UNDERSTANDING THE QUANTUM BEHAVIOUR OF MATTER AS A DERIVED PROPERTY. I. A COSMOLOGICAL IMPLICATION OF STOCHASTIC ELECTRODYNAMICS

L. de la Peña and A.M. Cetto, Instituto de Física, Universidad Nacional Autónoma de México

ABSTRACT

An old hope to get a better physical understanding of the quantum behaviour of matter has been connected with the observation that it is natural to consider the zeropoint radiation field as a real entity, instead of the virtual field considered by quantum electrodynamics. This random electromagnetic field pervades the entire universe and is in permanent interaction with matter. An order-of-magnitude estimate, considering the matter-field system to be in dynamical equilibrium at zero temperature, allows to establish a relation between Planck's constant –which determines the scale of the field fluctuations– and cosmological constants.

I. INTRODUCTION

No physicist on Earth would doubt that quantum theory gives a right description of those parts of the world that belong to its domain; that this domain extends much beyond the mere atomic scales which the theory originally was intended to address, and that the descriptions is not only excellent and can attain admirable precision, but has never failed to give the right answer as yet. However, a good part, if not all, of these same practicing physicists will also agree, openly or reluctantly, that the mysteries of the quantum world have not been cleared up, and that many of them remain on stage despite the considerable efforts invested in trying to get a clear understanding of what is going on in the real world.

Of course, whether or not one subscribes to the last sentence essentially depends on what one understands by *understanding*, as happens so often in physics. A well known historical example of what we have in mind is the Newtonian theory of gravitation: the clarity, simplicity and high precision of this theory made of it a paradigm, indeed a grandiose paradigm, and the theory reigned undisputed for over two centuries. The universal gravitational force soon became the accepted central element for innumerable terrestrial and celestial facts; no physicist could doubt of it. And this occurred despite the shortcomings of the theory in more than one essential aspects. Not only did it rest on the ageing concept of action at a distance, but the specific form of the force was selected *ad hoc*, with no theoretical support that could justify or explain it. Thus, from this more demanding point of view, what one can say at most is that the classical theory gives a precise and simple description, good enough for almost all of our daily (non-cosmological) requirements; but it hardly constitutes an explanation of those events. Just to get such an explanation the whole of general relativity was proposed. Nowadays we explain the falling of bodies as a causal event directed by the local structure of space-time; no more actions at a distance nor *ad hoc* properties are needed. We know that for all simple applications the predictions of both theories agree; but nevertheless, they are entirely different conceptual structures, so different that one could even say that general relativity explains the Newtonian theory.

Is not something similar happening today with quantum theory? We can calculate delicate corrections to some transition frequencies to within a hundred- or a thousand-millionth part, and carry out refined applications of the quantum properties of matter and the radiation field to construct marvelous and powerful devices that characterize our present day civilization. But have we got a real understanding of what is happening deep-down in the quantum world? A glance at the quantum literature dedicated to the discussion of its fundamental aspects, or even to some of its direct applications, is sufficient to perceive the tremendous

confusions and uncertainties that trouble our present-day knowledge. Of course, if the number predicted by the theory is taken as the test, just as was the case with Newtonian gravitation and has become customary among physicists due to the pragmatic viewpoints that pervade the scientific atmosphere, there seems to be no problem at all. But, for instance, what is the physical explanation of atomic stability, the origin of uncertainty, or of quantization, or of the so called wave-particle duality? The theory correctly predicts that the ground state is there; but it tells nothing about the mechanism that brings about such stability, ar just the right energy. In more general terms, it would be difficult to express our feeling about quantum theory better than Bell has already done,[1] by saying that quantum mechanics is a FAPP theory -alright *f* or all practical purposes- about measurements and observables, not *beables*.

As an example of the kind of essential doubts related to basic questions that are under active discussion in the current literature, take the problem of the calculation of the time that particles spend inside the a well or a potential barrier. Not only a number of mutually inconsistent definitions have been proposed, at least one of them giving complex times, but unbounded velocities are predicted and even alleged to have been measured and used to transmit useful information ar superluminal speed.[2]

Since the creation of quantum mechanics there has been a flood of papers and essays discussing these and related questions, and almost any conceivable argument or answer has been proposed or attempted, both from within physics and outside it, principally from the philosophy of science. And this has not been the endeavor of idle physicists or philosophers, since names as Einstein, Dirac, de Broglie, Dirac, Landé, Popper and Schrödinger enter in the unending list. Some of the more representative attempts of these unorthodox efforts have been reviewed in books, as those of Ballentine,[3] Jammer[4] or Wheeler and Zurek.[5] We do not intend in this talk to enter into such matters, due to the obvious lack of time and space, so we leave here our general considerations and go over to more specific questions, directly related to our work.

In the two parts of the present contribution, we deal with specific aspects of a theory that has been proposed since long ago in an attempt to get a causal and realistic understanding of quantum mechanics, known as *stochastic electrodynamics* (SED). This theory starts from the observation that the assumption that atomic electrons move in an empty space, occupied only by the Coulomb field of the nucleus, is contrary to the spirit of quantum theory itself. For we know from quantum theory that it would be inconsistent to ascribe a well defined value to the vacuum state of any (quantum) field, including very particularly the radiation field. Indeed, in quantum theory the fields are represented by operators, to which no definite value can be ascribed, and this should remain true even for the ground or vacuum state. This is recognized to be the case in QED, where it has become customary to recognize that the vacuum can produce observable effects (it contributes its bit, for instance. to the atomic Lamb shift), without, however, itself being observable.

This is the point where SED departs from the usual views, by considering the vacuum not as a virtual field, but as a *real* entity, as real as the electron itself. Hence, in SED one visualizes the atom as immersed in this pervasive stochastic electromagnetic background, with which it interacts continuously, thus acquiring a restless stochastic motion. Qualitatively speaking, one inmediately perceives here a possibility to explain atomic stability, as was envisaged long ago by Nernst[6]: the idea is to prove whether the average rate of energy radiated by the moving electrons can compensate the mean energy absorbed during a given time interval, leading thus to an average stationary condition. Elementary calculations considering only circular orbits validate this conclusion; however, more detailed calculations lead to difficulties. A detailed discussion of this and related matters, with an ample list of references, can be seen in de la Peña and Cetto.[7]

A revision of this literature shows that there are many positive results of the theory –among others the quantum properties of the harmonic oscillator, the diamagnetism of electrons, Casimir and other long-range forces, the kinematics of the Compton effect, the blackbody radiation spectrum and the specific heat of solids- indicating that its physical contents are by no means negligible. In essence, as is discussed in detail in Ref. [7], one can conclude that SED has not been refuted –contrary to what has been said so many times in different forms-, but that in its original form it was beset with problems that can be associated to the extra postulates introduced along its development. A careful revision of its detailed postulates seems indeed to correct the theory in the right sense, leading in particular to a stable atom with the whole set of its random orbital motions. Some aspects of this issue are the subject of part II of the present work, while in this first part,

we take as granted the legitimacy of SED as a fundamental theory of quantum matter in principle, and use it – or rather, its basic principle- to reach an interesting conclusion about the fundamental constants of nature.

II. A COSMOLOGICAL IMPLICATION

An interesting article published recently by Calogero[8] has moved us to disentomb an estimation which we made many years ago,[9] just when the above mentioned difficulties with SED started to become apparent. Having to face those difficulties at the time, we left aside the calculation and almost forgot about it; stimulated now by the new form of SED referred to above that frees us from the difficulties of principle, and by the results of Calogero that parallel our old estimations, we come back to our neglected work.

Let us consider a world made of harmonic oscillators, representing both matter and the radiation field. Such a crude model should be appropriate for the purpose of performing only an order-of-magnitude estimate of certain quantities. Matter and field oscillators, taken in equilibrium at zero temperature, are nevertheless interchanging energy, in such a form that in the mean the oscillators absorb as much as they radiate. Under the assumption that all oscillators of a given frequency, irrespective of their nature, are equivalent, they all must have the same mean ground state energy. Thus, the matter oscillators are radiating and contributing to the random field component, and the random fiel so generated should coincide with the vacuum field of that frequency; the background field is thus maintained. This is a kind of Cosmological Principle associated with SED, or, if preferred, a kind of Electromagnetic Mach Principle: the field produced at a given point by all dipoles should equal the random field acting at that point on the particles themselves. This requirement establishes a relationship between cosmological and atomic constants; in other words, the scale of quantum fluctuations, Planck's constant, becomes determined by cosmological parameters.

A similar reasoning, but dealing entirely with the gravitational field, is the subject discussed by Calogero in his recent work, where, restricting himself to order-of-magnitude considerations, he shows that the identification of the un-avoidable gravitational fluctuations with the quantum fluctuations of atomic systems leads indeed to a relationship between atomic and cosmological constants. Here one should recall also a similar attempt made by Puthoff some years ago[10] within SED, on the basis of a self-regenerating model, to prove that the zeropoint field is due to radiation from the zeropoint field-driven particle motion throughout the rest of the universe. Indeed, the basic ideas of Puthoff's paper are very much in line with the present ones, although in our roungh estimate we refrain from resorting to a specific cosmological model. Also, we leave aside any problem related with the infinite gravitational effects of the zeropoint field, just as is done in QED with all vacuum fields, simply because nobody knows how to escape this problem (a detailed discussion can be seen in the review by Weinberg[11]). By equating the radiation field predicted by the model at a given point with the corresponding component of the zero-point field, we obtain a prediction for the baryonic mass density of the universe, which gives the pursued relation between atomic constants and the Hubble constant. This relation happens to correspond essentially to the one discussed in Weinberg's book[12] (§16.4), which is usually taken as a numerical coincidence, of unknown meaning.

Consider the dipole $\alpha(\alpha = 1,2,\ldots,N)$ of frecuency ω, at a distance $r\alpha$ from us, i.e. from the origin of coordinates; the Fourier amplitude of the electric field produced by this oscillator at the origin is

$$\mathbf{E}\alpha(\omega) = -k^2 \mathbf{n}\alpha \times (\mathbf{n}\alpha \times \mathbf{p}\alpha)\frac{e^{ikr\alpha}}{r\alpha}, \quad k = \frac{\omega}{c}, \tag{1}$$

where $\mathbf{p}\alpha = (e/2)(\mathbf{q}_{0\alpha} + i\mathbf{q}_{0\alpha}/\omega)$ is the (complex) amplitude of the dipole moment $p\alpha e^{-i\omega t}$ and $n\alpha = r\alpha/r\alpha$ is the unit vector in the direction of $r\alpha$. Since the mean energy of the oscillator is $\frac{1}{2}\hbar\omega$, one has $\left\langle q_{0\alpha}^2 + i\dot{q}_{0\alpha}^2/\omega^2 \right\rangle = \hbar/m\omega$, where the average is taken ober the set of oscillators of frecuency ω, so that we write

$$\mathbf{p}\alpha = \frac{e}{2}\sqrt{\frac{\hbar}{m\omega}}\,\mathbf{B}\alpha \tag{2}$$

and consider the amplitudes $B_{\alpha i}$ to be statistically independent complex stochastic variables with zero mean and second moments given by

$$\left\langle B_{\alpha i} B^{*}_{\beta j} \right\rangle = \delta_{\alpha\beta}\delta^{*}_{ij}, \quad \left\langle B_{\alpha i} B^{*}_{\beta j} \right\rangle = 0 . \tag{3}$$

With these assumptions the mean square of equation (1) is (we omit the index α)

$$\left\langle |E(\omega)|^2 \right\rangle = \frac{\hbar e^2}{4mc^4}\frac{\omega^3}{r^2}\left\langle |n\times(n\times B)|^2 \right\rangle = \frac{\hbar e^2}{2mc^4}\frac{\omega^3}{r^2}, \tag{4}$$

since $\left\langle |n\times(n\times B)|^2 \right\rangle = \langle B\cdot B^{*} - (n\cdot B)(n\cdot B^{*}) \rangle = 2$. Here we have taken into account that the field amplitudes produced by statistically independent oscillators are uncorrelated. To evaluate the average energy content of the radiation field of frequency ω at the origin, we integrate equation (4) over a spherical volume of radius R, assuming an isotropic and homogeneous distribution of oscillators, of which there are $n(\omega)$ of frequency ω and a total number $N = \Sigma\, \omega n(\omega)$:

$$\langle \varepsilon(\omega) \rangle = \frac{n(\omega)}{4\pi}\int_V \left\langle |E(\omega)|^2 \right\rangle dV = \frac{\hbar e^2 \omega^3 R}{2mc^4} n(\omega) . \tag{5}$$

The Cosmological Postulate of SED asserts that this energy should correspond to the zero-point field energy of frequency ω, $\frac{1}{2}\hbar\omega$, one thus obtains

$$n(\omega) = \frac{mc^4}{e^2\omega^2 R} . \tag{6}$$

To estimate the total number of oscillators we integrate over all frequencies, using the rule $\frac{1}{V}\sum \omega \to (2\pi^2 c^3)^{-1}\int d\omega\, \omega^2$, which gives

$$N = \sum \omega n(\omega) = \frac{mc^4}{e^2 R}\sum \omega \frac{1}{\omega^2} \to \frac{mcV}{2\pi^2 e^2 R}\int_0^{\Omega} d\omega = \frac{mc\Omega V}{2\pi^2 e^2 R} . \tag{7}$$

Since the integral is divergent we have introduced a cutt of frequency for the material oscillators. Indeed, the material oscillators are transparent at arbitrarily high frequencies, and one can take a cutt of around the pair-creation frequency $\Omega = 2mc^2/\hbar$ as physically meaningful, so that (7) becomes

$$\frac{N}{V} = \frac{m^2 c^2}{\pi^2 \alpha \hbar^2 R} , \tag{8}$$

where $\alpha = e^2/\hbar c$ stands for the fine-structure constant. Here V must be taken as the volume of the visible universe, as this is the part that contributes to the radiation field, and thus N/V is to be identified with the cosmological density of charged particles, which multiplied by m_N (the nucleon mass or any typical baryon mass) gives for the baryonic density of the universe the following estimate:

$$\rho \cong \frac{m^2 m_N c^2}{\pi^2 \alpha \hbar^2 R} . \tag{9}$$

Before going further let us add a couple of remarks with regard to this expression. Firstly, we have not taken into account any absorption process, the reason being that we are dealing with the *zeropoint* field, a

field that is not absorbed by matter in equilibrium with it. Of course it is scattered by matter, but for a uniform and homogeneous universe the final distribution remains the same. Thus equation (9) needs *no* correction from Thomson scattering.

