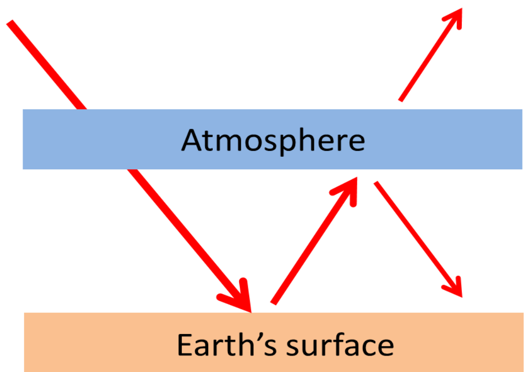

Figure 1. Arrhenius picture of the Earth climate. The Earth receives energy from the sun and radiates as a black body. The atmosphere, also radiates, to the Earth, and to the space.

For simplicity, they considered the layer representing the atmosphere as a vertical one-dimensional system on top of the Earth's surface –remember that, back in 1967, computers were not particularly powerful. This layer had the usual radiative properties, but they included on it, the effect of water vapor convection. This allowed them to trace the influence of the humidity profile of the atmosphere in the temperature. In practice it means that the temperature dynamics of the Earth would depend on two coupled equations, one for the temperature itself, and the other for the distribution of the humidity in the layer. To check the relevance of their model, they compared their results in two different cases: i) considering the actual concentration of specific gases in the atmosphere and ii) removing these gases. They found that the equilibration temperature of the Earth was larger in the first case.

With the knowledge that we have today on the Greenhouse effect, the result may look as expected, but it was not in 1962. Actually, it remained unnoticed for almost ten years, only to be now recognized as one of the first modeling approaches supporting the impact of mankind in the heating of the Earth.

More than 10 years later Manabe and Wetherald [6] published another breakthrough paper introducing a Global Climate Model (GCM) that included the dynamics of heat, mass, momentum and the radiation around the globe where again the studied the role of CO_2 on the Earth's temperature.

II.2. Klaus Hasselmann: Weather and Climate

If the GCM introduced by Manabe and collaborators can be considered as a deterministic theory for the fields (temperature, density of gases, water vapor, etc.) describing the dynamics of the Earth's climate, the work of Pr. Hasselmann went a step further, coupling these equations –actually an abstract version of them– to the Earth's weather [7].

In Hasselmann's picture, the weather would act as a noise in the framework of Global Climate Models. Let us call c_i the variables describing the climate, as above, and w_i new variables that will describe the weather, also locally. Thus, in a very general way we can write:

$$\dot{c}_i = f_i(\vec{w},\vec{c}) \quad (4)$$

$$\dot{w}_i = g_i(\vec{w},\vec{c}) \quad (5)$$

$$(6)$$

The intuition is that w_i and c_i vary within different time scales. The weather variables, w_i change faster than c_i. Then, when studying the dynamics of the fast variables one can assume that the slow variables are constant. On the contrary, in the study of the dynamics of the slow variables, the fast variables can be substituted for their average values and some noise representing the fluctuations.

In short:

$$\dot{c}_i \approx f_i(\langle\vec{w}\rangle_{\vec{c}} + \vec{w}_{fluc}, \vec{c}) \quad (7)$$

where, $\langle\vec{w}\rangle_{\vec{c}}$ represents the average of the variable $\vec{w}$ given the state of the climate, while $\vec{c}$ and $\vec{w}_{fluc}$ represent the fluctuations of the weather. Expanding the expression above, we get:

$$\dot{c}_i \approx f(\langle\vec{w}\rangle_{\vec{c}},\vec{c}) + \frac{\partial f(\langle\vec{w}\rangle_{\vec{c}},\vec{c})}{\partial\vec{c}}\vec{w}_{fluc} = -\frac{\partial V(\vec{c})}{\partial\vec{c}} + \sigma(\vec{c})\eta(t) \quad (8)$$

that takes the form of a Langevin-like equation, see equation (3). Within this picture the weather acts as a noise in the framework of Global Climate Models. The relevance of this work can not be overestimated, it brought to the models of the climate, all the machinery of the stochastic systems of differential equations.

