CAN THE BLACKETT CONJECTURE DIRECTLY ACCOUNT FOR THE MAGNETIC FIELDS OF CELESTIAL BODIES AND GALAXIES? AND, IS A LAB-BASED TEST FOR THE BLACKETT CONJECTURE FEASIBLE?

¿PUEDE LA CONJETURA DE BLACKETT EXPLICAR DIRECTAMENTE EL CAMPO MAGNÉTICO DE LOS CUERPOS CELESTES Y LAS GALAXIAS? Y, ¿ES POSIBLE REALIZAR UN TEST DE LABORATORIO PARA LA CONJETURA DE BLACKETT?

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According to the Blackett conjecture, any neutral rotating body acquires a magnetic moment proportional to its angular momentum. Using the data on the dipolar magnetic field of Mars, we put a stringent upper limit on the value of the Blackett's constant, the dimensionless constant that relates the magnetic moment to the angular momentum. As a consequence, the Blackett effect cannot directly account for the magnetic fields of celestial bodies and galaxies. We also show that the Blackett effect cannot be tested in a laboratory since the magnetic moment of any rotating lab-scale object would be much smaller than the one produced by the well-known Barnett effect.

La conjetura de Blackett establece que cualquier cuerpo neutro en rotación adquiere un momento magnético proporcional a su momento angular. Empleando datos del momento dipolar magnético de Marte encontramos una cota superior para la constante de Blackett que relaciona el momento magnético con el momento angular. Como consecuencia, el efecto Blackett no puede justificar directamente los campos magnéticos de los cuerpos celestes y las galaxias. También demostramos que el efecto Blackett no puede probarse a nivel de laboratorio puesto que los momentos magnéticos de cualquier cuerpo rotatorio que podamos emplear serían mucho menores que aquellos que producen el efecto Barnett.

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I. INTRODUCCÍON

The Blackett hypothesis [1] is a conjecture according to which any neutral massive rotating macroscopic body should possess a magnetic moment $\boldsymbol{\mu}$ proportional to its angular momentum $\mathbf{J}$ according to

$$\boldsymbol{\mu} = \beta \, \frac{\sqrt{G}}{2c} \, \mathbf{J}. \tag{1}$$

Here, G is the Newton constant, c is the speed of light, and β is a free dimensionless constant of order unity. (In this paper, we use Gaussian-cgs units.) This effect would have his origin in a putative fundamental unified theory of gravitation and electromagnetism, in which the "gravitational magnetism" would emerge.

The plausibility of the Blackett hypothesis reposed entirely on empirical "evidences". Indeed, Blackett observed in his 1947-paper [1] that the magnetic field calculated from Eq. (1) agrees with its observed value for the Earth, Sun, and 78 Virginis (a spectral type B2 star). About 32 years later, Sirag [2] tested the Blackett conjecture using new available data for Mercury, Venus, Jupiter, Saturn, the Moon, and the pulsar Her X-1. Equation (1), once again, seemed to be relatively successful in explaining the observed magnetic fields of celestial bodies.

It is important to stress two points. First, at those times there was no satisfactory explanation for the existence of the Earth magnetic field and in general of magnetic fields of planets and stars, while, today, the presence of magnetic fields in celestial bodies is successfully explained as the result of a dynamo action (for reviews on dynamo theory, stellar dynamo action, and planetary dynamos see [3], [4], and [4], respectively.).

Second, although one could expect a correlation between the angular momentum and the magnetic field of a rotating magnetized body, the impressive feature was that such a (linear) correlation extended over 15 orders of magnitude in both J and μ. Therefore, the Blackett hypothesis, renewed by Sirag, was perhaps a legitimate tentative to give a theoretical explanation for the magnetization of so vastly different celestial bodies.