The second comment refers to the naivete of the model. As already stated, the idea is to make a qualitative test of the SED Cosmological Principle, and for such purpose the present rough estimate should suffice. For example, a somewhat more realistic model would take into account the expansion of the universe, which produces a redshift, so that instead of the original frequency ω radiated when the universe had a scale factor R(t), we see the frequency

$$\omega_0 = \frac{R(t)}{R_0}\omega, \tag{10}$$

where the subindex 0 refers to the present moment and place of observation. Thus, if $\nu(\omega)$ represents the spatial density of oscillators of local frequency ω at a distance r from us, instead of equation (5) we would write

$$\langle \varepsilon(\omega) \rangle = \frac{e^2\hbar}{2mc^4}\omega_0^3 R_0^3 \int_0^{R_0} \frac{\upsilon(\omega_0 R_0 / R(t))}{R^3(t) r^2} r^2 dr, \tag{11}$$

where, using Weinberg's notation,[12] one must put $dr = (\sqrt{1-kr^2}/R(t)dt$. To go further one would have to specify the cosmological model; however, no fundamental qualitative change seems to ocurr, although there appear of course numerical factors, which do not alter the essential contents of equation (9). Thus, up to such numerical factors we take our former result (9) as a reasonable relation among the relevant constants of nature.

Let us investigate now how well equation (9) works. For this purpose we introduce an auxiliary mass defined by

$$\overline{m} = \left(\frac{m^2 m_N}{\pi^2 \alpha}\right)^{1/3} \cong 30m. \tag{12}$$

Equation (9) can then be rewritten in the form (observe that $R = R_0$, so we add the subindex 0 to mark the present values of the cosmological parameters)

$$\frac{\rho_0 R_0^3}{\overline{m}} = \frac{\overline{m}^2 c^2 R_0^2}{\hbar^2} = \left(\frac{R_0}{\lambda_{\overline{m}}}\right)^2, \tag{13}$$

where $\lambda_{\overline{m}}$ is the Compton wavelength (divided by 2π) associated with the mass $\overline{m}, \lambda_{\overline{m}} = \hbar / \overline{m}c$. We recognize in each side of equation (13) one of the 'large numbers' of cosmology, namely (H_0 is the present value of Hubble constant, $H_0 = c/R_0$)

$$N_1 = \frac{\hbar c}{G m_N^2} \sim \frac{1}{6} 10^{39}, \tag{14}$$

$$N_2 = \frac{mc^2}{\hbar H_0} = \frac{mcR_0}{\hbar} = \frac{R_0}{\lambda_m} \sim \frac{1}{3} 10^{39}, \tag{15}$$

$$N_3 = \frac{\rho_0 c^3}{m_N H_0^3} = \frac{\rho_0 R_0^3}{m_N} \sim 10^{79}, \tag{16}$$

Except for the differences in the masses, equation (13) reads

$$N_3 = N_2^2, \tag{17}$$

which is one of the well-known numerical coincidences among these large numbers. The surprising content of this expression is that it relates cosmological parameters with Planck's constant, which is a highly non-trivial result (recall that Weinberg[12] qualifies it as mysteriuos). The second independent relation among these numbers, which can be taken to be $N_1 N_2 \cong N_3$, relates cosmological parameters only and can be obtained from cosmological models, as the Friedmann model.

We conclude that the SED Cosmological Principle, namely that the energy of the vacuum fluctuations corresponds to the energy radiated by all dipoles of the universe, seems to hold and serves to explain the relation $N_3 = N_2^2$ up a constant factor of at most a few orders of magnitude.

Let us now recast equation (9) in a different form. In terms of the dimensionless gravitational coupling constant $\alpha_G = Gmm_N/\hbar c$ equation (9) becomes

$$\alpha_G R_0 \cong \frac{3\pi}{8}\alpha\lambda_m,$$

which we write simply as

$$\alpha\lambda_m \cong \alpha_G R_0, \tag{18}$$

where the value of the common length $l = \alpha\lambda_m = e^2/mc^2$ equals the classical electron radius. Equation (18) can be extended by observing that for the nuclear forces one can take $\alpha_N \cong 1$ and $R_N \cong \hbar / m\pi c \cong \alpha\lambda_m / 2$ (numerically), so that

$$\alpha_G R_0 \cong \alpha\lambda_m \cong \alpha_N R_N. \tag{19}$$

Equation (18) explains why Calogero's gravitational arguments and the present electromagnetic ones lead to equivalent results, and equation (19) extends this equivalence to nuclear forces, suggesting a kind of universality of the vacuum effects on matter. This is, in our view, another form of saying that it should be feasible to represent the effects of the zero-point field as a fluctuating metric field, a possibility that was already studied by Einstein himself.[13] Some additional comments on this subject and related references can be seen in the cited book on SED.[7] It is interesting to observe that equation (17) (or Ed. (18)) cannot be obtained solely from the usual quantum formalism; it is within the conceptual frame of SED where the cosmological principle leading to equation (17) finds its natural place.

REFERENCES

[1]BELL, J.S. (1985) CERN preprint TH-2053, known as "The theory of local beables". Reprinted in **Dialectica** 39.

[2]See, e.g., (1994): The recent reviews by V.S. OLKHOVSKY and E. RECAMI, **Phys. Rep.** 214, 339 (1992), R. Landauer and Th. Martin, **Rev. Mod. Phys.** 66, 217 (1994), and S. BROUARD, R. SALA and J:G. MUGA, **Phys. Rev.** A 49, 4312.

[3]BALLENTINE, L.E. (1989): "Quantum Mechanics", Prentice Hall, New York.

[4]JAMMER, M. (1974): "The Philosophy of Quantum Mechanics". J. Wiley, New York.

[5]WHEELER, J.A. and W. ZUREK, eds. (1983): "Quantum Theory and Measurement", Princeton University Press, New Jersey

[6]NERNST, W. (1916): Verhandl. Deutsch. **Physik. Gesell.** 18, 83.

[7]PEÑA, L. de la and A.M. CETTO (1996): "The Quantum Dice, An introduction to stochastic electrodynamics", Kluwer Academic Publishers, Dordrecht.

[8]CALOGERO, F. (1997): Phys. **Letters** A.

[9]PEÑA, L. de la and A.M. CETTO (1984): "A cosmological implication of stochastic electrodynamics", Preprint IFUNAM, México.

[10]PUTHOFF, H.E. (1989): **Phys Rev.** A 40, 4857; see also P.S. WESSON, **Phys. Rev.** A 44, 3379, E. SANTOS, **Phys. Rev.** A 44, 3383, and H.E. PUTHOFF, **Phys. Rev.** A 44, 3385 (1991).

[11]WEINBERG, S. (1989): **Rev. Mod. Phys.** 61, 1.

[12]WEINBERG, S. (1972): "Gravitation and Cosmology". J. Wiley, New York.

[13]EINSTEIN, A. (1924): Schweiz. Nat. Gesell. **Ver.** 25, 85.

UNDERSTANDING THE QUANTUM BEHAVIOUR OF MATTER AS A DERIVED PROPERTY. II. RECOVERY OF THE QUANTUM FORMALISM

A.M. Cetto and L. de la Peña, Instituto de Física, Universidad Nacional Autónoma de México

ABSTRACT

The atomic problem is approached a new within the framework of SED, allowing the zeropoint field to be modified by its interaction with matter. By requiring the complete matter-field system to satisfy a principle of stability against fluctuations, that characterizes the stationary regime, one is led in a natural form to the Heisenberg equations of motion for the description of the mechanical part.

I. BASIS FOR A NEW SED

This is the second part of our contribution to this Conference, and should be read as the continuation of part I, which deal with a relationship between Planck's constant and cosmological constants involving the large-numbers relation $N_2^2 \sim N_3$. As anticipated in the introduction contained in part I, we now present a modified version of SED that is suited to deal with the atomic problem and leads to results which are fully consistent with the quantum formalism.

To make a long story short, we start as usual with the equation of motion for a particle subject to the simultaneous action of an external binding force and the zeropoint radiation field. In an approximate form, which is sufficient for the present analysis, one may write the corresponding Abraham-Lorentz equation (usually renamed in the context of SED after Braffort and Marshall, two pioneers of the theory)

$$m\ddot{x} = m\tau\, \dddot{x} + F(x) + eE(x,t). \qquad (1)$$

where $\mathbf{E}(x,t)$ represents the electric zeropoint field and is of course a stochastic variable with zero mean; the magnetic force term is neglected, and the constant factor that appears in the radiation reaction term is the time parameter $\tau = 2e^2/3mc^3$. Given the external force $\mathbf{F}(x)$, the problem is simply stated: look for the stationary solutions of equation (1) and find out if they have something to do with the corresponding quantum description, after assigning adequate statistical properties to the (vacuum) field $\mathbf{E}(x,t)$.

This kind of program has been carried out for some particular problems with results that go from satisfactory to excellent. For example, excellent results are obtained in the study of van der Waals and Casimir forces, the diamagnetic properties of electrons, etc.; we speak only of satisfactory results for the harmonic oscillator, because the excited levels, even if they are there, appear in an entirely formal way, more or less as they do in usual quantum theory; thus, the physics remains obscure. As was said before, when dealing however with the hydrogenic atom, the results of such a direct approach to the problem are disastrous: no bound ground state is predicted.

A more careful consideration of the above examples shows that the different problems have not been treated with equity. Indeed, one usually takes for granted the statistical properties of the vacuum field, and uses these same properties from the beginning to the end of the calculation, as if the vacuum field were exactly *the same* for all problems, and equal to the free field. However, in other cases, for instance when dealing with macroscopic bodies, as in the calculation of Casimir forces, one takes into consideration the effects of the boundaries (which are of course made of atoms), so that the vacuum field is adjusted to the

specific problem. Another such example is the calculation of the modification of the background field within a dielectric material, in equilibrium with the atoms of the material. One may argue that a similar consideration should be applied to *all* cases, including the single atom. Indeed, the complete problem refers to the coupled matter-field system, and in principle both parts of the system become mutually affected by the interaction , as soon as they start to interact. Of course, to state the problem in these terms practically amounts to surrendering, because it becomes intractable; however we may be somewhat less ambitious and consider the effects of the adjustment of the field to the *final* equilibrium situation, by looking for a self-consistent stationary solution of equation (1) that satisfies appropriate conditions, assuming that the matter-field system reaches a situation of dynamical equilibrium –which is supposed to be precisely the situation dealt with by the quantum formalism. In other words, one would expect a close correspondence between the self-consistent stationary solutions of SED and the stationary solutions of quantum mechanics.

As a first approach in this direction, we consider a particular kind of stationary solutions which are characterized by the fact that they are the less random possible solutions, as independent of the *specific* realization of the random field, or, what amounts to the same, as stable against small perturbations, as possible.[1,2] These stable stationary solutions turn out to be characterized by two properties, namely, they correspond precisely to the quantum mechanical solutions given by the Heisenberg equations of motion, and moreover some 'relevant' field components of the modified vacuum field end up having correlated phases, which indicates a central aspect in which this field differs from the free vacuum field, characterized by statistically independent phases.

II. SELF-CONSISTENT SED SOLUTION FOR A SIMPLE NONLINEAR PROBLEM

According to the previous discussion and contrary to standard practice in usual SED, along the treatment of the mechanical part of the SED problem we consider the statistical properties of the vacuum field initially unknown, and let them become determined by the requirements of the system itself under stationary conditions, that is, once the quantum regime has been attained. For arbitrarily small time intervals the conditions of the system are largely arbitrary, and may even be inconsistent with the usual quantum behavior; this is the reason we consider a system that has already evolved towards the quantum regime. We represent the field as a time Fourier transform in the form

$$E(x,t) = \sum_{\sigma,k} \tilde{E}_k\sigma(x) a^0_{k\sigma} e^{-i\omega_k t} + \text{c.c.}, \quad k = \omega_k / c. \tag{2}$$

The functions $\tilde{E}_k\sigma(x)$ carry all the space dependence of the field, whereas the coefficients $a_k\sigma$ are the stochastic variables that determine its statistical properties. Strictly speaking, the sum must be understood as an integration, because the field contains all frequencies from zero to infinity; the point is that, as we shall soon see, there are some specific frequencies to which the system responds with particular intensity (we call them the relevant frequencies); the rest of components taken together play the role of a noise, with little influence on the determination of the basic equilibrium properties. This noise is already being neglected in (2) but it must be reintroduced when performing a more detailed analysis (it gives rise, for instance, to radiative corrections[1,2]). In what follows we further simplify the treatment by avoiding everywhere the question of the polarization.

In the long-wavelength approximation one neglects the spatial dependence of the field; the $\tilde{E}_k\sigma$ are therefore taken as constants, whose value is determined by the energy of the field (see equation (5) below). Since the Fourier components $\tilde{E}_k a^0_k$ correspond to an infinite number of modes, each with the same average energy $\frac{1}{2}\hbar\omega_k$, the energy per mode of each of these components has an extremely sharp distribution, as follows from the central limit theorem, and the random amplitudes $a^0_{kj} = \left(a^0_k\right)_j$ have surely a (very nearly) Gaussian distribution sharply peaked around ω_k; thus, for all practical purposes one can consider them as having fixed amplitudes but random phases. For the *free vacuum field* they are taken as statistically independent, so that

$$\left\langle a_{ki}^{0}\right\rangle = 0, \quad \left\langle a_{ki}^{0} a_{k'i'}^{0*}\right\rangle = \delta_{kk'}\delta_{ii'}, \tag{3}$$

$$a_{ki}^{0} = e^{i\varphi_{ki}}, \tag{4}$$

with the random phases φ_{ki} uniformly distributed over $(0,2\pi)$. Equation (3) is alright only for the noise (which we are not taking into account), but not for the relevant frequencies, the properties of which are to be generated by the theory itself (the general rule will be given in section III below). Equation (4), in contrast, holds in general; all the stochasticity of the field is now expressed in the random phases. With the selection (4), we must take

$$\tilde{E}_k = i\sqrt{\frac{\pi\hbar\omega_k}{V}} \tag{5}$$

for the energy per mode bo be $\frac{1}{2}\hbar\omega_k$. In the above expressions a superindex '0' has been added to the field amplitudes, with the purpose of introducing the following notation (we omit from now on the Cartesian indices wherever they are unneccessary):

$$a_k \equiv a_k(t) = a_k^0 e^{-i\omega_k t}. \tag{6}$$

Note that $a_k(t)$ and a_k^0 have the same statistical properties.

The Braffort-Marshall equation that will be used for the analysis of the quantum regime reads therefore, for a one-dimensional problem,

$$m\ddot{x} = m\tau\dddot{x} + F(x) + e\sum_k \tilde{E}(\omega) a\omega^0 e^{-i\omega t} + c.c., \tag{7}$$

with $\tilde{E}_k$ given by (5) and random amplitudes of the form (4).

