MATHEMATICAL MODEL FOR ESTIMATING SOIL REDISTRIBUTION RATE BY WATER EROSION USING THE NUCLEAR AND ISOTOPIC FRN TECHNIQUE

MODELO MATEMÁTICO PARA ESTIMAR LA TASA DE REDISTRIBUCIÓN DEL SUELO POR EROSIÓN HÍDRICA UTILIZANDO LA TÉCNICA NUCLEAR E ISOTÓPICA FRN

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The use of nuclear techniques for the evaluation of soil water erosion is based on empirical relations to evaluate inventories of radionuclides in the soil (^{137}Cs, ^{210}Pb and ^{7}Be). In applying the fallout radionuclides technique (FRN) to croplands, the most widely used model is the Mass Balance Model (MBM-II), which employs ^{137}Cs isotope inventories (Bq m^{-2}) to estimate soil redistribution rate. This work develops a mathematical model (MRS-2) to obtain this soil redistribution rate, yielding results comparable to those of MBM-II. The model estimates the soil redistribution rate based on areal activity density (AAD) values (Bq m^{-2}) used in FRN techniques. This linear model is notable for its internal and external validation, providing reliable estimates of soil redistribution rate down to values of -60 t ha^{-1} y^{-1}. Its use is therefore recommended.

El uso de técnicas nucleares para evaluar la erosión hídrica del suelo se fundamenta en el establecimiento de relaciones empíricas para evaluar los inventarios de radionucleidos en el suelo (^{137}Cs, ^{210}Pb y ^{7}Be). Al aplicar la técnica de radionucleidos de precipitaciones (FRN) a las tierras cultivadas, el modelo más utilizado es el Balance de Masa (MBM-II), que emplea inventarios de isótopo ^{137}Cs (Bq m^{-2}) para estimar la tasa de redistribución del suelo. Este trabajo desarrolla un modelo matemático (MRS-2) para obtener esta tasa de redistribución del suelo, produciendo resultados comparables a los del modelo MBM-II. El modelo estima la tasa de redistribución basándose en valores de densidad superficial de actividad (DSA, en Bq m^{-2}) utilizados en la técnica FRN. Este modelo lineal es notable por su validación interna y externa, proporcionando estimaciones fiables de la tasa de redistribución del suelo hasta -60 t ha^{-1} y^{-1}. Por lo tanto, se recomienda su uso.

Keywords: water erosion (erosión hídrica), fallout radionuclides (radionúclidos de precipitación), soil redistribution (redistribución del suelo), conversion model (modelo de conversión)

I. INTRODUCTION

Soil loss due to water erosion and its subsequent sedimentation in the landscape and water reservoirs is a global environmental problem affecting agriculture and hydraulic infrastructure. The negative impacts include threats to agricultural sustainability and water resources, requiring comprehensive assessments using reliable tools such as nuclear techniques [1,2] to characterize soil health [3]. Water erosion, the most common problem, is caused by surface runoff from soil. The nuclear technique (FRN) assesses water erosion directly and quantitatively, as its isotopic tracers move coupled with soil particles transported by water.

Numerous traditional techniques and universal equations exist to estimate soil loss, such as USLE [4], RUSLE [5], and MUSLE [6]. Their use, combined with the Geographic Information System (GIS), is widespread [7]. Additional model estimates redistribution in cultivated soils [8] and evaluates the effect of crop rotation and land-use changes on sediment transport [9]. Compared to these models, practical experience with nuclear and isotopic techniques has significant advantages by overcoming their limitations, moving beyond predictive frameworks to deliver more precise and concrete results.