But Hasselmann went farther. In a series of papers [8,9] that span almost 20 years he approached a similar but conceptually different problem. How to compare the results of the models with those of observation? The problem is specially difficult because both are prone to errors and fluctuations and are defined over spatial and temporal scales. Mathematically it can be defined starting from a regression equation:

$$\vec{c} = \mathbf{X}\vec{a} + \vec{w} \quad (9)$$

where $\vec{c}$ represents the measurement of the fields for the climate, the matrix $\vec{X}$ contains the estimate response patterns obtained from Global Climate Models and $\vec{w}$ is usually a Gaussian noise reflecting the weather impact on the measurements. The goal is to obtain $\vec{a}$ as a function of these *known* parameters. The vector $\vec{a}$ indeed gives us information about the importance of the different variables $\mathbf{X}$ in the measurement $\vec{c}$.

III. DISORDER

The second half of the Nobel Prize in Physics of 2021 was awarded to Pr. G. Parisi for his discovery of special types of *order* in disordered systems. This discovery has found deep implications in fields as diverse as condensed matter, mathematics, biology, neuroscience and machine learning.

III.1. Giorgio Parisi and replicas

Giorgio's more original contribution to science –and there were many– can be introduced starting from the most celebrated model of Statistical Physics, the Ising model, whose Hamiltonian has the form:

$$H = -J \sum_{i,j} s_i s_j \tag{10}$$

where s_i and s_j are spins, and J represents an interaction. The corresponding equilibrium distribution his Boltzmann-like:

$$P(\{s\}) = \frac{e^{\beta J \sum_{i,j} s_i s_j}}{Z} \tag{11}$$

This equation is reminiscent of (2).

The physics of this model is quite well understood. Landau's mean field solution [10] provides a qualitative picture of the continuous phase transition from the paramagnetic phase at high temperatures to the ferromagnetic phase at low temperatures. Already in 1944 Onsager, with an unequal tour-de-force of mathematical physics, solved the problem in two dimensions. Later on, the celebrated Renormalization Group Theory for which Wilson was awarded in 1982 the Nobel Prize, provided the conceptual framework to understand and approach this and other models with continuous phase transitions.

Figure 2 provides a simple picture of the physics behind this model.

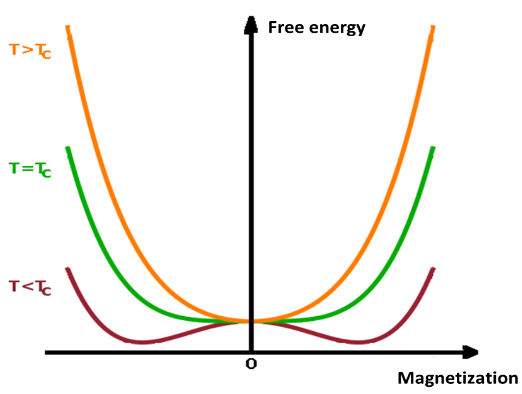

Figure 2. Landau's Free Energy for the Ising model.

At very high temperatures, the free energy has one minimum at zero magnetization, $m = 0$. Below a critical temperature T_c the system magnetizes spontaneously and at every temperature T the magnetization can be both positive ($m > 0$) or negative ($m < 0$). These two solutions correspond to the two minima of the Free Energy represented in Figure 2.

However, the presence of disorder in the Hamiltonian (10) may change dramatically the physics of the problem. Consider for example when $H = -\sum_{i,j} J_{ij} s_i s_j$ where J_{ij} is 1 or -1 , with probability 1/2. In Figure 3, we show a simple representation of the problem for 4 spins in a square lattice.