It should be clear that since a partial averaging is being performed over all modes of a given frequency, the description afforded by equation (7) refers not to an individual particle, but to an ensemble of equivalent particles, each one acted on by a specific mode of such frequency. Alternatively, we may refer to a partly averaged behaviour of a given particle. Thus, the ensuing theory will be *essentially statistical*, just as is the case with quantum mechanics. Also, the emerging mechanical variables are to be seen as partly averaged random variables, and not as strictly individual local variables; in particular, the correlations between variables may be poorly described.

To be above limitations and features of the ensuing theory, we must add that the detailed behaviour in terms of the single field modes has not only becomes hidden, but is irretrievably lost for the description to follow (hence also for the quantum description). We cannot, any more recover a fully deterministic picture by 'adding the hidden variables' or any similar simple procedure; it would not be a matter of embedding the ensuing description into a larger theory, but of constructing a new one from scratch.

To solve equation (7) in general[1,2] one starts by writing both the stationary x(t) and the force as Fourier series in the form

$$x = \sum_k \tilde{z}_k(\omega) e^{-i\omega_k t} + cc, \tag{8}$$

$$F(x) = \sum_k \tilde{\Phi}_k(\omega) e^{-i\omega_k t} + cc, \tag{9}$$

so that from (7) one gets the set of equations

$$m(-\omega_k^2 + i\tau\omega_k^3)\tilde{z}_k = \tilde{\Phi}_k + e\tilde{E}_k a_k^0. \tag{10}$$

Both $\tilde{z}(\omega)$ and $\tilde{\Phi}(\omega)$ may depend on arbitrary combinations of the random amplitudes a_k^0 of the different relevant frequencies ω_k, which are stochastic variables; further, $\tilde{\Phi}(\omega)$ depends in general on $\tilde{z}(\omega)$ in a complicated way, with the obvious exception of the linear-force problem. Observe that no speficific relation among the relevant frequencies is being considered, as is done beforehand, for example, in the treatment of classical multiply periodic systems.

To simplify the exposition it is convenient to consider a simple nonlinear example that contains already the typical features of the general case. A natural example is the anharmonic oscillator consisting of a linear oscillator plus a cubic force term and governed by the equation

$$\ddot{x} + \omega_0^2 x + \kappa x^3 = \tau \dddot{x} + \frac{e}{m} E(t). \tag{11}$$

The core of the problem resides in the nonlinear part of the force; its Fourier coefficient is, from equations (8) and (9),

$$\tilde{\Phi}_k^{(3)} = \lim_{T\to\infty} \frac{\kappa}{2T} \int_{-T}^{T} x^3(t) e^{i\omega_k t} dt = \kappa \sum_{n'n''n'''} \tilde{z}_{n'} \tilde{z}_{n''} \tilde{z}_{n'''}, \tag{12}$$

where the sum extends over the set of indices for which the frequencies satisfy the condition

$$\omega_k = \omega_{n'} + \omega_{n''} + \omega_{n'''} \tag{13}$$

coming from the delta function that is generated by the time integration of $e^{i(\omega_{n'}+\omega_{n''}+\omega_{n'''})t}$ extended to infinite limits. Note that this condition reduces the number of independent Fourier indices to two for a given frequency ω_k.

With the dissipative effect of the radiation reaction term $m\tau \dddot{x}$ fully taken into account, only one stationary solution of the form (8) should exist, corresponding to the ground state if the appropriate stationary field is used. However, in the radiationless approximation –which corresponds to the quantum mechanical description–, the system admits more than one stationary solution, and usually an infinite number of them. An extra index (α) is therefore required to distinguish between different solutions:

$$x\alpha(t) = \sum_n \tilde{z}_{\alpha n} e^{-i\omega_{n\alpha} t} + c.c. \tag{14}$$

Note the reverse order of the subindices of ω in the exponent; we have adopted this convention to adjust the final results to the normal conventions of quantum mechanics. It will be seen below that the two indices (here taken as different and denoted by n and α) play actually a symmetric role; therefore Greek letters will be used for both. Further, it will turn out that $\omega_{\beta\alpha} = -\omega_{\alpha\beta}$ (see equation (55)), so that one may change the sign of the time exponents by inverting the order of the indices, and write equation (14) in the form

$$x\alpha(t) = \sum_\beta \tilde{z}_{\alpha\beta} e^{i\omega_{\alpha\beta} t} + c.c. \tag{15}$$

The Fourier coefficient of the nonlinear term of frequency $\omega_{\alpha\beta}$ is then

$$\tilde{\Phi}^{(3)}(\omega)\Big|_{\omega_{\alpha\beta}} = \kappa \sum (\tilde{z}_{\mu'\upsilon'}\tilde{z}_{\mu''\upsilon''}\tilde{z}_{\mu'''\upsilon'''})_{\omega_{\alpha\beta}}, \tag{16}$$

where the summation is to be performed over the set of indices such that

$$\omega_{\alpha\beta} = \omega_{\mu'\nu'} + \omega_{\mu''\nu''} + \omega_{\mu'''\nu'''}, \tag{17}$$

according to (13). Although the number of subindices has increased, it should be kept in mind that in equation (16) there is only a double sum, which means that the additional indices must be related to the summation indices, as will be clarified below.

With (16), equation (10) becomes for this problem

$$(-\omega_{\alpha\beta}^2 + i\tau\omega_{\alpha\beta}^3 + \omega_0^2)\tilde{z}_{\alpha\beta} + \kappa \sum (\tilde{z}\tilde{z}''\tilde{z}''')_{\alpha\beta} = \frac{e}{m}\tilde{E}(\omega_{\alpha\beta})a_{\alpha\beta}^0. \tag{18}$$

Now given the structure of the right-hand-side term in this equation, it is convenient to rewrite the Fourier amplitudes of the position coordinate and the force in the form

$$\tilde{z}_{\alpha\beta} = \tilde{x}_{\alpha\beta}a_{\alpha\beta}^0. \tag{19}$$

$$\tilde{\Phi}_{\alpha\beta} = \tilde{F}_{\alpha\beta}a_{\alpha\beta}^0. \tag{20}$$

In the particular case of the linear oscillator ($\kappa = 0$), the resulting coefficients $\tilde{x}_{\alpha\beta}$ and $\tilde{F}_{\alpha\beta}$ are independent of the $a_{\alpha\beta}^0$, and are thus nonrandom numbers; but with $\kappa \neq 0$ the situation is different; in this case it follows from equation (18) that the coefficient $\tilde{x}_{\alpha\beta}$ depends on the set amplitudes $\{a_{\alpha\beta}^0$ in a quite complicated way, for on substituting (19) there appear products of three a^0's containing all those frequencies which combine to give just $\omega_{\alpha\beta}$, as demanded by equation (17). Explicitly, and on dividing by the common factor $a_{\alpha\beta}^0$, one gets the set of coupled stochastic equations

$$(-\omega_{\alpha\beta}^2 + i\tau\omega_{\alpha\beta}^3 + \omega_0^2)\tilde{x}_{\alpha\beta} + \kappa \sum (\tilde{z}\tilde{z}''\tilde{z}''')_{\alpha\beta} = \frac{(a^{0'}a^{0''}a^{0'''})\alpha\beta}{a_{\alpha\beta}^0} = \frac{e}{m}\tilde{E}(\omega_{\alpha\beta}). \tag{21}$$

These equations determine the *response amplitudes* $\tilde{x}_{\alpha\beta}$ and *characteristic frequencies* $\omega_{\alpha\beta}$ for the problem; these latter correspond to what we have termed relevant frequencies.

The solutions of (21) are functions of the random amplitudes $a_{\alpha\beta}^0$, and thus are in principle stochastic numbers by themselves; they represent a different set of stationary solutions for every realization of the field. Here is where we deviate from the original SED approach. We observe that for *certain* random fields there are solutions to (21) that are nonrandom numbers and thus independent of the specific realization of the field. These particularly simple solutions occur when the set of equations that determine them contains no random coefficients, i.e., when all explicit dependence on the a^0's vanishes from the equations, which in the case under study occurs only when the following equality holds for *each* relevant frequency,

$$(a^{0'}a^{0''}a^{0'''})_{\alpha\beta} = a_{\alpha\beta}^0, \tag{22}$$

except for a possible constant factor of proportionality (which must be set equal to 1, as was already done here). For those fields that satisfy these conditions, the characteristic (Fourier) frequencies $\omega_{\alpha\beta}$ and the

corresponding response amplitudes $\tilde{x}_{\alpha\beta}$ of the stationary states of motion are essentially insensitive to the fluctuation of the random variables a^0, and hence independent of the specific realization. They are thus remarkably stable solutions. We propose to consider seriously these solutions, determined by demanding the frequencies $\omega_{\alpha\beta}$ to become *non-stochastic.*

Let us now investigate the consequences of this demand. Firstly, with (22), the system of algebraic equation (21) reduces to

$$(-\omega_{\alpha\beta}^2 + i\tau\omega_{\alpha\beta}^3 + \omega_0^3)\tilde{x}_{\alpha\beta} + \kappa\sum(\tilde{x}\tilde{x}''\tilde{x}''')_{\alpha\beta} = \frac{e}{m}\tilde{E}(\omega_{\alpha\beta}), \tag{23}$$

whose solutions $(\tilde{x}_{\alpha\beta}, \omega_{\alpha\beta})$ are obviously deterministic. A further important consequence is obtained as follows.

Introducing (19) into (15) we write with the help of equations (17), (20) and (22), in a synthetic notation,

$$x^3 = \sum \tilde{z}'\tilde{z}''\tilde{z}''' e^{i(\omega'+\omega''+\omega''')t} = \sum \left(\sum \tilde{x}'\tilde{x}''\tilde{x}'''\right)_{\alpha\beta} \left(a^{0'}a^{0''}a^{0'''}\right)_{\alpha\beta} e^{i(\omega'+\omega''+\omega''')t}$$

$$= \sum \left(\sum \tilde{x}'\tilde{x}''\tilde{x}'''\right)_{\alpha\beta} a^0_{\alpha\beta} e^{i\omega_{\alpha\beta}t} = \sum (\tilde{x}^3)_{\alpha\beta} e^{i\omega_{\alpha\beta}t}, \tag{24}$$

where the element $(\tilde{x}^3)_{\alpha\beta}$ involves a double summation over indices such that (17) and (22) are satisfied; in the previous shortand notation this reads

$$(\tilde{x}^3)_{\alpha\beta} = \left(\sum \tilde{x}'\tilde{x}''\tilde{x}'''\right)_{\alpha\beta}. \tag{25}$$

The detailed meaning of this and other similar rules involving constrained sums will be discussed shortly. What should be noted here is that a force which is nonlinear in x has become a linear function of the field amplitudes $a^0_{\alpha\beta}$, as shown by equation (24). This is a most important result: despite the presence of nonlinearities, the system *responds linearly to the field* and behaves as a set of linear oscillators of frequency $\omega_{\alpha\beta}$ and amplitude $\tilde{x}_{\alpha\beta}$. We stress that no linear approximation is being made, but it is the system's response to the field that is linear under the assumed stability conditions, with sharply defined frequencies and response coefficients determined by the nonlinear equations (23). This is the reason why we call the present theory *linear* SED.

To see the meaning and implications of the above results let us use them to recast equation (23) in the algebraic form

$$-m\omega_{\alpha\beta}^2\tilde{x}_{\alpha\beta} = \tilde{F}_{\alpha\beta} - im\tau\omega_{\alpha\beta}^3\tilde{x}_{\alpha\beta} + e\tilde{E}(\omega_{\alpha\beta}), \tag{26}$$

with $\tilde{F}_{\alpha\beta} = (\tilde{F}(\tilde{x}))_{\alpha\beta}$ including of course the nonlinear terms. For the specific case of the anharmonic oscillator the force coefficients are given by

$$\tilde{F}_{\alpha\beta} = -\omega_0^2\tilde{x}_{\alpha\beta} - \kappa\sum(\tilde{x}\tilde{x}''\tilde{x}''')_{\alpha\beta}, \tag{27}$$

where the (double) sum in the triple product of $\tilde{x}$ must be such that equation (17) is satisfied.

Observe from (15) and (19) that one can use alternatively time-dependent coefficients defined by

$$\tilde{x}_{\alpha\beta}(t) = \tilde{x}_{\alpha\beta} e^{i\omega_{\alpha\beta} t}, \tag{28}$$

$$\tilde{F}_{\alpha\beta}(t) = \tilde{F}_{\alpha\beta} \exp(i\omega_{\alpha\beta} t), \tag{29}$$

$$\tilde{E}_{\alpha\beta}(t) = \tilde{E}_{\alpha\beta} \exp(i\omega_{\alpha\beta} t), \tag{30}$$

so that

$$\frac{d^2}{dt^2} \tilde{x}_{\alpha\beta}(t) = -\omega_{\alpha\beta}^2 \tilde{x}_{\alpha\beta}(t) \tag{31}$$

and so on, and equation (26) once multiplied by $\exp(i\omega_{\alpha\beta} t)$ becomes

$$m \frac{d^2 \tilde{x}_{\alpha\beta}(t)}{dt^2} = \tilde{F}_{\alpha\beta}(t) + m\tau \frac{d^3 \tilde{x}_{\alpha\beta}(t)}{dt^3} + e\tilde{E}_{\alpha\beta}(t). \tag{32}$$

This is a set of (nonlinear) deterministic equations of motion for the $\tilde{x}_{\alpha\beta}(t)$; all random quantities have vanished from the description, once the fundamental effects of the random radiation field have been taken into account. We can thus take now the radiationless approximation by writing the above equation to zero order in *e*,

$$m \frac{d^2 \tilde{x}_{\alpha\beta}(t)}{dt^2} = \tilde{F}_{\alpha\beta}(t). \tag{33}$$

Despite its form this (unperturbed) equation is *not* a classical equation of motion, owing to the specific meaning and algebraic properties of the terms $\tilde{x}_{\alpha\beta}$ and $\tilde{F}_{\alpha\beta}$. In fact, it is a Heisenberg equation of motion (as will be shown), with solutions that correspond to those of the quantum mechanical description of stationary states. It is astonishing that equation (33), which can be (and is) taken as the fundamental law for the problem, does not contain any element whatsoever reminding us of its stochastic origin, and no trace whatsoever of the background field that generates and sustains the stationary solutions.