A key advancement is the use of fallout radionuclides (FRN) to evaluate soil redistribution rate [10–12]. This technique uses the artificial anthropogenic isotope from nuclear explosions (^{137}Cs), the natural cosmogenic isotope (^{7}Be), and the natural geogenic isotope (^{210}Pb) to define erosion and sedimentation zones. These radionuclides provide information on soil redistribution across different time scales: ^{7}Be (several months) [13], ^{137}Cs (approximately 50 years) [14,15], and ^{210}Pb (approximately 100 years) [16]. The IAEA has been actively involved in developing these nuclear techniques [17,18].

To convert ^{137}Cs inventories into soil erosion/deposition rates, software has been developed as a standard add-in for Microsoft Excel. These models are complex and tedious, involving an evaluation process based on laboratory-derived mass activity density (MAD-Bq kg^{-1}). For ^{137}Cs, temporal variation is assessed using the traditional conversion Mass Balance Model II (MBM-II), which accounts for time-varying ^{137}Cs inputs and the fate of newly deposited precipitation before its incorporation into the plow layer. Currently, FRNs enable the evaluation of erosion processes and extreme climatic events [19] and have been used as tracers in aquatic systems to indicate sediment residence times and to identify their sources [20,21].

The main objective of this work was to obtain, develop, and validate a mathematical model that would allow for the evaluation of soil redistribution per unit area in a shorter time, thereby improving the efficiency of the assessment process in the short and medium term and providing a practical replacement for the lengthy traditional calculation methods.

II. MATERIALS AND METHODS

The use of fallout radionuclides (FRN) to assess water erosion is based on the conversion models that consider radionuclide inventories in the soil (^{137}Cs, ^{210}Pb, and ^{7}Be) to estimate soil redistribution rate [7–9,16,22].

Radionuclide measurements were performed using a hyperpure germanium gamma spectrometer with 35 % efficiency. The expanded uncertainty ranged from ±0.13 to ±8.24, determined by multiplying the combined standard uncertainty by a cover factor. The stability of the gamma spectrometer system was monitored weekly by measuring energy, resolution, and efficiency values for point sources of Am^{-2}41 (59.54 keV), Cs-^{137} (661.66 keV), and Co-60 (1332.50 keV), as well as the background count rate across the entire analytical range. The Monte Carlo statistical simulation was used to accurately solve the measurement problem [22], and the system remained stable throughout the sampling process. The activity concentrations for the radionuclides in the IAEA-300, IAEA-306, and IAEA-314 certified reference materials were determined. The measurement results were compared with target values, revealing no significant differences.

The results of this work led to a new mathematical model (MRS-2) for estimating soil redistribution rate based on areal activity density (AAD) values (Bq m^{-2}) and other factors used in the FRN technique, as part of the integrated methodology of the nuclear techniques [1,2]. This model is less complex, yielding results more quickly from AAD values and making the evaluation process more efficient in the short and medium term. It practically eliminates the need for a reference site. The development of the MRS-2 model is based on results from previous research involving 151 samples from the studied sites in the Hanabanilla reservoir and the National Site for the Conservation of Water, Soil, and Forests (Havana). Soil sampling followed expert criteria and a combination of random, stratified, and systematic protocols [12,17]. Detailed knowledge of the study areas and the use of composite sampling prevented cross-contamination.

II.1. *The methodological strategy used is described below:*

1) Analysis of bivariate correlations between the following variables: areal activity density (AAD, Bq m^{-2}), soil redistribution rate (t ha^{-1} y^{-1}), soil density (kg cm^{-3}), geology, soil type and use, height (m), rainfall (mm), soil moisture, and soil slope (%).

2) Hierarchical classification analysis of all variables involved to determine their grouping based on results similarity.

3) Selection of the variable(s) with the highest correlation linked to the dependent variable (soil redistribution rate (t ha^{-1} y^{-1})).

4) Analysis of bivariate model fits related to the dependent variable (soil redistribution rate (t ha^{-1} y^{-1})).

5) Internal validation of the model based on data from the study region and external validation in the National Site for Soil, Water, and Forest Conservation, located in Havana Province. In this area, soil redistribution results from applying the integrated methodology were analyzed using 50 sample sites and statistically compared with the results estimated by the model, using the Kolmogorov-Smirnov statistical test to compare two distributions and the mean absolute error.

