MALUS' LAW REVISITED

LA LEY DE MALUS, REVISITADA

Vladimir Ivchenko

Department of Natural Sciences Training, Kherson State Maritime Academy, Kherson 73000, Ukraine reterty@gmail.com†
† corresponding author

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Malus's law is generalized for elliptical and partially polarized light. A simple way to determine the degree of polarization of a polarizer was also proposed.

Se generaliza la ley de Malus para luz eliptica y parcialmente polarizada. También se propone una sencilla v'ia para determinar el grado de polarización de un polarizador.

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I. INTRODUCTION

The study of polarized electromagnetic radiation is of great importance in many areas of science. This analysis is based on the discovered in 1809 Malus' law [1]. Malus' law is widely used in making polaroids that are applied to control the intensity of light, as in sunglasses, window panes, and sometimes photographic and 3-d movie cameras.

There are many many articles in physics education journals devoted to the description of the experimental verification of the Malus' law in the physics classroom. For example, in references [2] and [3] a simple setup using a cell phone is proposed to quantify Malus' law inexpensively.

It should be noted that the derivation of this law is based on some model assumptions, namely, it is assumed that the light after passing through the polarizer is strictly linearly polarized, and in addition, the analyzer also produces only polarized light at the output.

In the present work, we generalize Malus' law to cases where these conditions are not met. The issues outlined in this article will be useful for undergraduates studying the basics of wave optics.

II. MALUS' LAW FOR THE ELLIPTICALLY POLARIZED LIGHT

In the most general case, the elliptically polarized light consists of two mutually perpendicular waves of unequal amplitude which differ in phase by δ ($0 \leq \delta \leq 2\pi$). The components of the electric field $\vec{E}$ along these transverse directions are [4]:

$$E_x = \frac{E_0}{\sqrt{1+\varepsilon^2}}\cos\omega t, \tag{1}$$

$$E_y = \frac{E_0\varepsilon}{\sqrt{1+\varepsilon^2}}\cos(\omega t-\delta), \tag{2}$$

where E_0 is the amplitude value of $\vec{E}$; ε is the ellipticity of the polarization ellipse, that is, the ratio between the lengths of its minor and major axes; ω is the angular frequency. The irradiance of this light (that is the radiant flux received by a surface per unit area) is

$$I_0 = n\varepsilon_0 c\langle E_x^2 + E_y^2\rangle \tag{3}$$

where n is the refractive index of the medium of propagation; ε_0 is the vacuum permittivity; c is the speed of light in vacuum. The angular brackets in equation (3) mean the time-averaging. For example,

$$\langle E_x^2\rangle = \frac{1}{\tau}\int_t^{t+\tau} E_x^2(t')\,\mathrm{d}t' = \frac{E_0^2}{2(1+\varepsilon^2)}, \tag{4}$$

where the integration is taken over an interval τ. The result (4) is valid only we assume that this interval is much greater than the period of the electromagnetic wave $T = 2\pi/\omega \sim 10^{-15}$ s. This is mandatory, to guarantee that the calculated irradiance be comparable to the measured quantity with a suitable sensor that integrates and averages the energy per unit area (see, for instance, Ref. [5]). Therefore,

$$I_0 = \frac{n\varepsilon_0 c E_0^2}{2} \tag{5}$$

Let the angle between the transmission axis of the analyzer and the Ox axis be equal to θ. Then, upon exiting the analyzer, the resulting linear oscillation is:

$$E_\xi = E_x\cos\theta + E_y\sin\theta. \tag{6}$$

Then, the output irradiance of the light will be:

$$I = n\varepsilon_0 c\langle E_\xi^2\rangle = \frac{n\varepsilon_0 c E_0^2}{2(1+\varepsilon^2)}\left(\cos^2\theta + \varepsilon^2\sin^2\theta + \varepsilon\sin 2\theta\cos\delta\right) \tag{7}$$

Taking into account equation (5), we finally obtain:

$$I = \frac{I_0}{1+\varepsilon^2}\left(\cos^2\theta + \varepsilon^2\sin^2\theta + \varepsilon\sin 2\theta\cos\delta\right) \tag{8}$$

The function $I(\theta)$ is a periodic function with the period equal to π. The distance between the adjacent maxima and minima is exactly equal to $\pi/2$. If $\delta = \pi/2$, then maxima and minima of I are reached at $\theta = 2k\pi$ and $\theta = (2k+1)\pi/2$, respectively. In general, $I_{\min}$ does not vanish (Fig. 1).