III. THE GENERAL BOUND PROBLEM

We now turn our attention to the solution of the general equation

$$m\ddot{x} = m\tau \dddot{x} + F(x) + e\sum_k \tilde{E}_k a_k^0 e^{-iw_k t} \tag{34}$$

in the quantum regime, and show that the results of the previous section are applicable in general. For simplicity, in equation (34) the c.c. terms of the zero-point field have been embodied in the sum, which now runs over positive and negative frecuencies. Following equations (15) and (19) one writes.

$$x\alpha(t) = \sum \beta \tilde{z}_{\alpha\beta} e^{i\omega_{\alpha\beta} t} = \sum \beta \tilde{x}_{\alpha\beta} a_{\alpha\beta}^0 e^{i\omega_{\alpha\beta} t} \tag{35}$$

and introduces this expansion into equation (34), to obtain

$$x\alpha = -\frac{e}{m} \sum \beta \frac{\tilde{E}_{\alpha\beta}}{\Delta(\omega_{\alpha\beta})} = a_{\alpha\beta}^0 e^{i\omega_{\alpha\beta} t}, \tag{36}$$

where

$$\Delta(\omega) = \omega^2 + \frac{1}{m}\frac{\tilde{\Phi}(\omega)}{\tilde{z}(\omega)} - i\tau\omega^3 \tag{37}$$

still depends on the random amplitudes. As before, $\tilde{\Phi}(\omega)$ represents the Fourier transform of the external force. We assume that all relevant singularities are simple isolated poles, and that they give the dominant contributions to $x\alpha(t)$. The system is assumed to behave resonantly at the frequencies corresponding to the poles; the smallness of the parameter τ entering in the radiation reaction force guarantees the sharpness of the resonance, so that the contributions from the poles are clearly dominant. For bounded motions, the equations for the poles $\Delta(\omega) = 0$, or

$$\frac{1}{m}\frac{\tilde{\Phi}(\omega)}{\tilde{z}(\omega)} = -\omega^2 + i\tau\omega^3, \tag{38}$$

is satisfied only for certain (discrete, and in general stochastic) frequencies. For instance, for the harmonic oscillator of natural frequency ω_0 there are poles at + ω_0 and - ω_0, but in the case of particles bound by nonlinear forces, the values of ω at the poles will in general depend on the state of motion. Further, since for all frequencies ω of interest $|\tau\omega| \ll 1$, we take the radiationless approximation, so that equation (38) becomes

$$-\frac{1}{m}\frac{\tilde{\Phi}_{\alpha\beta}(\omega)}{\tilde{z}_{\alpha\beta}(\omega)} = \omega_{\alpha\beta}^2. \tag{39}$$

Now we introduce the requirement of nonrandom values for the characteristic frequencies $\omega_{\alpha\beta}$. The ratio $\tilde{\Phi}_{\alpha\beta}/\tilde{z}_{\alpha\beta}$ must then be independent of $a^0_{\alpha\beta}$, and equations (35), (36) and (39) lead to $\tilde{z}_{\alpha\beta}$ and $\tilde{\Phi}_{\alpha\beta}$ linear in $a^0_{\alpha\beta}$, so that one can write

$$\tilde{z}_{\alpha\beta} = \tilde{x}_{\alpha\beta}a^0_{\alpha\beta}, \quad \tilde{\Phi}_{\alpha\beta} = \tilde{F}_{\alpha\beta}a^0_{\alpha\beta} \tag{40}$$

with $\tilde{x}_{\alpha\beta}$ and $\tilde{F}_{\alpha\beta}$ nonrandom coefficients, related by the system of algebraic equations contained in (39), namely,

$$\tilde{F}_{\alpha\beta} = -m\omega_{\alpha\beta}^2\tilde{x}_{\alpha\beta}. \tag{41}$$

>From this expression and the second one in (40) it follows that the Fourier transform of the external force is a linear function of the stochastic amplitudes; the linear response to the field is thus extended to any binding external force. It is remarkable that this general property follows as a consequence of the demand of nonrandom values for the characteristic frequencies $\omega_{\alpha\beta}$ of the stationary solutions.

Let us now investigate the properties that the field amplitudes $a^0_{\alpha\beta}$ must have for equations (40) to nold. We assume that the external force can be expressed as a power series in x; this leads to a sum of terms containing any number n of factors of $\tilde{z}_{\mu\nu}$, of the type

$$\tilde{z}_{\lambda_1\mu_1}\tilde{z}_{\lambda_2\mu_2}\cdots\tilde{z}_{\lambda_n\mu_n} = \tilde{x}_{\lambda_1\mu_1}\tilde{x}_{\lambda_2\mu_2}\cdots\tilde{x}_{\lambda_n\mu_n}a^0_{\lambda_1\mu_1}a^0_{\lambda_2\mu_2}\cdots a^0_{\lambda_n\mu_n}, \tag{42}$$

each of which should correspond to a fixed frequency, say $\omega_{\alpha\beta}$, so that

$$\left(\tilde{z}^n\right)_{\alpha\beta} = \left(\tilde{x}^n\right)_{\alpha\beta}a^0_{\alpha\beta}. \tag{43}$$

It follows the *any* product of *relevant* amplitudes a^0 must reduce to a single a^0, so that the product terms remain linear in a^0,

$$a^0_{\lambda_1\mu_1} a^0_{\lambda_2\mu_2} \cdots a^0_{\lambda_n\mu_n} = a^0_{\alpha\beta}. \tag{44}$$

Of the 2n indices appearing on the left hand side, two are fixed (α and β) and n – 1 are summation indices (due to the implicit δ-functions, as in equation (12)), so that n – 1 indices remain free; but since this product of a^0's should be just the required $\alpha^0_{\alpha\beta}$, and not another independent random amplitude, the indices must repeat themselves (otherwise independent random phases would appear). In particular, for n = 2 one can write either

$$a^0_{\alpha\beta} a^0_{\mu\mu} = a^0_{\alpha\beta} \tag{45}$$

or

$$a^0_{\alpha\beta} a^0_{\mu\beta} = a^0_{\alpha\beta} \tag{46}$$

for arbitrary α, β and μ. From the first of these equations we get

$$a^0_{\mu\mu} = 1, \tag{47}$$

whereas the iteration of (46) to arbitrary n gives:

$$a^0_{\alpha\mu_1} a^0_{\mu_1\mu_2} \cdots a^0_{\mu_{n-1}\beta} = a^0_{\alpha\beta}. \tag{48}$$

With the a^0's having constant magnitude equal to 1,

$$a^0_{\alpha\beta} = e^{i\varphi_{\alpha\beta}}, \tag{49}$$

equation (48) means that the phases must satisfy

$$\varphi_{\alpha\beta} = \varphi_{\alpha\mu_1} + \varphi_{\mu_1\mu_2} + \cdots + \varphi_{\mu_{n-1}\beta}, \text{ modulo } 2\pi \tag{50}$$

and, as follows from equations (46) and (45),

$$\varphi_{\mu\mu} = 0, \qquad \varphi_{\alpha\beta} = -\varphi_{\beta\alpha}, \tag{51}$$

It terms of the amplitudes, the latter constraint implies the relations

$$a^0_{\alpha\beta} = \left(a^0_{\beta\alpha}\right)^* = \left(a^0_{\beta\alpha}\right)^{-1}, \tag{52}$$

As follows from equation (17), the frequencies of the time coefficients corresponding to the a^0's will in their turn satisfy the equality

$$\omega_{\alpha\beta} = \omega_{\alpha\mu_1} + \omega_{\mu_1\mu_2} + \cdots + \omega_{\mu_{n-1}\beta} \tag{53}$$

and, in particular,

$$\omega_{\alpha\alpha} = \omega_{\alpha\beta} + \omega_{\beta\alpha}. \tag{54}$$

Hence with $(\alpha\alpha)$ denoting the field mode of zero frequency (a mode which is actually absent from the pure radiation field), $\omega_{\alpha\alpha} = 0$, the above equation implies the important symmetry property:

$$\omega_{\alpha\beta} = -\omega_{\beta\alpha}. \tag{55}$$

which was used in advance in connection with equation (15).

The general solution to equation (50), taking into account (51), is

$$\varphi_{\alpha\beta} = \varphi\alpha - \varphi\beta \mod 2\pi, \tag{56}$$

with $\varphi\alpha$ and $\varphi\beta$ independent random phases uniformly distributed over the interval $(0,2\pi)$, so that $\varphi_{\alpha\beta} = \varphi\alpha - \varphi\beta \mod 2\pi$, is also uniformly distributed over the same interval. Thus the amplitudes have the general form

$$a^0_{\alpha\beta} = e^{i(\varphi\alpha - \varphi\beta)}, \tag{57}$$

which shows that the statistically independent random quantities are not the amplitudes $a^0_{\alpha\beta}$ with the combined index $(\alpha\beta)$, but the single-index phases $\varphi\alpha$. The characteristic frequencies share this important property of separability, i.e.,

$$\omega_{\alpha\beta} = \Omega\alpha - \Omega\beta, \tag{58}$$

where $\Omega\alpha$ are (nonrandom) numbers. One thus gets for the Fourier coefficient of a typical term $(\tilde{x}^n)$ corresponding to frequency $\omega_{\alpha\beta}$:

$$(\tilde{x}^n)_{\alpha\beta} a^0_{\alpha\beta} = \sum_{\mu_i} \tilde{z}_{\alpha\mu_1} \tilde{z}_{\mu_1\mu_2} \cdots \tilde{z}_{\mu_{n-1}\beta} = \left(\sum_{\mu_i} \tilde{x}_{\alpha\mu_1} \tilde{x}_{\mu_1\mu_2} \cdots \tilde{x}_{\mu_{n-1}\beta} \right) a^0_{\alpha\beta} \tag{59}$$

where the sum is performed over all allowed values of the (repeated) intermediate indices. One recognized here the multiplication rule for matrices:

$$(\tilde{x}^n)_{\alpha\beta} = \sum_{\mu_i} \tilde{x}_{\alpha\mu_1} \tilde{x}_{\mu_1\mu_2} \cdots \tilde{x}_{\mu_{n-1}\beta}, \tag{60}$$

so that the solution arrived at is naturally expressed in terms of matrices. For example, $\tilde{x}_{\lambda\mu}$ is the $\lambda\mu$-element of a matrix $\hat{x}$, and so on.

IV. THE HEISENBERG EQUATIONS OF MOTION

The result just obtained can be used to write the equations of motion in matrix form. For this purpose, recall that one may associate the time factor $\exp(i\omega_{\alpha\beta}t)$ of $\tilde{z}_{\alpha\beta}$ either with $\tilde{x}_{\alpha\beta}$, as in (28), or with $a_{\alpha\beta}$ (see (6)):

$$\tilde{x}_{\alpha\beta} a^0_{\alpha\beta} e^{i\omega_{\alpha\beta}t} = \tilde{x}_{\alpha\beta}(t) a^0_{\alpha\beta} = \tilde{x}_{\alpha\beta} a_{\alpha\beta}(t),$$

where

$$\tilde{x}_{\alpha\beta}(t) = \tilde{x}_{\alpha\beta} e^{i\omega_{\alpha\beta}t}, \; a_{\alpha\beta}(t) = a^0_{\alpha\beta} e^{i\omega_{\alpha\beta}t} \tag{61}$$

and use whatever expression is convenient in each case. In particular, from equations (33) and (60) one has in the radiationless approximation

$$m\frac{d^2\hat{x}(t)}{dt^2} = \hat{F}(\hat{x}), \tag{62}$$

where $\hat{x}(t)$ and $\hat{F}(\hat{x})$ are now time-dependent matrices with elements $\tilde{x}_{\alpha\beta}(t)$ and $\tilde{F}_{\alpha\beta}(\hat{x})$, respectively. To complete the description we define a matrix $\hat{p}(t)$ with elements that follow from the time derivative of equation (28),

$$\tilde{p}_{\alpha\beta}(t) = im\omega_{\alpha\beta}\tilde{x}_{\alpha\beta}(t), \tag{63}$$

or in matrix notation,

$$\hat{p} = m\frac{d\hat{x}}{dt}. \tag{64}$$

This definition is suitable for making contact with quantum mechanics, which is a zero-order theory as regards the radiative terms and hence unable to distinguish between the mechanical moment and the canonical moment with respect to the zero-point field (recall equations (32) and (33)). Combining with equation (62) it follows then that

$$\frac{d\hat{p}}{dt} = \hat{F}(\hat{x}). \tag{65}$$

Equations (64) and (65) are evidently the Heisenberg equations of motion, and $\hat{x}_{\alpha\beta}(t)$ are the elementary oscillators of matrix mechanics. The matrix algebra of quantum mechanics follows therefore as *the algebra that guarantees stable, nonrandom values for the characteristic frequencies of the stationary* SED *system in the radiationless and long-wavelength approximation.*

V. THE SCALE OF QUANTUM PHENOMENA

Observe that the above equations of motion (64), (65) do not yet fully determine $\hat{x}_{\alpha\beta}$ and $\omega_{\alpha\beta}$. In writing the stationary solutions (39) in the form of (35), only the positions of the poles were taken into account, without ever really solving the equation (34) which contains the full information, and, in particular, fixes x(t) in terms of Planck's constant. This means that we have to come back to the equations that describe the *complete* SED system before the radiationless approximation is made and the zero-point field is dropped altogether, in order to fix the scale of the solutions.

Instead of dealing with the complicated original equation of motion, however, a simpler procedure can be used, observing that what is lacking in the above formalism to complete the full system of Heisenberg equations is the value of some fundamental commutator, such as, e.g., $[\hat{x},\hat{p}]$. This is not the place to enter into details, which requiere a lenghty consideration, so we refer the reader to the cited references and content ourselves here with a sketch of the procedure and the final result. The basic idea is to consider the translation into the above language, of the fundamental Poisson brackets of the original theory. It is possible to demonstrate that in particular, the SED Poisson bracket $[x_i,p_j] = \delta_{ij}$ transforms in the new language into

$$-\sum\beta\omega_{\alpha\beta}\left|\tilde{x}_{\alpha\beta}\right|^2 = \frac{\hbar}{2m}. \tag{66}$$

This result can be identified as the Thomas-Reiche-Kuhn sum roule of quantum mechanics, which is just the quantization rule $[\hat{x},\hat{p}]=i\hbar$ expressed in terms of matrix elements. It is through this (and similar) results that $\hbar$ enters into the scheme.