6) External validation through the application of the integrated methodology, culminating in results graphs and thematic maps. All statistical analyses for obtaining, evaluating, and validating the linear model were performed using the software Statgraphics Centurion (version 19.5.01).

[1]Soil redistribution rate

III. RESULTS AND DISCUSSION

Bivariate correlation analysis among the studied variables revealed only one high and significant correlation: between areal activity density (Bq m^{-2}) and soil redistribution (t ha^{-1} y^{-1}), with a coefficient of 0.956. The remaining variables showed no significant influence on soil redistribution rate (see Table 1).

Hierarchical classification analysis showed a cluster formed by the AAD and the calculated soil redistribution rate, clearly separated by their Euclidean distances from the other variables. This confirms the bivariate correlation results, where both variables exhibited the strongest correlation (above 95 %).

Table 1. Results of the bivariate correlation analysis for all studied variables.

Studied variables	Correlation coefficients AAD (Bq m^{-2})	SRR[1] (t ha^{-1} yr^{-1})
AAD (Bq m^{-2})	—	0.9557
SRR (t ha^{-1} yr^{-1})	0.9557	—
Soil density (kg cm^{-3})	0.1230	0.1573
Geology	-0.0057	-0.0740
Land use	0.1927	0.2807
Soil type	-0.0798	-0.0865
Height (m)	0.4139	0.4942
Rainfall (mm)	0.3560	0.4005
Soil moisture index	-0.1709	-0.1747
Soil slope (%)	0.3123	0.3585

Based on these results, linear regression models were analyzed to select the optimal model for estimating soil redistribution rate from AAD values derived from laboratory mass activity density (MAD, Bq kg^{-1}) values.

The AAD (Bq m^{-2}) is the selected parameter for the model and is the main input for most conversion models applied in FRN. It is calculated as follows:

$$AAD = \frac{M \cdot MAD}{S} \tag{1}$$

where M is the total mass of whole soil core in kg, MAD is the mass activity density (Bq kg^{-1}) and S is the area of the core cross-section (m^2).

Simple linear regression analysis was performed. The best fit was obtained with a linear regression model using the square root of the areal activity density ($\sqrt{AAD}$) as the independent variable. The results are shown in Tables 2 and 3 and Fig. 1.

Table 2. Model coefficients.

Parameter	Least Squares Estimate	Standard Error	T Statistic	P-value
Intercept	-83.0182	1.40182	-59.2216	0.0000
Slope	2.71585	0.0399165	68.0683	0.0000

Table 3. Analysis of Variance..

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-value
Model	216814	1	216814	4629.21	0.0000
Residual	6978.58	149	46.8361		
Total (Corr.)	223793	150			

- Correlation coefficient = 0.9843
- R-squared = 96.8817 %
- R-squared (adjusted for d.f.) = 96.8607 %
- Standard error of the estimate = 6.8437
- Mean absolute error = 5.5039
- Durbin-Watson statistic = 1.8739 (P = 0.2202)
- Autocorrelation of residuals at lag 1 = 0.0602

The equation of the fitted model (MRS-2) is:

$$\text{SR (t ha}^{-1}\text{ y}^{-1}) = -83.0182 + 2.71585\,\sqrt{AAD}\ (\text{Bq m}^{-2}) \tag{2}$$

- SR: Soil redistribution

The P-value in the ANOVA Table (< 0.05) indicates a statistically significant relationship between the calculated soil redistribution and the AAD (Bq m^{-2}) at the 95.0 % confidence level. The fitted model explains 96.86 % of the variability in the calculated soil redistribution. The correlation coefficient of 0.9843 indicates a strong relationship between the variables.