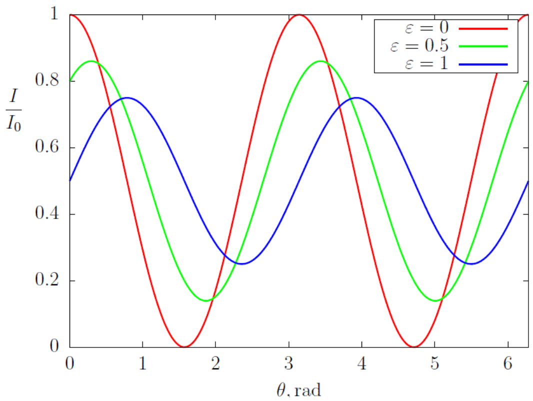

Figure 1. Dependence $I(\theta)$ at $\delta = \pi/3$.

The vanishing is possible only for linearly polarized wave, i.e. at $\varepsilon = 0$ or $\delta = k\pi$ $(k = 0, 1, 2, 3 \ldots)$. Under the same conditions, $I_{\max} = I_0$ (in all other cases $I_{\max} < I_0$, see Fig. 1). It is interesting that in all cases the averaged value of I around which the light intensity oscillates is equal to $I_0/2$.

As ε increases from 0 to 1, the extrema shift to the right on the θ-angle scale (Fig. 1). Herewith, the value of $I_{\max}$ decreases, while the value of $I_{\min}$ increases. For circularly polarized wave ($\varepsilon = 1$, $\delta = \pi/2$), we have $I = I_0/2$, that is the output irradiance does not depend on θ at all. Finally, at $\varepsilon = 0$ we deal with Malus's law in the standard form: $I = I_0 \cos^2\theta$.

MALUS' LAW FOR PARTIALLY POLARIZED LIGHT

A real polarizer produces only partially polarized light with a degree of polarization slightly less than unity. Any partially polarized light can be statistically described as a superposition of a completely unpolarized component (natural light) and a completely polarized one. Below, we present a derivation of the generalized Malus' law for partially polarized light.

The degree of light polarization p $(0 \le p \le 1)$ is the ratio of the irradiance of light polarized by the polarizer to the total light irradiance [6]:

$$p = \frac{I_p}{I_0} = \frac{I_p}{I_p + I_u}, \tag{9}$$

where I_u is the irradiance of the natural component of the light. Let such a partially polarized light passes through an analyzer completely identical in its optical properties to a polarizer (it means that the degree of polarization after passing the analyzer is the same as after the polarizer that created the partially polarized beam). Then the following equality takes place:

$$p = \frac{I_a}{I}, \tag{10}$$

where I_a is the irradiance of polarized light after passing through the analyzer; I is the total light irradiance after passing through the analyzer. It is evidently that

$$I_a = I_p \cos^2\theta + \frac{I_u}{2}, \tag{11}$$

where θ is the angle between the transmission axes of the polarizer and analyzer. Using equations (9)-(11), we finally obtain:

$$I = I_0 \left(p \cos^2\theta + \frac{1-p}{2} \right) < I_0. \tag{12}$$

The equation (12) is very similar to the usual Malus' law with the only difference that function $I(\theta)$ turns out to be "compressed" by the factor of $1/p$ and shifted up by $I_0(1-p)/2$ (Fig. 2).

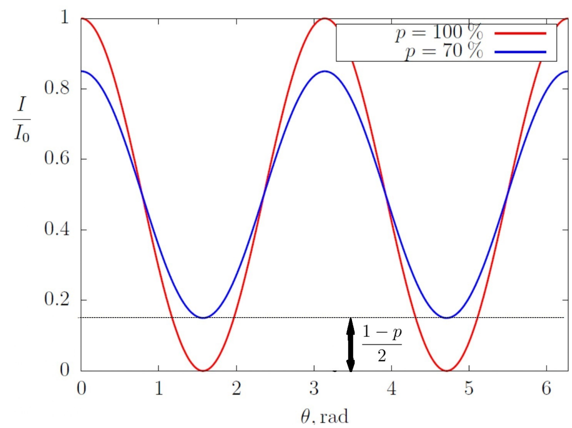

Figure 2. Dependence $I(\theta)$ for two different polaroids.

The averaged value of I around which the light intensity oscillates, is equal to $I_0/2$. The difference $I_{\max} - I_{\min}$ is $I_0 p$. This circumstance makes it easy to determine the degree of polarization of a polarizer by studying the experimental dependence $I(\theta)$.

III. CONCLUSIONS

The main contribution of this paper is to present the derivation and analysis of Malus' law for two important general cases which can be implemented in the learning process. To the author's knowledge, these derivations have not been reported before. The only attempt (known to us) to generalize Malus' law is the work by Damian [7]. Unfortunately, due to the use of a vague definition of the degree of polarization of light, the cited author obtains an incorrect final expression for the irradiance. It would be also an interesting task for the reader to consider the case of partially polarized light with the polarized component showing elliptical polarization.

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