Now we take advantage of the separability of $\omega_{\alpha\beta}$ expressed in equation (58), to write

$$\dot{\tilde{x}}_{\alpha\beta} = i\omega_{\alpha\beta}\tilde{x}_{\alpha\beta} = i(\Omega\alpha - \Omega\beta)\tilde{x}_{\alpha\beta} = -i\sum\mu(\tilde{x}_{\alpha\mu}\Omega\beta\delta_{\mu\beta} - \Omega\alpha\delta_{\alpha\mu}\tilde{x}_{\mu\beta}) \tag{67}$$

which can be recast as follows, with H the Hamiltonian of the mechanical system,

$$i\hbar\,\dot{\tilde{x}}_{\alpha\beta} = \sum\mu(\tilde{x}_{\alpha\mu}\tilde{H}_{\mu\beta} - \tilde{H}_{\alpha\mu}\tilde{x}_{\alpha\beta}), \tag{68}$$

whence a comparison gives

$$\tilde{H}_{\alpha\beta} = \hbar\Omega\alpha\delta_{\alpha\beta}. \tag{69}$$

This result shows that the matrix representing H in the present formalism is diagonal, which means that $H\alpha$ is not random,

$$H\alpha = \sum\mu H_{\alpha\mu}a_{\alpha\mu} = \hbar\Omega\alpha a_{\alpha\alpha} = \hbar\Omega\alpha, \tag{70}$$

and further, that the $\Omega\alpha$ introduced via equation (58) is proportional to $H\alpha$, identified in quantum mechanics as the energy of the particle in state α,

$$\varepsilon\alpha \equiv \langle H\alpha\rangle = H\alpha = \hbar\Omega\alpha. \tag{71}$$

>From this it follows that equation (58) is Bohr's formula for the transition frequencies,

$$\hbar\omega_{\alpha\beta} = \varepsilon\alpha - \varepsilon\beta. \tag{72}$$

Hence the characteristic or relevant frequencies of SED coincide with the transition frequencies of quantum mechanics. Analogously, from the above relations it follows that the response amplitudes $\tilde{x}_{\alpha\beta}$ are the transition amplitudes of quantum mechanics.

VI. HILBERT-SPACE FORMALISM

A correspondence has been established between the description of linear SED and quantum mechanics, via the Heisenberg equations. Now it is a relatively easy matter to further develop the new description, until a direct contact with the usual Hilbert-space formalism is reached. As a practical means to achieve this we introduce the *a-representation*, as follows.

Consider a set of square matrices $\hat{a}^{\alpha\beta}$, each of which has only the element $\alpha\beta$ different from zero,

$$\left(\hat{a}^{\alpha\beta}\right)_{\mu\nu} = a_{\alpha\beta}\delta_{\alpha\mu}\delta_{\beta\nu}. \tag{73}$$

The coefficients $a_{\alpha\beta} \equiv a_{\alpha\beta}(t)$ are given by (57) and (61); thus,

$$a_{\alpha\beta} = e^{i\varphi_{\alpha\beta}+i\omega_{\alpha\beta}t} = e^{i(\varphi\alpha-\varphi\beta)+i(\Omega\alpha-\Omega\beta)t} = e^{i(\varphi\alpha-\Omega\alpha)t}e^{-i(\varphi\beta+\Omega\beta)t}. \tag{74}$$

Note that the off-diagonal elements have random phases, whereas for $\alpha = \beta$ the phase is zero. A product of two of such matrices gives, as follows from (73) and (74),

$$(\hat{a}^{\alpha\beta}\hat{a}^{\gamma\delta})_{\mu\nu} = a_{\alpha\delta}\delta_{\alpha\mu}\delta_{\beta\gamma}\delta_{\delta\nu} = \delta_{\beta\gamma}(\hat{a}^{\alpha\delta})_{\mu\nu}. \tag{75}$$

The fact that this product differs from zero only for $\beta = \gamma$ makes these matrices especially suited for the present purposes, for they can be used as a basis to write the matrix representing an arbitrary dynamical variable. For instance for the variable x we write

$$\hat{x} = \sum_{\alpha,\lambda} \tilde{x}_{\alpha\lambda}\hat{a}^{\alpha\lambda} = \sum \alpha\hat{x}\alpha, \tag{76}$$

where

$$\hat{x}\alpha = \sum \lambda \hat{x}_{\alpha\lambda}\hat{a}^{\alpha\lambda}; \tag{77}$$

the matrix elements of $\hat{x}$ are then just $\hat{x}_{\alpha\lambda} = \hat{x}_{\alpha\lambda}a_{\alpha\lambda}$. Then from (75) and (76) one gets for instance for the square of $\hat{x}$

$$\left(\hat{x}^2\right)_{\mu\nu} = \sum \lambda \tilde{x}_{\mu\lambda}\tilde{x}_{\lambda\nu}a_{\mu\lambda}a_{\lambda\nu} = \left(\tilde{x}^2\right)_{\mu\nu}a_{\mu\nu}. \tag{78}$$

The matrix x^2 is thus again a linear combination of the $\hat{a}$'s, with coefficients $\left(\hat{x}^2\right)_{\mu\nu} = \sum \lambda \tilde{x}_{\mu\lambda}\tilde{x}_{\lambda\nu}$. Consequently the operator $\hat{x}$ reproduces the matrix properties which the variable x must possess according to the discussion above. The same applies of course to any other observable, which means that it applies to any variable in the quantum regime that can be expressed in the linear form (35).

Now observe from equation (74) that the matrix $\hat{a}^{\alpha\beta}$ can be written as the product of two vectors, namely, a column vector $|\alpha\rangle$ of the form

$$|\alpha\rangle = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ a\alpha \\ \vdots \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \end{pmatrix} a\alpha \tag{79}$$

and a row vector $\langle\beta|$ which is the adjoint of $|\beta\rangle$, $\langle\beta| = \left(|\beta\rangle\right)^{+}$, and is given by

$$\langle\beta| = (0 \quad 0 \quad \cdots \quad a\beta^* \quad \cdots) = (0 \quad 0 \quad \cdots \quad 1 \quad \cdots)a\beta^*, \tag{80}$$

with

$$a\alpha = e^{i(\varphi\alpha + \Omega\alpha t)}. \tag{81}$$

These vectors have as a many components as there are different indices α, which normally means a (denumerable) infinity of them. The only element of $|\alpha\rangle$ which is different from zero is in row α, i.e., $(|\alpha\rangle)\lambda = a\alpha\delta_{\alpha\lambda}$, and therefore

$$a\alpha a\beta^* = a_{\alpha\beta}, \tag{82}$$

is agreement with equation (74). Finally, from equations (73) and (79) follows the factorization rule

$$\hat{\alpha}^{\alpha\beta} = |\alpha\rangle\langle\beta|. \tag{83}$$

The vectors $|\alpha\rangle$ form a complete orthonormal basis, as follows from the equations

$$\langle\alpha|\beta\rangle = \delta_{\alpha\beta},\ \Sigma\,\alpha|\alpha\rangle\langle\alpha| = \Sigma\,\alpha\hat{a}^{\alpha\alpha} = \hat{1}. \tag{84}$$

They thus span the Hilbert space of the states of the system, and an observable f can be represented by any one of the following expressions:

$$\hat{f} = \sum \alpha\hat{f}\alpha = \sum_{\alpha,\beta} \tilde{f}_{\alpha\beta}\hat{a}^{\alpha\beta} = \sum_{\alpha,\beta} \tilde{f}_{\alpha\beta}|\alpha\rangle\langle\beta|, \tag{85}$$

with

$$\hat{f}_{\alpha\beta} = \langle\alpha|\hat{f}|\beta\rangle. \tag{86}$$

In particular for the Hamiltonian we have, as follows from (70),

$$\hat{H} = \sum \alpha\hat{H}_{\alpha\alpha}\hat{a}^{\alpha\alpha} = \sum \alpha\varepsilon\alpha|\alpha\rangle\langle\alpha|. \tag{87}$$

It is interesting to note that the vectors $|\alpha\rangle$ do not involve the $\omega_{\alpha\beta}$, but the quantities $\varepsilon\alpha/\hbar = \Omega\alpha$; the transition to a Hilbert-space formulation in terms of bras and kets has had the effect of shifting the accent from the relevant frequencies to the energy eigenvalues for the stationary states, and from the field amplitudes $a_{\alpha\beta}$ to the vector elements $a\alpha$. The proposed mathematical transformation has thus changed the conceptual framework into an entirely different one, in which the main objects are vectors in a Hilbert space and eigenvalue equations. Note further that in expressions such as (79) and (80) the factors $a\lambda$ contain random phases that remain hidden in the usual Hilbert-space formulation.

In the a-representation the dynamical equation take the form (see equations (61-66))

$$m\frac{d\hat{x}}{dt} = \hat{p}, \qquad \frac{d\hat{p}}{dt} = \hat{F} \tag{88}$$

and

$$[\hat{x},\hat{p}] = i\hbar\hat{1}. \tag{89}$$

Once more the time dependence can be attributed to $\tilde{x}$ by the writing $\sum\tilde{x}_{\alpha\beta}(t)\hat{a}^{\alpha\beta}$, or else to $\tilde{a}$ by writing $\sum\tilde{x}_{\alpha\beta}\hat{a}^{\alpha\beta}(t)$ and using equation (61). The passage from the first expression to the latter is equivalent to a transition from the Heisenberg to the Schrödinger picture.

VII. CLOSING COMMENTS

At this point it is appropriate to comment briefly on some of the features and implications of the theory just sketched.

The characteristics of the self-consistent solution show that the coupling between the atomic processes and the field due to radiation, generates in the long run phase correlations between the components of the

neighboring field, and between this field and the atomic motions, as was anticipated by Theimer and Peterson.[3] The method followed, though obviously successful, has the important shortcoming that the self-consistent solution must be accepted as a matter of fact. It is quite clear that without the use of an auxiliary principle or hypothesis it would have been impossible to identify the solution, due to the high complexity of the mathematical problem. The central question of deriving the solution from first principles remains open.

As already stated, an exact and detailed solution of the full problem would *not* lead to the present description; this is only reached after performing a series of approximations and simplifications., In the transition to such an approximate theory, some attributes of the starting description, as that of being genuinely statistical and local realistic, become much weakened and adopt their quantum guise. This can be identified as the main reason for the difficult interpretative issues characteristic of the quantum description. It is here in particular where we find an explanation to questions such as why the quantum formalism gives an incomplete and seemingly noncausal account of the behavior of mechanical systems: quantum mechanics appears from the present point of view not as a fundamental theory of matter, but as a derived, approximate, asymptotic theory. And approximate physical theories may not satisfy the same rigorous requirements that fundamental theories are supposed to fulfil; this is particularly true in regard to consistency with first principles.

Even if still unfinished, the theory presented allows already for a certain reinterpretation of some quantum issues. Of special interest is the finding that quantum operators such as $\hat{x}$, $\hat{p}$ and so on, should be interpreted as referring not to a single particle, but to the subensemble constructed through the coarse-graining process, and then in a highly abstract form, distant from any direct empirical meaning. The present theory is as intrinsically nonrelativistic as it is intrinsically statistical, and quantum mechanics inherits these peculiarities. In particular, the extended belief that the quantum variables can be readily identified with those describing the individuals, and with a meaning directly suggested by their classical counterpart, is not supported by this theory. Also, the fact that the theory is not constructed around the notion of trajectory, does not mean that individual trajectories do not exist, nor that the possibility of constructing a space-time description is cancelled forever. Simply, neither the present formulation nor quantum mechanics are such a theory.

Although the basic equations for the linear SED system in the quantum regime are stochastic by nature, they have been recast in nonrandom terms, which happen to be just those of matrix mechanics. In such a cryptic description (in terms of the Heisenberg equations of motion) the elements responsible for the stochasticity –the field amplitudes $a_{\alpha\beta}$- have vanished completely, resulting in a seemingly fully deterministic picture. This simple observation explains by itself much of the enduring interpretative problems of the usual quantum mechanics.

A question that invites us to indulge in further speculation refers to the possibility that under certain circunstances the system responds with random frequencies (as would be the case if the demand of detailed balance were removed), in which case the situation would be more chaotic than the one represented by usual quantum states. It is clear that for this to happen the system must leave the quantum regime, but it is unclear whether such a process means merely a return to a classical (stochastic) behaviour, or whether some new behaviour arises.

REFERENCES

[1]PEÑA, L. de la, and A.M. CETTO (1996): "The Quantum Dice, An Introduction to Stochastic Electrodynamics", Kluwer Academic Publishers, Dordrecht.

[2] _____ (1995): Found. **Phys.** 24, 917 (1994), 25, 573.

[3]THEIMER, O. and P.R. PETERSON (1977): **Phys. Rev.** A. 16, 2055.

ABOUT BLACK HOLES WITHOUT TRAPPING INTERIOR

Alejandro Cabo and Eloy Ayón
Instituto de Física, Universidad de Guanajuato, México
Grupo de Física Teórica, ICIMAF, Cuba

ABSTRACT
Physical arguments related with the existence of black holes solutions having a nontrapping interior are discussed. Massive scalar fields interacting with gravity are considered. Interior asymptotic solutions showing a scalar field approaching a constant value at the horizon are given. It is argued that the coupled Einstein-Klein-Gordon equations can be satisfied in the sense of the generalized functions after removing a particular regularization designed for matching the interior solution with an external Schwartzschild spacetime. The scalar field appears as just avoiding the appearance of closed trapped surfaces while coming from the exterior region. It also follows that the usual space integral over T_0^0 in the internal region just gives the total proper mass associated to the external Schwartzschild solution, as it should be expected.

1. RESUME

The questions about the final states of collapsing massive stars is a central issue in Astrophysics. Black holes are expected to be the ultimate configurations of stars being merely twenty times more massive than the Sun [1]. The general point of view asserts that after the collapse a central singularity develops which is surrounded by an interior region being causally isolated from outside [2].

That is the structure shown by all the known black hole solutions, and which also is strongly suggested by the so called singularity theorems [3]. It is a fact that in all the standard solutions, it is possible to find trapped surfaces at arbitrarily near distances from the horizon from the inside. However, it is not evident that all the physical solutions should have such a behavior.

The aim of this work is to discuss physical considerations related with the existence of black hole solutions having a "normal" interior space-time without closed ttrapped surfaces [3]. The Lagrangian system seeming to allow configurations being of interest in this sense corresponds to the massive scalar field interacting with gravity. Here we will consider a solution being spherically symmetric. It shows a scalar field tending to a constant at some "assumed" horizon in such a way that this horizon can be approached from the interior.

The plan of the work goes as follows.

In first place, a regular asymptotic solution of the Einstein and Klein-Gordon equations is obtained in the neighborhood of the origin and numerically extended to a probable horizon at some radius ρ_0. The solution has a scalar field which increases up to a finite value at ρ_0. The asymptotic field configuration was also discussed in the related work [4] in the preparation of which we started to investigate the problems addressed here.

The analytic behavior corresponding to the considered numerical solution in the neighborhood of the expected horizon is also determined. Afterwards, the question of matching the Schwartzschild solution at the exterior is considered. For this purpose the singular field configuration consisting of the considered solution art the interior and the Schwartzschild one at the exterior, is regularized. The scalar field is assumed as vanishing in the outside zone in accordance with the no-hair theorems. Therefore, the scalar field shows a rapid variation near the horizon. The inverse of the radial component of the metric tensor near the horizon tend to vanish on both sides. This property allows to design a regularization which can smooth out the strong singularities of the scalar field derivative in the kinetic energy terms of the Einstein equations. Effectively, after

properly selecting the regularization for the three fields involved, it can be argued that they solve Einstein-Klein-Gordon equations in the sense of the generalized functions.

It also follows that the marching conditions fully determine all the parameters determining the starting asymptotic solution in terms of the total black hole mass.