Applying the linear model obtained from linear regression analysis with the ($\sqrt{AAD}$), soil redistribution values were estimated and compared with those calculated using MBM-II to determine deviations. The mean absolute error was 5.003 t ha^{-1} y^{-1}. This value is acceptable for the proposed model, which estimates soil redistribution rates up to 60 t ha^{-1} y^{-1}, considering that loss levels up to approximately 7 t ha^{-1} y^{-1} are classified as low erosion levels.

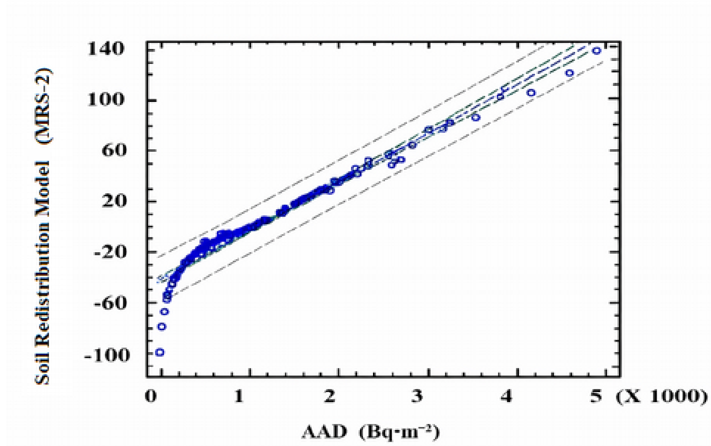

Figure 1. Calculated soil redistribution rate and its dependence on AAD (Bq m^{-2}).

The Kolmogorov-Smirnov test was used to compare both distributions. The results are shown below:

- Sample 1: Soil redistribution MBM II model (t ha^{-1} y^{-1})
- Sample 2: Redistribution MRS-2 model (t ha^{-1} y^{-1})

Kolmogorov-Smirnov Test:

- Statistic DN estimated = 0.0861
- Two-tailed K-S statistic for large samples = 0.7481
- Approximate P-values = 0.6304

In this case, the maximum distance is 0.0861 (visible in the Quantile Plot, Fig. 2). As the P-value is $\geq$ 0.05, there is no statistically significant difference between the two distributions, demonstrating the model's internal validity. However, this result needs to be complemented with its geographical expression of the territorial conditions where the variables studied in the correlational analysis are present.

Fig. 2 shows a close approximation between the two distributions. However, estimated soil redistribution rates below -60 t ha^{-1} y^{-1} begin to diverge from calculated values. The MRS-2 model provides optimal results up to the erosion range of -60 t ha^{-1} y^{-1}). High soil loss values correspond to very low concentrations of the studied radionuclide (^{137}Cs) in soil samples (values close to the detection limit of gamma-ray spectrometry), leading to greater uncertainty in the estimated analytical values for the mass activity density (Bq kg^{-1}). This error propagates when calculating the main model variable, areal activity density (Bq m^{-2}), which is used to estimate soil redistribution (t ha^{-1} y^{-1}).