Blackett conjecture, however, runs into difficulties. For the case of the Earth, for example, the Blackett effect would produce a magnetic dipole directed along the axis of rotation. Instead, not only the present dominant dipole of the Earth's magnetic field is offset about 10° from the rotational axis, but also episodic reversals in field polarity have been recorded. During the $\sim 10^4$ years over which reversals occur, the total magnetic field strength decreases and the field becomes multipolar. Therefore, the magnetic field produced by the Blackett effect cannot explain the Earth's magnetic field, whose origin is today successfully explained by a geodynamo taking place inside the inner and outer cores of the Earth [5]. The same conclusion applies to Jupiter, Uranus, and Neptune which have dipole tilt angles of 9°, 59°, and 45°, respectively [5].

Also, data from the Mars global surveyor and paleointensity data from Apollo samples have shown that Mars and the Moon do not presently have an active magnetic field. Yet a residual crustal magnetization has been detected, which points towards an extinct dynamo action. Venus, Io, Callisto, and Titan, instead, seem to have no surface magnetic field. The lack of magnetization is understood as the consequence of a complete absence of dynamo activity for these celestial bodies [5].

Moreover, the study of 32 magnetic stars [6] reveals that the intensity of the average surface magnetic field of such stars is consistently larger than the one predicted by Blackett [15 stars having magnetic fields one order of magnitude bigger than the one predicted by Eq. (1)] and dipole tilt angles consistently different from zero (27 stars have dipole tilt angles bigger than 10°).

Based on the above observations, the Blackett conjecture, as the direct origin of the magnetic fields of planets and stars, must be rejected (the magnetic field produced by the Blackett effect could act, eventually and at most, as a "seed" for celestial turbulent dynamos). Nevertheless, it could play a role in "galactic magnetism", namely it could in principle explain the presence microgauss, large-scale correlated magnetic fields observed in all types of galaxies (for a review on cosmic magnetic fields, see [7, 8]). Indeed, recently enough, Opher and Wichoski [9] have applied the Blackett conjecture to the study of galactic magnetic fields. Their results suggest that the Blackett effect could directly accounts for the magnetization of galaxies if the Blackett constant β were in the range $10^{-2} \lesssim \beta \lesssim 10^{-1}$. Jimenez and Maroto [10], on the other hand, have shown that the Blackett hypothesis naturally emerges in an electromagnetic theory that includes nonminimal couplings to the spacetime curvature. These analyses seem, then, not to rule out the Blackett hypothesis.[1]

The result of this paper is twofold. First we point out that, due to the smallness of the Blackett constant, Blackett conjecture cannot account for the origin of galactic magnetism. Second, we show that such a conjecture is not feasible for a direct lab-based test.

II. LIMIT ON BLACKETT'S CONSTANT

Planets and satellites of the solar system are neutral rotating systems which, according to the Blackett conjecture, should be magnetized, and indeed they are, as revealed by the data of a number of spacecrafts [5]. Approximating such systems as spheres of radius R, the average magnetic field $\mathbf{B}$ inside (and on the surface) is proportional to the magnetization, $\mathbf{B} = 2\boldsymbol{\mu}/R^3$ [12]. Outside the systems, the magnetic field is that of a magnetic dipole with magnetic moment $\boldsymbol{\mu}$. The angular momentum can be written as $J = 2\pi I/P$, where P is the intrinsic rotational period, $I = \frac{2}{5}kMR^2$ is the moment of inertia, M the mass, and k is the moment-of-inertia parameter (which for an homogeneous and perfectly spherical object is equal to 1).

A strong constraint on β is given by the non-observation of a dipolar magnetic field of Mars (yet a residual crustal magnetization has been detected, which seems to point towards an extinct dynamo action). Using the upper limit on the Martian magnetic dipole moment, $\mu \lesssim 2 \times 10^{20}\mathrm{G\,cm}^3$ [13],

$$\beta \lesssim 2 \times 10^{-5}, \tag{2}$$

where we used $M = 6.4 \times 10^{23}$kg, $R = 3390$km, $k = 0.925$, and $P = 1.03$d [5]. To our knowledge, this is the strongest constraint on the Blackett's constant. (In the model of Jimenez and Maroto, the model-dependent limit on the Blackett's constant comes from the constraints on the parameterized post-Newtonian parameters and turns to be of order of $\beta \lesssim 10^{-4}$ [10].)