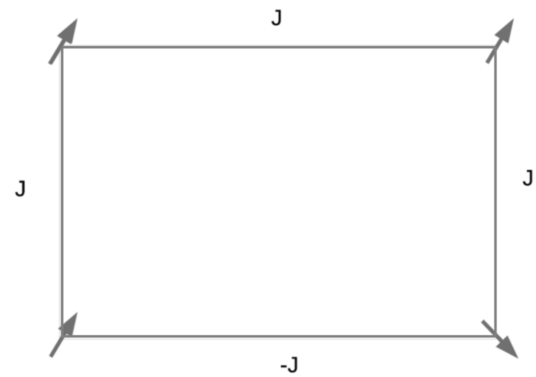

Figure 3. Four spins interacting in a frustrated square lattice. Notice that the right bond is not satisfied. The interaction is ferromagnetic, but the spins are oriented in opposite directions

Notice the impossibility to satisfy all the bonds at the same time. Independently on how we orient the spins, at least one of them is always unsatisfied. We call this phenomenon *frustration*, and is at the basis of the phenomenology behind what we now know as disordered systems. In these models it is hard –if not impossible– to minimize all the interactions between the elements, i.e., particles, spins, variables, of a system. In the case of the disordered Ising model presented above, the frustration arises as a result of the disorder, but in other models, like structural glasses, the frustration is a direct result of the dynamics of the model.

The picture emerging is summarized in Figure 4 where we sketch the kind of free energy landscape expected for this kind of models.

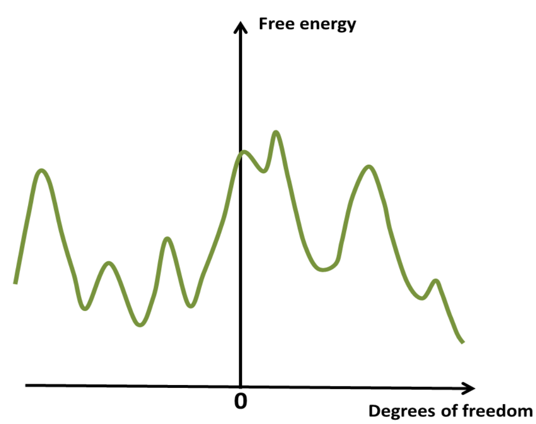

Figure 4. Schematic representation of the Free-Energy landscape for a frustrated system. The free energy is characterized by exponentially many minima.

The many minima, usually exponentially many, appear as a result of the frustration in the system.

To estimate the free energy of the problem one must compute:

$$F = -\langle \log \sum_{s} \exp^{\beta \sum_{ij} J_{ij} s_i s_j} \rangle_J \tag{12}$$

where $\langle \dots \rangle$ indicates the average over the disorder. This average over a logarithmic function is a difficult operation and to deal with it one must resort to the replica trick $\log x = \text{lím}_{n\to 0} \frac{x^n-1}{n}$. With this trick, all the mathematical difficulty is enclosed into the average over the n-power of the partition function

$$\langle Z^n \rangle_J = \langle \sum_{s^n} \exp^{\beta \sum_a \sum_{ij} J_{ij} s_i^a s_j^a} \rangle_J \tag{13}$$

After a lengthy algebra necessary to compute 13 the average free energy (12) will depend on a complex order parameter Q_{ab} that encloses all the information about the solution structure of the system. The celebrated Parisi's solution to this problem was a clever and original proposal for the structure of this matrix [11, 12]. A picture representing it is shown in Fig 5.

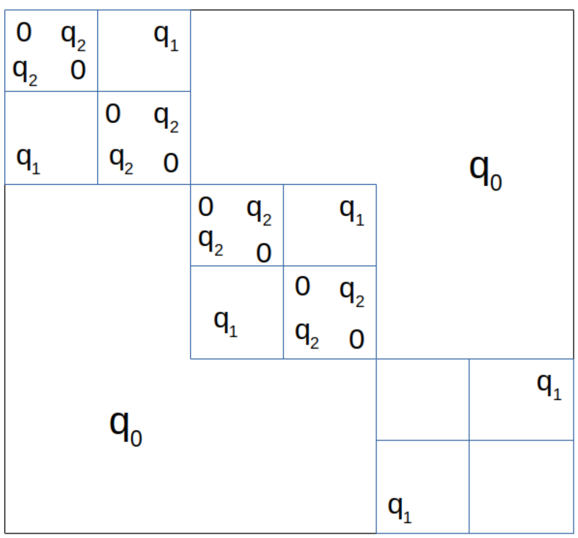

Figure 5. Schematic representation of the Parisi's ansatz, for the matrix $Q_{a,b}$. In its more general form it is parameterized by an infinite number of parameters q_0, q_1, q_2, etc...