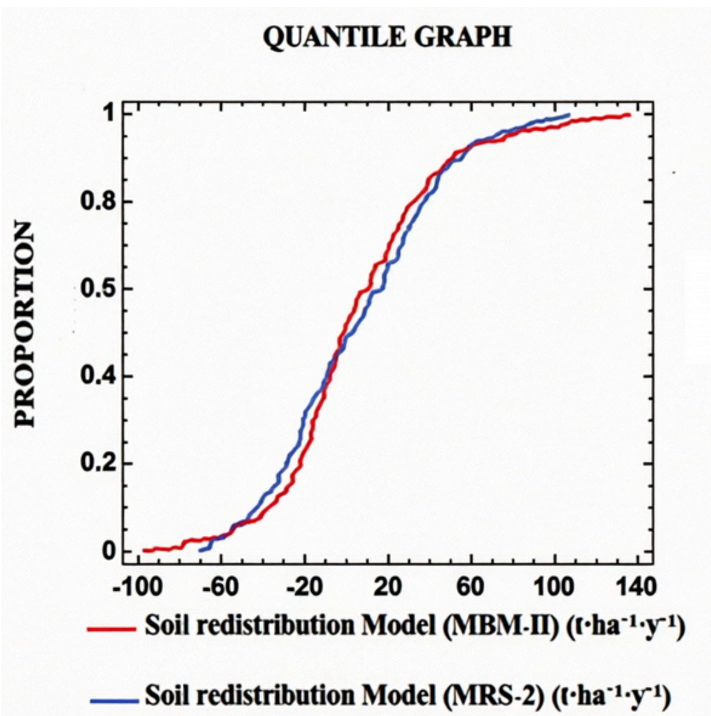

Figure 2. Comparison of calculated soil redistribution rate and soil redistribution rate estimated by the MSR-2 model

III.1. Practical application of the MRS-2 model in case studies.

The MRS-2 mathematical model is validated statistically and practically. The statistical analyses are presented above; practical validation is demonstrated through various case studies. Below is a comparison of soil redistribution results obtained by applying the traditional conversion model (MBM-II) and the new mathematical model (MRS-2). Both models were applied in the following study areas: crop fields in the Provincial site for Soil, Water, and Forest Conservation (Cienfuegos) and the Hanabanilla Reservoir (Cienfuegos and Villa Clara).

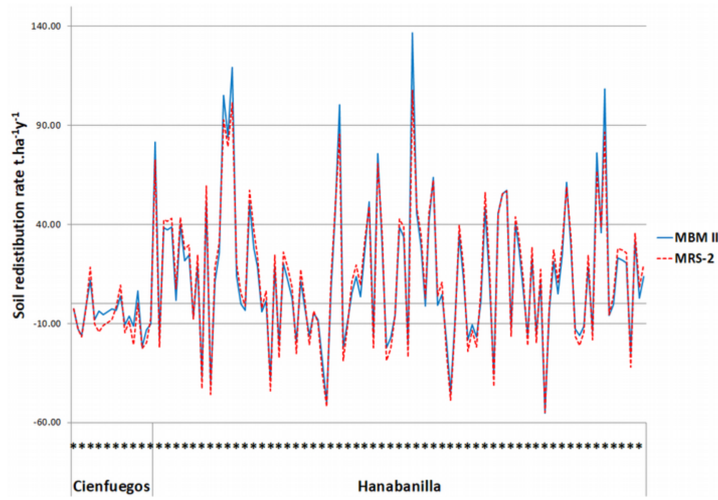

Figure 3. Comparison of soil redistribution estimates from the linear model obtained (MRS-2) with the MBM-II. Internal validation.

The research enabled the assessment of soil redistribution in the landscape and the reservoir through the application of both models. To validate the efficiency of the MRS-2 model, its results are compared with those from the traditional MBM-II model using GIS (QGIS, version 3.22.6) for mapping and OpenOffice (version 4.1.7) for graphics (see Fig. 3).

III.2. Validation of the mathematical model (MRS-2).

III.2.1. Results (Hanabanilla Sub-Basin).

The conversion models (MBM-II and MRS-2) were applied in the basin of the Hanabanilla Reservoir in Cienfuegos. The main land uses in the study site are forestry, coffee, livestock, and seasonal crops. The interpolation (kriging) results for 115 points are shown in figures 4A (MBM-II) and 4B (MRS-2). The areas of soil gain (positive values) and loss (negative values) are clearly identified. The same color palette was used for both models for comparison.