With such a low value for the Blackett's constant, planetary magnetic fields and magnetic fields in stars and galaxies cannot be directly explained by the Blackett conjecture. Moreover, the above limit on β makes not feasible, at the present time, a direct lab-based test of the Blackett conjecture, as we show below.

III. BARNETT EFFECT VS. BLACKETT EFFECT

It is well known that any (neutral) body rotating at an angular velocity $\boldsymbol{\omega}$ acquires a magnetic dipole moment. This effect of "magnetization by rotation" is known as Barnett effect [**?**]. For a homogeneous diamagnetic or paramagnetic solid occupying a volume V, the magnetic dipole $\boldsymbol{\mu}$ is [12]

$$\boldsymbol{\mu} = \frac{2m_e c}{e} \chi g^{-1} V \boldsymbol{\omega}, \tag{3}$$

where m_e and e are the mass and electric charge of the electron, χ is the volume susceptibility, and g is the gyroscopic g-factor.

For a sphere of radius R (the main results do not change if we consider different shapes), the ratio between the magnetic moment given by the Blackett conjecture and the one given by the Barnett effect is then

$$\frac{\mu\,(\mathrm{Blackett})}{\mu\,(\mathrm{Barnett})} \sim 10^{-3} \left(\frac{\beta}{10^{-5}}\right)\left(\frac{10^{-6}\mathrm{cm}^3/\mathrm{g}}{\chi_m}\right)\left(\frac{R}{1\mathrm{m}}\right)^2, \tag{4}$$

where $\chi_m = \chi/\rho$ is the mass susceptibility and ρ the density. To our knowledge, there are not known solid materials with mass susceptibility below $10^{-6}\mathrm{cm}^3/\mathrm{g}$. Equation (4), then, shows that the Blackett effect is always subdominant with respect to the Barnett one for lab-scale objects. Our conclusion is that, at the present time, the Blackett effect cannot be tested in a laboratory.

The Blackett effect could eventually be tested if a material with a mass susceptibility as low as $10^{-9}\mathrm{cm}^3/\mathrm{g}$ were synthesized. This is in principle possible if one combines two or more inert

[1] Barrow and Gibbons [11] have somehow "relaxed" the Blackett conjecture by suggesting that the Blackett's constant is bounded above by a number of order unity, and have verified their conjecture for (classical) charged rotating black holes in theories where the exact solution is known.

materials with different magnetic properties. According to the Wiedemann's additivity law [15], the mass susceptibility of a mixture of its paramagnetic and diamagnetic components would be $\chi_m = \sum_i m_i \chi_m^{(i)} / \sum_i m_i$, where m_i and $\chi_m^{(i)}$ are the mass and mass susceptibility of the component i. Thus, an appropriate choice of the mass percentage of each constituent in powder form would give the possibility of obtaining a material with magnetic susceptibility as low as desired. The resulting powder could be then sintered and made into a solid. It is worth noticing that such a procedure has been already applied by Khatiwada et al. to produce a solid material with very low (volume) susceptibility composed by tungsten and bismuth [16]. However, even if the resulting solid pellets were compact enough to stay together they were delicate. According to Khatiwada et al., the pressing procedure could be further enhanced by using higher pressures and temperatures to produce strong solids. Even if this were possible, however, the resulting solid material should have a sufficiently large volume, and be dense and strong enough in order to produce a detectable magnetic field once it is put into rotational motion, as we discuss below.