The possibility for the exchanging of real probe particles across the horizon is also conjectured after remarking that: 1) Any arbitrarily small (but real) probe particle would make a nonvanishing back-reaction on the metric being able to locally disrupt the horizon surface when the particle is sufficiently near it. 2) Outgoing particle trajectories of the Schwartzschild space-time always exist for arbitrarily small but finite distances from the horizon from the outside. 3) If the back-reaction is able to allow a falling from the inside probe particle to pass out just a little, then it could go far away the horizon following an outgoing particle trajectory. This can be so because the particle accumulated sufficient kinetic energy during the falling out at the interior.

In Section 2 the interior solution is discussed. Section 3 is devoted to discuss the matching with the Schwartzschild space-time. Finally in a final section conclusions and possibilities for the continuation of the work are given.

2. CONCLUSIONS

Physical arguments about the possibility for the existence of black holes with non-trapping interior are given. The internal solution consists of an interacting with gravity scalar field which is bounded at the horizon. On the contrary, the derivative of the scalar field diverges ar the border. This interior solution, after extended to the external region with the Schwartzschild space-time, is regularized in the neighborhood of the horizon in a particular way. It allows to show that the external sources which are needed for the regularized fields to solve the equations, tend to vanish in the generalized functions sense after removing the regularization. Thus, the studied configuration suggests an alternative equilibrium limit for the collapse of matter described, not by dust particles, but by continuous scalar field configurations. As all the particles are ultimately described by fields, the possibility for the physical relevance of such solutions seems not out of place. It can be also speculated that a kind of behavior of the wave equations coupled with gravity can exists, in which the formation of trapped surface could be rejected dynamically. These questions will be considered elsewhere.

Finally, we would like to remark on other possibilities for the further continuation of the work. A first task which is imagined consists in to numerically solve time dependent solutions produced by a sudden disappearance of the external sources associated to the regularized fields. Such an study could give information about the stability of the singular solution. The stability would be reflected by a tendency of the time dependent fields to reproduce the singular configuration.

ACKNOWLEDGMENTS

We are grateful Jorge Castiñeiras, Víctor Villanueva and Iraida Cabrera who have participated in the earlier stages of the work and also acknowledge the helpful discussions with Augusto González, Daniel Sudarsky, Rapahel Sorkin, Erik Verlinde, José Socorro, Juan Rosales and the members of the Group of Theoretical Physics of the Instituto de Matemática, Cibernética y Física. The support of the TWAS through the Research Grant 93-120 RG/PHYS/LA is also deeply acknowledged.

REFERENCES

[1] ELLIS, G. (1994): Summer School in High Energy Physics and Cosmology, Lecture SMR 762-39, "Black Holes and General Relativity". ICTP, Trieste, Italy.

[2] WEINBERG, S. (1972): "Gravitation and Cosmology", (John Wiley & Sons, Inc.)

[3] HAWKING; S. (1993): "Hawking on the Big Bang and Black Holes", (World Scientific).

[4] AYON, E. (1993): "Asymptotic Behavior of Scalar Fields Coupled to their Own Gravitational Field". Graduate Diploma Dissertation, Faculty of Physics, Havana University.

[5] LANDAU, L. (1962): "The Classical Theory of Fields", (Pergamon Press).

[6] ELLIS, G. (1994): Summer School in High Energy Physics and Cosmology, Lecture SMR 762-41, "Black Holes and General Relativity". ICTP, Trieste, Italy.

[7] VLADIMIROV, V.S. (1979): "Generalized Functions in Mathematical Physics" (Mir Publishers, Moscow).

CHALLENGES FROM SOME KNOWN RESULTS IN QUANTUM MECHANICS

V. Gupta, Departamento de Física Aplicada, CINVESTAV – Unidad Mérida, Yucatán, México

ABSTRACT
Generalization of some known results for the Schrödinger equation still need rigorous proofs wich provide a challenge to the practicioners of quantum mechanics.

The discovery of quark-antiquark ($q\bar{q}$) atoms like charmonium ($c\bar{c}$) and bottonium ($b\bar{b}$) led to general investigations of the Schröndinger equation with a central potential V(r) representing the ($q\bar{q}$) potential. The motivation was to prove results based on general properties of V(r), like its shape, since its precise form was (and still is) not known. Some questions of interest were, for example, the ordering of bound state energy levels [1] and the dependence of the value of the S-state wave function ar the origin (r = 0) on the reduced mass m [2] and the principal quantum number n [3]. This knowledge was of practical use in predicting decay rates and understanding quarkonium spectra.

Some of the results I will mention have been arround for a long time. For some reason they are not as widely as they should be to students and teachers of quantum mechanics. Rigorous proofs of the generalizations of these results still provide a theorerical challenge.

For a central potential V(r), the reduced radial equation for bound states is (u´= du/dr etc.):

$$-\frac{\hbar^2}{2m}u''+\frac{\hbar^2 l(l+1)}{2mr^2}u+V(r)u=Eu \qquad (1)$$

where $u = u_{nl}$ and the radial wave function $R_{nl} = 1/r u_{nl}$ with the normalization:

$$\int_0^\infty R_{nl}^2(r)r^2dr=\int_0^\infty u_{nl}^2(r)dr=1 \qquad (2)$$

Multiplying equation (1) by $r^q u'_{nl}(r)$ (q integer) and integrating over r from 0 to ∞ leads to various interesting results through simple manipulations like integration by parts. These give recursion relations between expectation values of V(r) and its derivatives, V´(r) = dV/dr, V"(r), etc.

The most well known of these results is the Virial Theorem obtained for q = 1. However, the results obtained for q = 0 are very interesting and for some reason are not common knowledge. Applying $\int_0^\infty dr u'_{nl}(r)$ to both sides of equation (1) one obtains:

$$-\frac{\hbar^2}{2m}\left(u'_{nl}(r)\right)\Big|_0^\infty+\left(W_l(r)u_{nl}^2(r)\right)\Big|_0^\infty$$

$$\int_0^\infty W_l' u_{nl}^2 dr - E_{nl} u_{nl}^2 \Big|_0^\infty = 0 \tag{3}$$

where the effective potential:

$$W_l(r) = \frac{\hbar^2 l(l+1)}{2mr^2} + V(r) \tag{4}$$

Since u_{nl} represents a bound-state there is no contribution for $r = \infty$. One has to be careful about the contribution for $r = 0$.

For the potentials wich satisfy $\lim_{r\to 0} r^2 V(r) = 0$ one has the general result $u_{nl} \sim r^{l+1}$ as $r \to 0$. This means that foll all $l = 0, 1, 2,\ldots$, the second and fourth terms of equation (3) give no contribution. The first term contributes only for $l = 0$ since $u'_{nl}(0) = R_{nl}(0)$. For S-wave states one has:

$$R_{n0}^2(0) = \frac{2m}{\hbar^2} < V'(r) > n0 \tag{5}$$

This is a very interesting result wich we use below. Historically, no one knows who derived it first but it is safe to attribute it to Fermi. For $l \neq 0$ [4], the first term vanishes giving:

$$< W_l' >_{nl} = 0$$

or

$$< V' >_{nl} = \frac{\hbar^2 l(l+1)}{2m} < 1/r^3 >_{nl} \tag{6}$$

In the first form it states average of the effective force in $l \neq 0$ states is zero. In the second form it connects $< V' >$ with $< 1/r^3 >$ for a large class of functions V(r). For power law potentials it gives simple relations. For example, it checks for the Coulomb case.

Two useful results applied to quarkonium phenomenology wich emerged for S-waves states, for potentials with definite curvature were [4]:

$$|R_{n+1,0}(0)| - |R_{n0}(0)| \gtrless 0 \quad V''(r) \gtrless 0 \ \forall r,$$

and

$$\frac{\partial}{\partial m}(1/m(R_{n0}^2)) \gtrless 0 \quad V''(r) \gtrless 0 \ \forall r. \tag{8}$$

These results for the lowest value $n = 1$ were proved [2,3] rigurously starting from the radial equation. The validity of eqs. (7) and (8) for arbitrary but large n, using the WKB approximation was proved [5] for a large class of potentials with finite V(0), $V'(r) > 0$ for all r and which have definite curvature i.e. same sign for V"(r) for all r.

Note that for a linear potential $V(r) \sim r$ ($V''(r) = 0$ for all r) results in eqs. (7) and (8) for all n follow inmediately from equation (5)! Furthermore, exactly solvable potentials $-1/r$ ($V''(r) < 0$ for all r) and r^2 ($V''(r) > 0$ for all r) satisfy eqs. (7) and (8).

Thus the conjecture is that equations (7) and (8) are valid for all n. The challenge is to give a rigorous proof.

AKNOWLEDGMENT

I am grateful to the Organizing Committee of the Workshop and CINVESTAV, Unidad Mérida for financial support.

REFERENCIAS

1. MARTIN, A. (1977): **Phys. Lett.** 67 B, 330, H. Grosse, **Phys. Lett**. 68 B, 343.

2. _____ (1977): **Phys. Lett.** 70 B, 194.

3. ROSNER, J. L; C. QUIGG and H.B. THACKERS (1978): **Phys. Lett**. 74B, 350.

4. GUPTA, V. and A. KHARE, (1977): Unpublished.

5. GUPTA, V. and R. RAJARAMAN (1979): **Phys. Rev.** D19, 697

NEW PERTURBATIVE APPROACH TO QUANTUM FIELD THEORY

V. Gupta, Departamento de Física Aplicada, CINVESTAV, Unidad Mérida, México

1. I will present a new approach to perturbation theory for renormalizable quantum fields theories (QFTs) which gives renormalization scheme (RS) independent predictions for observable and other quantities of interest (eg. Green's functions.) The resulting **RE**normalization **S**cheme **I**ndependent **PE**rturbation theory will be called RESIPE for short. I will illustrate how RESIPE works for a renormalizable QFT with one dimensionless coupling constant (see ref. 2). Applications of 2nd order RESIPE to some specific physical measurables, for massless QCD, are to be found in ref. 3. Generalization of the RESIPE formalism to QFT's with masses and more than one coupling constant and its connection with the renormalization group (RG) formalism is given in ref. 4. Here, in addition, a new scheme independent perturbation expansion, without reference to RG techniques, is given which is valid for the general case with masses, several kinematics variables and more than one coupling constant. These and references 5 and 6 may be consulted for more detail.

2. **RESIPE Formalism for a Renormalizable QFT with one Coupling constant**. Consider a QFT wich is renormalizable and has one dimensionless bare coupling constant g_0 (eg. QCD). For simplicity, consider a physical quantity wich depends on only one external energy scale Q. Corresponding to it one can always construct a dimensionless measurable quantity R, such that its regularized unrenormalized perturbation expansion is of the form

$$R = a_0 + r_{10}a_0^2 + r_{20}a_0^3 + \cdots \qquad (1)$$

Here the bare couplant $a_0 \equiv g_0^2 / 4\pi^2$ and the subscript '0' denotes bare or unrenormalized quantities. The bare perturbation series is not well defined since the coefficients of expansion are infinite. In a renormalizable theory finite results are extracted by absorbing the infinities in the base parameters (coupling constants, masses, etc.) and the fields present in the Lagrangian. The definitions of the renormalized fields and parameters in terms of the corresponding bare quantities are, however, not unique because of the posibility of finite renormalizations. After renormalization, since the measurable R has no anormalous dimensions, equation (1) becomes

$$R = a + r_1a^2 + r_2a^3 + \cdots, \qquad (2)$$

where the renormalized couplant $a \equiv g^2 / 4\pi^2$ and g = renormalized contant. The coefficients r_n are finite but their values depend on the RS used to define g. Consequently, finite order predictions for R in the renormalized procedure gives predictios for R which, although finite, are still ambiguous. Can this problem of RS-dependent perturbative predictions (present for all QFT's) be solved? Does the fact that the perturbative predictions base on equation (1) or equation (2) are not well defined mean that R itself is not directly computable in the theory, but instead the theory predicts some function f(R) of R uniquely? How and in what form does the theory determine f (R)? RESIPE provides the answers. We will see that for a renormalizable QFT with a single dimensionless coupling constant g_o the theory, at best, determines the Q dependence of R through the differential equation.

$$Q\frac{dR}{dQ} \equiv R'(Q) = f(R(Q)) = -f_0R^2(1 + f_1R + f_2R^2 + \cdots). \tag{3}$$

The last term expresses f(R) as a series in R with finite RS-invariant coefficients f_0, f_1, ... Each term in this series is RS-invariant and therefore so is any finite order truncation. The convergence of perturbative approximations to f(R) is nowcontrolled by magnitude of R itself. For practical applications, one may approximate the r.h.s. by the first 2 or 3 terms if $|f_n R^n| << 1$ for $n \geq 2$ or 3. These would give the second or third order RESIPE prediction. Since these finite order predictions are RS-independent, their confrontation with eperiment provides an unambiguous probe for higher order corrections.

2a Determination of RS-invariants f_n's. Since the coefficients $r_{n\,0}$ depend on Q through the regularization scale (eg., an ultraviolet cut off), equation (1) gives

$$R' = r'_{10}a_0^2 + r'_{20}a_0^3 + \ldots \qquad r'_{n0} \equiv Q\frac{\partial r_{n0}}{\partial Q}. \tag{4}$$

Eliminate a_0 between equations (1) and (4) to express R´ as a in series R and compare with equation (3). Or equivalently, substitute equation (1) in equation (3) and compare the resulting series in a_0 for R´ with equation (4). The resulting expressions for f_n´s in terms of r'_{n0} and r_{n0} are given in equation (6) below.

Since the theory is renormalizable, one can start with equation (2) to obtain

$$R' = r'_1a^2 + r'_2a^3 + \ldots, \qquad r'_n \equiv Q\frac{\partial r_n}{\partial Q}. \tag{5}$$

Manipulating equations (2), (3) and (5) as indicated above yields expressions for f_n s in terms of r_n and r'_n. Note the algebra is the same whether one starts with equation (1) or equation (2). Thus, we find:

$$-f_0 = r'_{10} = r'_1, \tag{6.1}$$

$$-f_0f_1 = r'_{20} - 2r'_{10}r_{10} = r'_2 - 2r'_1r_1, \tag{6.2}$$

$$-f_0f_2 = r'_{30} - 3r'_{20}r_{10} - 2r'_{10}r_{20} + 5r'_{10}r_{10}^2 = r'_3 - 3r'_2r_1 - 2r'_1r_2 + 5r'_1r_1^2, \tag{6.3}$$

etc. Since r_{n0} and r'_{n0} are RS-independent, while r_n and r'_n are finite (by definition) equation (6) proves that f'_n are both finite and RS-invariant. These properties for the f_n's are, in a sense, obvious from equation (3), since both R and R' posses these two properties being measurables. Note that f_0, f_1,... etc. can be directly calculated from the combinations of the bare series coefficients (in equation (6)) without having to renormalize them. The finiteness of f_n's is guaranteed by renormalizability of the theory. Note that f_0 and f_1 are universal in the sense that they are independent of the process under consideration. Of course, f_n, $n \geq 2$, do depend on the process, that is R, though this has not been explicity indicated in equation (3) for notational simplicity.