The Fig. 4A show the MBM-II model estimates a soil redistribution range from -43 to 119 t ha^{-1} y^{-1}. The MRS-2 model (Fig. 4B) estimates a range of soil redistribution values from -60 to 118 t ha^{-1} y^{-1}, very similar to the MBM-II, reflected in the proximity of the spatial redistribution pattern. The distribution of positive and negative values on the map shows a high coincidence, as do the soil redistribution rates isolines. Analysis of the residuals (t ha^{-1} y^{-1}) between model's prediction shows a mean (-0.4) and standard error (0.6) close to zero, indicating the closeness of both models.This confirms the internal validation from the Kolmogorov-Smirnov test.

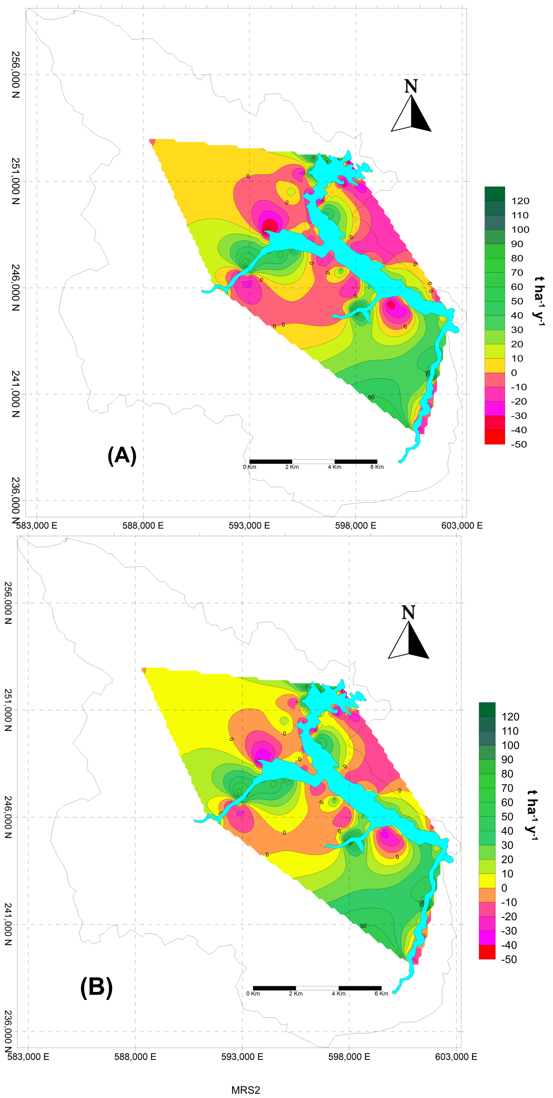

Figure 4. Interpolation results of the (A) MBM-II model and (B) MRS-2 model in Hanabanilla areas.