IV. BLACKETT-TYPE EXPERIMENT

Let us consider a homogeneous rotating sphere made of a hypothetical material whose mass susceptibility is such that the Blackett effect is dominant with respect to the Barnett one. The maximum safe angular speed ω can be found as follows. The stress tensor in spherical coordinates r, θ, ϕ can be written as $\sigma_{ij} = c_{ij}(\nu, \theta, r) \rho\, \omega^2 R^2$, where $c_{ij}(\nu, \theta, r)$ is a dimensionless tensor, ν is the Poisson's ratio [17], and $i, j = r, \theta, \phi$. Here, σ_{rr} is the radial stress, $\sigma_{r\theta}$ is the shear stress, and $\sigma_{\theta\theta}$ and $\sigma_{\phi\phi}$ are the angular normal stresses (all the other components of the stress tensor are zero by symmetry). Using the results of [18] we find that, for given density, angular speed, and radius, the maximum stress corresponds to the angular normal stresses and in particular $\text{máx}_{\theta,r}\, c_{\theta\theta} = \text{máx}_{\theta,r}\, c_{\phi\phi} = c_\nu$, where $c_\nu = (5\nu^2 - \nu - 12)/(25\nu^2 + 10\nu - 35)$. (Here, we assumed $0 \leq \nu \leq 1/2$, which is certainly true for all metals and known alloys. Theoretically, the Poisson's ratio is in the range $-1 \leq \nu \leq 1/2$ [17].) The function c_ν is an increasing function of ν such that $c_0 = 12/35 \simeq 0.34$ and $c_{1/2} = 9/19 \simeq 0.47$.

The stress generated by rotation must be smaller that the ultimate tensile stress $\sigma_{\max}$, $|\sigma_{ij}| < \sigma_{\max}$. This, in turn, determines the maximum possible value for the angular speed, $\omega_{\max} = (\sigma_{\max}/c_\nu \rho)^{1/2}/R$. Inserting this value of ω in Eq. (1), we find the maximum magnetic field that can be generated by a rotating sphere,

$$B_{\max} \sim 10^{-13} \left(\frac{\beta}{10^{-5}}\right) \left(\frac{\sigma_{\max}}{1\text{MPa}}\right)^{1/2} \left(\frac{\rho}{1\text{g/cm}^3}\right)^{1/2} \left(\frac{R}{1\text{m}}\right) \text{G}. \tag{5}$$

For a given radius R, then, the maximum magnetic field is large for materials with high density and ultimate tensile stress, such as metals and alloys (the dependence of $B_{\max}$ and $\omega_{\max}$ on the Poisson's ratio is very week).

In order to detect $B_{\max}$, or to put a limit on the Blackett's constant more stringent than the one in Eq. (1), the hypothetical material must have a sufficiently high ultimate tensile stress and density. Indeed, taking $\beta = 10^{-5}$, a 2-meter sphere ($R = 1$m) would produce a maximal magnetic field of order of $B_{\max} \sim 10^{-17} (\sigma_{\max}/1\text{MPa})^{1/2} (\rho/1\text{g cm}^{-3})^{1/2}$ T. The most sensitive magnetometers are SQUID magnetometers, with maximum sensitivities of order of 1 $f\ T/\sqrt{\text{Hz}}$ [19], and SERF magnetometers, with maximum sensitivities of about 0.2 $f\ T/\sqrt{\text{Hz}}$ [20]. Even taking the maximum theoretical sensitivity of a SERF magnetometer, estimated to be 2aT [21], the hypothetical material must satisfy the mechanical condition $(\sigma_{\max}/1\text{MPa})^{1/2} (\rho/1\text{g cm}^{-3})^{1/2} \gtrsim 0.1$ in order to be of any relevance. (As a reference, a relatively low-density material with relatively low ultimate tensile stress is the "normal strength Portland" cement concrete for which $\rho \simeq 2.3\text{g/cm}^3$ and $\sigma_{\max} \simeq 3.5$MPa [22], while a very strong and very dense material is tungsten for which $\sigma_{\max} \simeq 1510$MPa and $\rho \simeq 19.25\text{g/cm}^3$ [23].)

V. CONCLUTIONS

The Blackett effect is a hypothetical effect consisting in the magnetization by rotation of a rigid neutral body that should emerge from a unified theory of gravitation and electromagnetism.

We have derived a stringent constraint on the Blackett's constant, the dimensionless constant of proportionality between the magnetization and the angular momentum of a body, by using the data on the dipolar magnetic field of Mars. This constraint excludes the possibility that the Blackett effect could directly account for planetary, stellar, and galactic magnetic fields.

We have also pointed out that the Blackett effect is similar but subdominant for lab-scale objects with respect to the well-known and experimentally tested Barnett effect, according to which any rotating object acquires a magnetic moment proportional to its angular velocity. The Blackett effect, then, cannot be tested in a laboratory.

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