2b Testing RESIPE. Equation (3) requires the knowledge of R at some $Q = Q_0$ (which has to be obtained from experiment) to predict it at any other Q. This boundary condition on equation (3) provides the process dependent scale Λ_R for R to have a non trivial dependence on Q. Dependence of R on the RS-independent scale Λ_R (undetermined by the theory) is consistent with the fact that the starting Lagrangian contained the undetermined parameter g_0. The dependence of R on the dimensionless g_0 has now appeared, by "dimensional transmutation" (ref. 7), through Λ_R. In RESIPE different physical quantities R, $\bar{R}, \cdots$ will automatically have scales $\Lambda_R, \Lambda_{\bar{R}} \cdots$ which are specific to them. Does that mean the theory has many independent scales? The aswer is no (ref. 2). For the massless case, one can integrate equation (3) for process R and the corresponding equation

$$\tilde{R}' = -f_0\tilde{R}^2(1 + f_1\tilde{R} + \tilde{f}_2\tilde{R}^2 + \cdots), \tag{7}$$

for the process $\tilde{R}' = a + \tilde{r}a^2 + \cdots$, since the RS-invariant f_n's and $\tilde{f}_n$'s are constants independent of Q. One can show that the two scales Λ_R and $\Lambda_{\tilde{R}}$ are related:

$$\Lambda_{\tilde{R}} = \Lambda_R \exp\left[f_0^{-1}(\tilde{r}_{10} - r_{10})\right] \text{ and } \Lambda_R = \Lambda \exp\left[f_0^{-1}(r_1)_{\mu=Q}\right], \tag{8}$$

where Λ is the usual RS-dependent scale parameter and μ is the renormalization point. Note r_n's and $\tilde{r}_n$'s are functions of Q/μ only and $\tilde{r}_{10} - r_{10} = \tilde{r}_1 - r_1$. To test the theory using RESIPE one can extract Λ_R and $\Lambda_{\tilde{R}}$ to a given order and see how well Equation (8) is satisfied. Alternatively, one can compare the value of Λ obtained in the two cases.

CONCLUSION

The central idea of RESIPE is to use some observable quantity as the perturbation expansion parameter instead of the usual RS-dependent coupling constant, as is normally done in conventional renormalized perturbation theory (CRPT). This central idea can be implemented in different ways depending on the techniques used (ref. 8). RESIPE can be considered as a full-fledged RS-independent substitute for CRPT.

ACKNOWLEDGEMENTS

I am greatful to the Organizing Committee of the Workshop and CINVESTAV, Unidad Mérida for financial support.

REFERENCES

[1] DHAR, A. (1983): **Phys. Lett.**, 128 B, 407.

[2] _____ and V. GUPTA (1984): **Phys. Rev.**, D29, 2822.

[3] _____ and _____ (1983): **Pramana**, 21, 207.

[4] GUPTA, V.; D.V. SHIRKOV and O.V. TARASOV (1991): **International Journal of Modern Physics**, A6, 3381.

[5] The formulation of the RESIPE program based on the RG is the equivalent to the RG formalism developed by N.N. Bogoliubov and D.V. Shirkov, **Nuovo Cimento** 3, 845 (1956); Introduction to the theory of Quantized Fields, 3rd Ed. Wiley Interscience, N.Y. (1980) Chap. 9.

[6] For an explicit example of the emergence of RS-invariants when masses are present see S.G. Gorishny, A.L. Kataev, S.A. Larin and L.R. Surguladze, **Phys. Rev.** D 43, 1633 (1991).

[7] COLEMAN, S. and E. Weinberg (1973): **Phys. Rev.** D7, 1888; D.J. Gross and A. Neveu (1974), ibid 10, 3235.

[8] GRUNBERG, G. (1980): **Phys. Lett** 95 B, 70; S.J. Maximov and V.I. Vovk, **Phys. Lett.** 199B, 443 (1987).

THE PHASES OF TWO-DIMENSIONAL QED AND QCD

Roya Mohayaee
International Centre of Theoretical Physics, Trieste, Italy,
Institute of Physics, University of São Paulo, São Paulo, Brazil

ABSTRACT
The semi-classical phase structure of two-dimensional QED and QCD are briefly reviewed. The non-abelian theory is reformulated to closely resemble the Schwinger model. It is shown that, contrary to the abelian theory, the phase structure of two-dimensional QCD is unaffected by the structure of the theta vacuum. We make parallel calculations in the two theories and conclude that massless Schwinger model is in the screening and the massive theory is in the confining phase, whereas both massless and massive QCD are in the screening phase.

1. INTRODUCTION

Massless Schwinger model is an exactly solvable theory [1]. The phase structure of this theory has been studied extensively. It is well-established that the theory is in the Higgs or screening phase. On the other hand, massive Schwinger model is not an exactly solvable theory. Nevertheless, it has also been studied intensely and it is known that, under certain approximations, the theory is confining. There are various ways of establishing the phases of the Schwinger model. A rather simple method, which are shall demonstrate here, is to use the bosonised version of the theory and introduce external probe charges into the system. For a semi-classical theory, the intercharge potential binding the test particles can be easily computed. Classically, the Coulomb potential is expected to rise linearly with the inter-charge separation. However, in the bosonised theory, vacuum polarization effect can shield the probe charges. As a result, in the massless Schwinger model, the confining Coulomb interaction is replaced by a screening potential. The massive theory, on the other hand, survives the polarization effects and is in the confining phase.

The same method can be applied to two-dimensional QCD [1]. Although an exact bosonisation formulae is not available for the non-abelian theory and the available bosonisation methods are perturbative in the mass parameter [9], the techniques developed for Schwinger model can be used to infer informations about the phase of two-dimensional QCD. We introduce external probe colour charges into the theory and evaluate their inter-charge potential. Unlike two-dimensional QED, both the massless and massive non-abelian theories are in the Higgs phase.

2. SCHWINGER MODEL

We start with the lagrangian for two-dimensional QED [4],

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \psi(i\gamma^{\mu}\partial_{\mu} - e\gamma^{\mu}A_{\mu} - m)\psi. \tag{1}$$

This lagrangian can be re-written in terms of the bosonic variables, by using Mandelstam bosonisation formula [8]

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{2}$$

$$\psi_1 \sim : e^{i\beta\phi + \frac{i}{\beta}\tilde{\phi}} :, \tag{3}$$

$$\psi_2 \sim : e^{-i\beta\phi + \frac{i}{\beta}\tilde{\phi}} : . \tag{4}$$

The bosonised lagrangian is,[1]

$$L = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + e^{\mu\nu}A_\mu\partial_\nu\phi + m^2\gamma(\cos(2\sqrt{\pi}\phi) - 1), \tag{5}$$

where γ is a normalisation constant [7]. Next, we place external probe charges q and –q at L/2 and –L/2. For the purpose of evaluating the inter-charge potential, it suffices to restrict ourselves to static fields (i.e. $\partial_0 = 0$). The static lagrangian, incorporating the probe charges, is

$$L = \frac{1}{2}(\partial_1 A_0)^2 - \frac{1}{2}(\partial_1\phi)^2 + m^2\gamma[\cos(2\sqrt{\pi}\phi) - 1]$$
$$+ \frac{e}{\sqrt{\pi}}A_0\partial_1\phi + A_0 q[\delta(x - L/2) - \delta(x + L/2)] \tag{6}$$

The equations of the motion corresponding to the above lagrangian are

$$\frac{e}{\sqrt{\pi}}\partial_1\phi + q(\delta(x - L/2) - \delta(x + L/2)) - \partial_1^2 A_0 = 0 \tag{7}$$

$$-2\sqrt{\pi}m^2\gamma\sin(2\sqrt{\pi}\phi) - \frac{e}{\sqrt{\pi}}\partial_1 A_0 + \partial_1^2\phi = 0 \tag{8}$$

The equation of motion of A_0 can be integrated to give an expression for the scalar field in terms of the electric field E ($\partial_1 A_0$), i.e.,

$$\phi = \frac{\sqrt{\pi}}{e}[\partial_1 A_0 - q(T(x - L/2) - T(x + L/2)) - \alpha], \tag{9}$$

where T is the step function and α is the integration constant[2]. This can be inserted into the equation of motion for ϕ to yield,

$$\partial_1^2\tilde{E} - \frac{e^2}{\pi}\tilde{E} - \frac{eq}{\sqrt{\pi}}(T - \bar{T}) - 2\sqrt{\pi}m^2\gamma\sin\left(2\sqrt{\pi}\tilde{E} - \frac{2\pi\alpha}{e}\right) = 0 \tag{10}$$

where

$$\tilde{E} = \frac{\sqrt{\pi}}{e}[E - q(T - \bar{T})], \tag{11}$$

$\theta = 2\pi\alpha/e$ is the theta vacuum and $(T - \bar{T})$ denotes $(T(x - L/2) - T(x + L/2))$.

In order to obtain the potential binding q and –q, we solve the above equation to first find the inter-charge electric field. The equation can be solved for two different cases: when the dynamical fermions are massless and when they are massive.

---

[1] Unlike the original lagrangian (1), the bosonised lagrangian does not describe a purely classical system. In the bosonisation procedure, one takes into account the vacuum polarization effects by evaluating the contributions from the one-loop vacuum functionals.

[2] Note that in four dimensions, this integration constant is zero. However, in two dimensions energetics allow for a non-vanishing background electric field [6].

2.1. Massless Schwinger model

For massless dynamical fermions, the equation of motion for $\tilde{E}$ (10) reduces to

$$\partial^2\tilde{E} - \frac{e^2}{\pi}\tilde{E} - \frac{e}{\sqrt{\pi}}q(T-T) = 0. \tag{12}$$

This equation is exactly solvable and its solutions are

$$\tilde{E}_I = a\exp\left(\frac{-e}{\sqrt{\pi}}x\right), \qquad x > \frac{L}{2} \tag{13}$$

$$\tilde{E}_{II} = b\exp\left(\frac{e}{\sqrt{\pi}}x\right), \qquad x < \frac{L}{2} \tag{14}$$

$$\tilde{E}_{III} = c\exp\left(\frac{-e}{\sqrt{\pi}}x\right) + d\exp\left(\frac{e}{\sqrt{\pi}}x\right) + ,\frac{\sqrt{\pi}q}{e}, \qquad \frac{-L}{2} < x < \frac{L}{2} \tag{15}$$

The solutions and their derivatives can be matched at the boundaries to easily obtain the unknown coefficients. Having obtained $\tilde{E}$, we use the expression (11) to obtain the electric field. The inter-charge potential V(L) can then be simply obtained by integrating the electric field over the inter-charge separation[3], i.e.,

$$V(L) = -q\int_{-L/2}^{L/2} E_{III}dx = \frac{q^2\sqrt{\pi}}{2e}\left(1 - e^{-eL/\sqrt{\pi}}\right) \tag{16}$$

The expression for the inter-charge potential shows that for small inter-charge separations, the potential risess linearly with L, as is expected from a classical coloumb potential. However, as the inter-charge separation is increased the polarization effects set up[4] and lead to the screening of the probe charges. For very large inter-charge separations, the potential eventually reaches the constant value $q^2\sqrt{\pi}/2e$.

So far, the charge of the probe particles has not been specified. Therefore, even the screening of the fractional probe charges by integer dynamical charges is allowed. This is puzzling since one would expect the confinement to prevail in such cases. The situation can be clarified by evaluating the conserved charge Q associated with the integer dynamical charges. This charge is given in terms of the conserved current $\partial_1\phi$, i.e.,

$$Q = \int_{x_1}^{x_2} dx\partial_1\phi\,. \tag{17}$$

To unravel the mechanism of screening, we consider the screening of one of the probe charges and evaluate the conserved charge along the axis from 0 to ∞ (i.e., a solitonic configuration.) That is,

$$Q = \phi(\infty) - \phi(0) = -q\,. \tag{18}$$

---

[3] This can be shown to be equivalent to evaluating the change in the hamiltonian caused by the probe charges (see [5], Chapter 10).

[4] Recall that we have taken into account the polarization effects in the bosonisation scheme. In replacing the original fermionic lagrangian by the bosonic lagrangian, one incorporates the contributions from the one-loop Feynman diagrams in the bosonic action.

This shows that although the dynamical charges are of integer values, the charge associated with their solitonic configuration can be non-integer. Specifically, this charge is opposite to the charge of the probe particles and accounts for the shielding phenomena.

2.2. Massive Schwinger model

For massive dynamical fermions, the Schwinger model is not exactly solvable. The equation of motion (10) is non-linear and can only be solved after expanding the sine-terms[5] . In this approximation, the equation of motion (10) reduces to

$$\partial_1 \tilde{E} - \left(\frac{e^2}{\pi} 4\pi m^2\gamma\right)\tilde{E} - \left(\frac{e}{\sqrt{\pi}} q(T-T) + 2\sqrt{\pi} m^2\gamma\theta\right) = 0 \qquad (19)$$

This equation resembles (12) for the massless Schwinger model and can be solved in the same manner. By using the expression (11) relating $\tilde{E}$ to the electric field, we obtain the electric field and subsequently the inter-charge potential. This is now given by

$$V(L) = \frac{e^4}{2\pi(e^2/\pi + 4\pi m^2\gamma)^{3/2}}\left(1 - e^{-\sqrt{(e^2/\pi + 4\pi m^2\gamma)}L}\right)$$

$$+ \frac{e}{2}\left(1 - \frac{e^2}{e^2 + 4\pi^2 m^2\gamma)}\right)\left(q - \frac{e\theta}{\pi}\right)L \qquad (20)$$

Therefore, for the massive fermions, the inter-charge potential has both a screening and a confining term. However, for long separations the confinement term dominates. In addition, for integer probe charges and for $\theta = \pi$ a phase transition ocurrs; the confinement term disappears and the screening phase is restored. This can be explained by recalling that the theta vacuum, which was introduced in equation (9) as an integration constant, is basically a non-vanishing background electric field. As the theta angle is increased, pair production sets up helping the screening of the probe charges. This continues until the net electric field falls below the threshold required for pair production. This circle repeats itself and therefore the dynamics of the system is a periodic function of the theta angle.

2.3. Two-dimensional QCD

In the preceding sections, we have studied the screening and confining phases in two-dimensional QED. We saw that although massless Schwinger model is in the screening phase, the massive theory exhibit confinement. In this section, we ask the same question for two dimensional QCD and examine whether the conclusions drawn for two-dimensional QED can be generalized to its non-abelian counterpart.