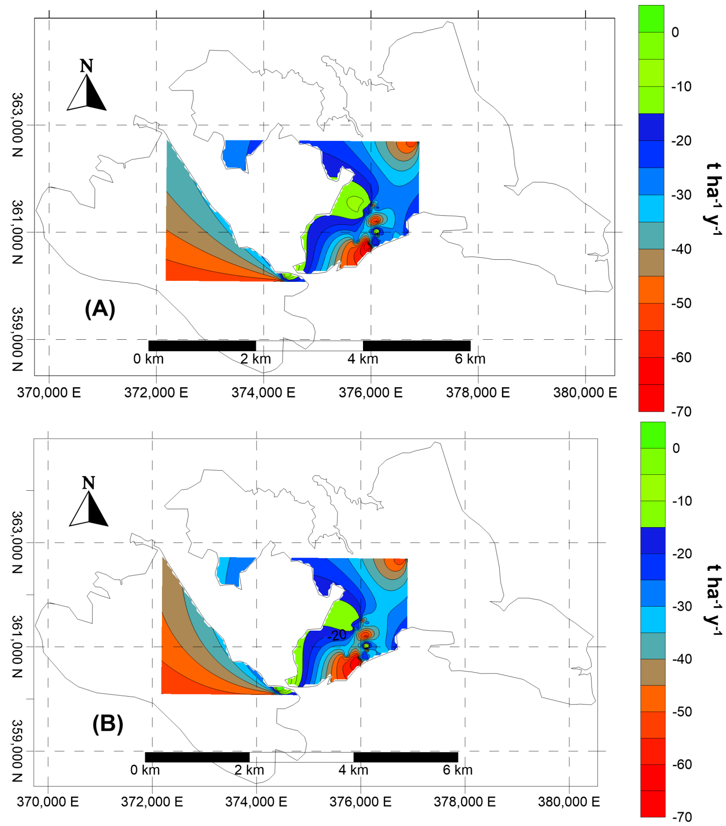

Figure 5. Interpolation results of the (A) MBM-II model and (B) MRS-2 model in the national site.

III.3. External validation of the model obtained

Crop fields in the National Site for Soil, Water, and Forest Conservation (Havana Province) were used for the external validation of the MSR-2 model. The model behaves similarly to the MBM-II model, following the pattern of maximum and minimum soil redistribution estimates. Differences appear for high erosion loss levels exceeding -60 t ha^{-1} y^{-1}. The MRS-2 model adequately fits the MBM-II results at most evaluated points.

The study site is highly eroded, as represented in Fig. 5A for the ^{137}Cs study. The main land uses are livestock and seasonal crops. The majority of the area shows negative redistribution rates (soil losses) with values below -15 t ha^{-1} y^{-1}; only a small green strip (center) identifies areas with tolerable soil loss levels (-10 t ha^{-1} y^{-1}). Three areas with very high erosion levels are shown in orange to red bands in the southwest, south, and northeast of the site.

For comparison, MRS-2 results in the same national site areas were mapped using the kriging interpolation technique with the same soil redistribution values scale as MBM-II (Fig. 5B). This linear model shows results close to the reference model, and the spatial distribution shows very similar soil redistribution isolines. Residuals (t ha^{-1} y^{-1}) between model predictions have a low mean (1.2) and a standard error (0.03) close to zero, demonstrating the proximity of both models for assessment.

This comparative analysis confirms the statistical results and establishes greater confidence in the MRS-2 model. Fig. 6 illustrates the impact of these differences between the models when evaluating their spatial representation, a fundamental step in interpreting FRN technique results.

The external validation was conducted at the national site on transects across several plots with diverse conservation measurements. Results are shown in Table IV.

Table 4. Comparison of soil redistribution models

Redistribution Models.	**Sample 1 (MBM-II)**	**Sample 2 (MRS-2)**
Real Soil redistribution (national site)	50 values (-61.81 to 18.22)	-
Soil Redistribution of (Linear model MRS-2)	-	50 values (-59.0146 to 23.7892)

Kolmogorov-Smirnov Test:

- Estimated DN statistic = 0.2
- Two-tailed K-S Statistic for large samples = 1.0
- Approximate P-value = 0.2710

In this case, the maximum distance is 0.2 (Fig. 6, Quantile Plot). As the P-value is > 0.05, there is no statistically significant difference between the two distributions at 95.0 % confidence levels (Fig. 7).

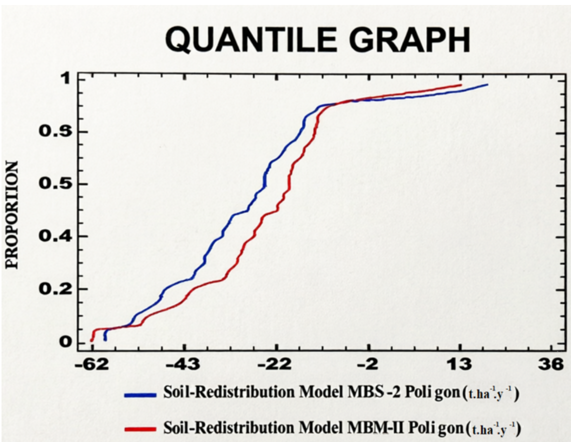

Figure 6. Comparison of soil redistribution rates: calculated (MBM II) vs. obtained by the MSR-2 model at national test site.