But Parisi went further, providing an interpretation for the elements of this matrix:

$$Q_{a,b} = \frac{1}{N} \sum_i \langle s_i \rangle_a \langle s_i \rangle_b \tag{14}$$

where a and b are replica indices, and $Q_{a,b}$ could be interpreted as the overlap of the states within different replicas. It turns out, that this is also a good representation of the properties of the states of the original system, such that:

$$P(q) = \sum_{a,b} w_a w_b \delta(q - Q_{a,b}) \tag{15}$$

where w_a and w_b represent the Boltzmann weights of the states a and b, and $P(q)$ is the disorder-average distribution of overlaps, and plays the role of the order parameter. Possible forms of $P(q)$ appear in figure 6:

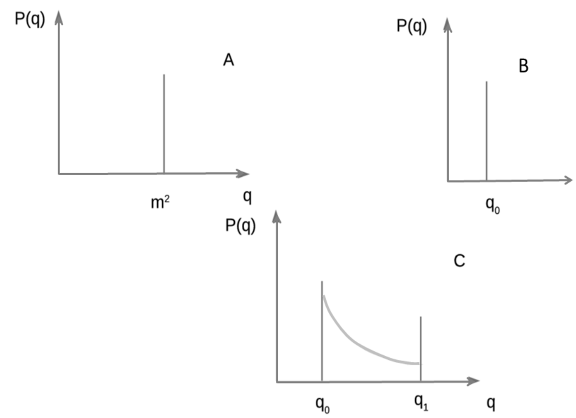

Figure 6. Order parameter $P(q)$ for different models A: Ferromagnet. B: Model with 1-Step Replica Symmetry Breaking. C: Model with a full Replica Symmetry Breaking solution

Although all this machinery reflects the solution of a mean-field model designed to study a rare material, it was soon discovered that it could be used in the study of many more systems. Combinatorial Optimization problems [13–15], Neural Networks [17], Granular Materials [16], Disordered Lasers [18], and more recently also interacting Metabolic Networks [19], are all problems that found in this approach a mathematical and conceptual framework testable in real experiments or in computer simulations.

IV. CONCLUSIONS

I must confess that, when the Editor asked me to write this article, I had some doubts. I was very familiar with Parisi's work, but I literally knew nothing about Earth's climate. I was even surprised by the combination of the names and fields of research involved in the Nobel Prize this year. While Parisi is an *all around* statistical physicist, with contributions that span over a wide range of fields, Climate Physics was not a subject in which he spent too much effort. The work of Manabe and Hasselmann is definitively at the foundation of our modern understanding of Earth's climate, but its connection with Statistical Physics was essentially circumstantial. However, after a couple of weeks reading intermittently the works of these climate scientists, and after reviewing the original works of Parisi, I think that I understand better the combination of names and the true meaning of this selection. Manabe devoted his efforts to build a proper theory of Earth's climate; he did so by introducing simple models, original ideas, but also advanced computational techniques. Hasselmann imported into Climate Physics important mathematical ideas previously developed in other fields. With their work, they contributed in a fundamental way to crack one of the most complex systems we know, *i.e.* Earth's climate. Furthermore, their work had important implications in our understanding of the impact of human kind in the planet. Parisi, on the other hand, devoted the last 30 years of his career to shed light into a large collection of problems from different fields enclosing all of them within a single conceptual framework. In my opinion, these are the three scientifically sound ways to attack scientific problems. To increasingly improve the models describing them, keeping the physics/science comprehensible, to attack them by importing techniques developed in other fields, or to enlarge our understanding looking for concepts and techniques that could be shared by many of them at the same time. I am glad that the first Nobel Prize in Physics specifically awarding the study of Complex Systems embraces all these approaches.

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