At the national site, all soil redistribution values are negative, representing different soil loss levels. The linear model shows the same behavior as the MBM-II model for the different points evaluated (Fig. 7). The closest values between models are for estimates above -50 t ha^{-1} y^{-1}. Beyond -60 t ha^{-1} y^{-1} (maximum soil loss values), the models show the greatest differences.

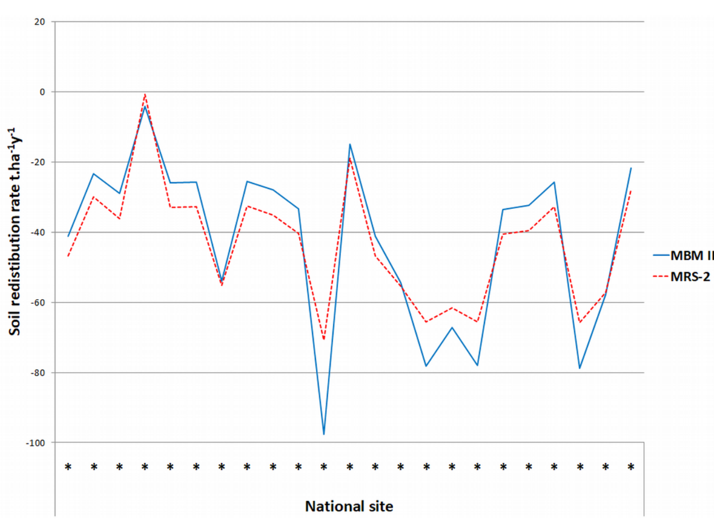

Figure 7. Comparison of soil redistribution estimates from the linear model (MRS-2) with the MBM-II model. External validation.

III.4. Potentials for practical application of the mathematical model (MRS-2).

Considering the stated premises, advantageous background, and the statistical behavior of the MRS-2 model, its application is proposed as a tool for estimating soil redistribution rates in landscape and water reservoirs. Its results exhibit a very good approximation with the traditional Conversion Model (particularly MBM-II), except where the erosion exceeds -60 t ha^{-1} y^{-1}. Applying this linear model simplifies the use of FRN techniques by eliminating the need to define and measure a reference site. This is particularly useful where defining a reference site to estimate reference AAD is not possible.

The application potential of the mathematical model MRS-2 is high. Applying a modified version of the Universal Soil Loss Equation (RUSLE) to estimate soil loss across the European Union (study area: 3941 x 103 km^2), with an emphasis on cultivated soils, identified that over 99 % of the assessed area falls within the range of 0 to -50 t ha^{-1} y^{-1}. This indicates the MRS-2 model could have potential use throughout this region. A study in northeast Ethiopia using Geographic Information Systems (GIS) and remote sensing techniques (RUSLE) classified soil erosion risk from low (0 - 7 t ha^{-1} y^{-1}) to severe (45 - 60 t ha^{-1} y^{-1}) and very severe (> 60 t ha^{-1} y^{-1}). Results showed that 98 % of the total studied area was in the 0 - 60 t ha^{-1} y^{-1} range, again confirming the potential use of the MRS-2 model from low to severe erosion levels (according to this classification).

IV. CONCLUSION

- The MRS-2 linear mathematical model simplifies soil redistribution estimations using areal activity density (AAD) and can replace complex traditional methods during initial screening.
- Its reliability and applicability have been demonstrated in diverse landscape contexts and agroecosystems and verified using maps, graphs, and case studies.

V. ACKNOWLEDGEMENT

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V.1. Data availability

The authors are willing to provide the database used to generate the MRS-2 code upon reasonable request. Access is granted solely for the purpose of reproducing or validating the results reported in this study. Any other use of the data requires prior authorization from the authors.

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