The action of two-dimensional QCD is

$$S = \int d^2x\left[-\frac{1}{4} + \mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}_i^f\left(i\partial\delta^{ij} - e\mathbb{A}^{ij}\right)\psi_j^f - m\delta^{ij}\overline{\psi}_i^f\psi_j^f\right], \qquad (21)$$

where i, j are the usual colour indices and f = 1,...,k is a flavour quantum number. Unlike the Schwinger model, for which an exact bosonisation scheme (3) exist, two-dimensional QCD cannot be exactly bosonised. However a perturbative bosonisation scheme by means of which the massless theory is first bosonised and the mass term is introduced and bosonised perturbatively exists [9]. Writing down the generating function for

[5] The validity of this approximation can be checked by evaluating the argument of the sine, using the approximate solution for the electric field. One verifies that, for a large mass to charge ratio, the argument of the sine is much smaller than $\pi/4$. Thus, the expansion of the sine term is only meaningful for m >> e.

the above lagrangian and integrating out the fermions in the path integral measure leads to the well-known Wess-Zumino-Written action

$$S_{eff} = \sum_f \Gamma[g_f] - (c_v + k)\Gamma[\Sigma] + k\Gamma[\beta] + S_{YM}$$

$$+ m^2 \int d^2z \, tr\left[\sum_f \left(g_f \Sigma^{-1}\beta + g_f^{-1}\Sigma\beta^{-1}\right)\right], \tag{22}$$

where

$$S_{YM} = \int d^2z \left[\frac{1}{2}(\partial_+ A_0)^2 + \lambda A_0\left(\beta^{-1} i\partial_+ \beta\right)\right], \tag{23}$$

$$\Gamma[g] = \int \frac{d^2x}{8\pi} tr\left(\partial_\mu g^{-1}\partial^\mu g\right) + \int \frac{d^3y}{12\pi} \epsilon^{\alpha\beta\gamma}\, tr\left(g^{-1}\partial_\alpha g g^{-1}\partial_\beta g g^{-1}\partial_\gamma g\right), \tag{24}$$

the light-cone coordinate is x^+, $\lambda = \frac{e}{2\pi}(c_v + k)$ and c_v is given by $c_v = f^{ad} = f_{abc}f^{cbd}$ which vanishes for the abelian group[6]. The g field is the gauge-invariant bosonic field corresponding to the original fermionic excitations. The field Σ is a negative-metric field and the fields β and A_0 are the massive sector fields. The equations of motion corresponding to this action are

$$\frac{1}{4\pi}\partial_+\left(g_f \partial_- g_f^{-1}\right) = m^2\left(g_f\Sigma^{-1}\beta - \beta^{-1}\Sigma g_f^{-1}\right), \qquad f = 1\cdots k, \tag{25}$$

$$-\frac{(c_v + k)}{4\pi}\partial_+\left(\Sigma\partial_-\Sigma^{-1}\right) = m^2\sum_{f=1}^{k}\left(\Sigma g_f^{-1}\beta^{-1} - \beta g_f\Sigma^{-1}\right), \tag{26}$$

$$-\frac{k}{4\pi}\partial_-\left(\beta^{-1}\partial_+\beta\right) + i\lambda\left[\beta^{-1}\partial_+\beta, A_0\right] + i\lambda\partial_+ A_0 = m^2\sum_{f=1}^{k}\left(g_f\Sigma^{-1}\beta - \beta^{-1}\Sigma g_f\right), \tag{27}$$

$$\partial_+^2 A_0 = \lambda\left(\beta^{-1} i\partial_+\beta\right). \tag{28}$$

Unlike the Schwinger model, where the equations of motion of the bosonised theory, (7) and (8), were given in terms of simple scalar fields, all the above fields are matrices. To obtain a set of solvable equations, we parametrise the matrix-valued fields and rewrite them as elements of the gauge group. That is,

$$g = e^{i\sigma_2\varphi}, \quad \Sigma = e^{i\sigma_2\eta}, \quad \beta = e^{i\sigma_2\varsigma}, \tag{29}$$

where the fields φ, η and ζ are scalars σ_2 and is a generator of SU(2) group[7]. Rewriting the lagrangian (22) in terms of the above variables, introducing external colour probe charges q^a, with fixed colour charges a, and taking the static limit ($\partial_0 = 0$), we obtain the equations of motion

$$\partial^2\varphi = 8\pi m^2 \sin(\varphi + \eta + \zeta), \tag{30}$$

[6] We shall make frequent use of this limit and make parallels with the Schwinger model by taking the limit $c_v \to 0$.

[7] This simple parametrisation can be easily extended to SU(N).

$$\partial^2\eta = \frac{-8\pi m^2}{(c_v+1)}\sin(\varphi+\eta+\zeta), \tag{31}$$

$$\partial E = \partial^2 A_0 = \frac{(c_v+1)}{2\pi} q^a\big(\delta(x-L/2)-\delta(x+L/2)\big)-\lambda\partial\zeta, \tag{32}$$

$$E = \frac{-1}{4\pi\lambda}\partial^2\zeta + \frac{2m^2}{\lambda}\sin(\varphi+\eta+\zeta)- \tag{33}$$

The integration constant α arising from the integration of equation (32) can be interpreted as the background electric field, i.e., as the theta vacuum; $\theta = (2\pi\alpha/e)$ (see equations (9) and (10) of the Schwinger model for comparison.)

By making the substitution $\phi = \varphi + \eta$, the above four equations are reduced to the coupled equations,

$$\partial^2\tilde{E} = 4\pi\lambda^2\tilde{E} + 8\pi m^2\sin\left(\phi+\tilde{E}+\frac{\alpha}{\lambda}\right)$$

$$-2\lambda q^a(c_v+1)\big(T(x-L/2)-T(x+L/2)\big), \tag{34}$$

$$\partial^2\phi = \left(\frac{8\pi m^2 c_v}{c_v+1}\right)\sin\left(\phi+\tilde{E}+\frac{\alpha}{\lambda}\right), \tag{35}$$

where $\tilde{E} = \zeta - \alpha/\lambda$ and $E = -\lambda\tilde{E} + \frac{c_v+1}{2\pi} q^a (T-T)$ (similar to (11) of the Schwinger model).

3. MASSLESS QCD

For massless dynamical fermions, equation (34) simplifies to

$$\partial^2\tilde{E} - 4\pi\lambda^2\tilde{E} + 2q^a\lambda(c_v+1)\big(T(x-L/2)-T(x+L/2)\big) = 0. \tag{36}$$

This equation is similar to the expression (12) of the Schwinger model and can be solved by the same techniques. The inter-charge potential, obtained in the fashion of the Schwinger model, is

$$V(L) = \frac{(c_v+1)\sqrt{\pi}q^{a2}}{2e}\left(1-e^{\frac{-(c_v+1)eL}{\sqrt{\pi}}}\right). \tag{37}$$

Thus, massless QCD exhibits screening. It is worth mentioning that the screening potential of the Schwinger model (16) can be easily obtained by taking the limit $c_v \to 0$.

4. MASSIVE TWO-DIMENSIONAL QCD

The massive equations of motion, (34) and (35), are not exactly solvable. We first expand the sine term and solve the coupled equations for the field ϕ. The quartic equation for the field ϕ is

$$\partial^4\phi - \left(\frac{8\pi m^2 c_v}{c_v+1} + 4\pi\lambda^2 + 8\pi m^2\right)\partial^2\phi - \frac{32\pi^2\lambda^2 m^2 c_v}{c_v+1}\phi$$

$$- \left(\frac{32\pi^2\lambda c_v}{c_v+1}\alpha + 16\pi m^2 c_v \lambda q^a (T-T)\right) = 0 . \qquad (38)$$

The electric field can be obtained from the solutions of the above equation by using expression (35) and the relation between E and $\tilde{E}$. Subsequently, the inter-charge potential is

$$V(L) = \frac{(c_v+1)^2 q^{a2}}{2}$$

$$\times \left[\left(\frac{4\pi\lambda^2 - m_-^2}{m_+^2 - m_-^2}\right)\left(\frac{1-e^{-m_+ L}}{m_+}\right) + \left(\frac{m_+^2 - 4\pi\lambda^2}{m_+^2 - m_-^2}\right)\left(\frac{1-e^{-m_- L}}{m_-}\right)\right], \qquad (39)$$

where the mass scales $m_\pm$, arising from the solutions of the quartic equation (38), are given by

$$m_\pm^2 = 2\pi\left[\lambda^2 + \left(1+\frac{c_v}{(c_v+1)}\right)2m^2\right]$$

$$\pm 2\pi\left[\sqrt{\left(\lambda^2 + \left(1+\frac{c_v}{(c_v+1)}\right)2m^2\right)^2 - 8\frac{c_v}{(c_v+1)}\lambda^2 m^2}\right] \qquad (40)$$

The expression (39) for the inter-charge potential contains no confining term. Thus, massive QCD is in the screening phase. The confining potential of the massive Schwinger model is obtained by taking the limit $c_v \to 0$. In this limit, the mass scale m_- goes to zero and we recover expression (20) of the Schwinger model for $\theta = 0$. We also observe that, unlike the Schwinger model, the theta vacuum, i.e., the constant α in (35), does not appear in the expression for the inter-charge potential. Thus, the phase structure of the non-abelian theory is not affected by the values of the theta angle.

5. CONCLUSION

By a simple semi-classical treatment of two dimensional QED and QCD, we have determined the phases of these theories. The introduction of the mass parameter in the Schwinger model causes a transition from the screening to the confining phase. This transition does not ocurr in the non-abelian theory where the screening phase prevails. It is worth mentioning that similar analysis were recently done in higher dimensions [2]. It has been shown that both massless and massive (for very large fermion masses) three-dimensional QED and in the screening phase [2].

6. ACKNOWLEDGEMENTS

This article is a simpler presentation of the results obtained in vollaboration with E. Abdalla and A. Zadra [3]. I would like to thank E. Abdalla for a critical reading of the manuscript and numerous helpful discussions. This work was done under financial support from Fundação de Amparo a Pesquisa do Estado de São Paulo (FAPESP).

REFERENCES

[1] ABDALLA, E. (1997): "Two-dimensional quantum field theory: examples and applications", Talk given at the 1st. Carribean workshop on quantum mechanics, particles and fields, Havana, Cuba, 24-28, Mach, hep-th/9704192.

[2] ABDALLA, E. and R. BANERJEE: "Screening in three-dimensional QCD", hep-th/9704176.

[3] ABDALLA, E.; R. MOHAYAEE and A. ZADRA: "hep-th/9604063 To appear in **Int. J. Mod. Phys.**

[4] SCHWINGER, J. (1959): **Phys. Rev. Lett** 3, 1296.

[5] ABDALLA, E.; M.C. ABDALLA and K.D. ROTHE (1991): "Non-perturbative methods in two dimensional quantum field theory", World Scientific.

[6] COLEMAN, S. (1976): **Annals of Physics** 101, 239.

[7] GROSS, D.J.; I.R. KLEBANOV; A.V. MATYTSIN and A.V. SMILGA (1996): **Nucl. Phys. B461**, 109.

[8] MANDELSTAM, S. (1975): **Phys. Rev.** D10, 3026.

[9] ABDALLA, E.; M.C.B. ABDALLA and K.D. ROTHE: hep-th/9511191.

ABOUT THE MOTION AND THE EQUATIONS OF A POINT PARTICLE WITH SPIN (POINT STRUCTURAL PARTICLE)

E. Entralgo, B. Cabrera y J. Portieles, Instituto Superior de Ciencias y Tecnología Nucleares

I. INTRODUCTION

Starting with the work of Schrödinger [1], there have been many attempts (see for example the work [2]) to construct a classical theory of a point-spin particle. The main difficulty of these models [2], is that the notion of spin is introduced in mathematical formalisms together with other "hidden" variables which have no explicit physical meaning. Besides, in the models mentioned in [2], either the particles are not point objects, or the theory can be called classical only formally. In the papers [2, 3, 4] the theory of the point structure particle (p.s.p.) was developed, that is, a classical theory for a point particle with spin, and other variables that have a clear physical meaning.

In the present paper, after a brief review of the general theory of p.s.p. in stationary fields, we present the generalization for non stationary fileds, the spin-orbit interaction in the one body problem, the spin-spin and the spin-orbit interaction in the two bodies problem and the motion of a neutral p.s.p. in some concrete external stationary fields.

II. PHYSICAL QUANTITIES AND EQUATIONS OF MOTION FOR A p.s.p. IN STATIONARY FIELDS

In works [2, 3, 4] the concept of p.s.p. as a cluster of point subparticles with masses m_j charges e_j coordinates $\vec{q}_j$, linear momentum $\vec{p}_j = m_j\dot{\vec{q}}_j$, was introduced. This cluster in the presence of external gravitational, electric and magnetic fields, with potentials $\phi(\vec{r}), \varphi(\vec{r}), \vec{A}(\vec{r})$ and respective intentisities:

$\vec{G} = -\vec{\nabla}\phi(\vec{r}), \vec{E} = -\vec{\nabla}\varphi(\vec{r})$ and $\vec{H} = \vec{\nabla}\times\vec{A}(\vec{r})$, is described by the system of equations:

$$m_j\ddot{q}_{ja} = m_j\cdot G_a(\vec{q}_j) + e_j E_a(\vec{q}_j) + \frac{e_j}{c}e_{\alpha\beta\gamma}\dot{q}_{j\beta}\cdot H_\gamma(\vec{q}_j) - \frac{\partial}{\partial q_{ja}}W(\cdots,\vec{q}_j-\vec{q}_k,\cdots), \quad \alpha = x,y,z \tag{1}$$

$$\text{with the energy } E = \sum_j\left\{\frac{\vec{p}_j^{\,2}}{2m_j} + m_j\phi(\vec{q}_j) + e_j\varphi(\vec{q}_j)\right\} + W(\cdots,\vec{q}_j-\vec{q}_k,\cdots), \tag{2}$$

where W is the potential energy of interaction of the point subparticles, and it is related to the intensities of the external fields in such a way that the confinement condition is fulfilled, i.e.:

$$|\vec{q}_j(t)-\vec{q}_k(t)| \le l_o, \text{ if } |\vec{q}_j(t_o)-\vec{q}_k(t_o)| \le l_o, \tag{3}$$

for any j, k, t_o and $t > t_o$, where l_o is a small inaccessible, for the direct experimental observation interval of length.

Conditions (3) impose limitations to the intensities of the external fields and the structure of the confinement potential. As it will be seen, in certain conditions (for example for magnetic and gravitational perpendicular fields), the length of the p.s.p. growth up, and for a time $t = \tau_0$ condition (3) ceases to be fulfilled, and in this way it is possible to explain the disintegration of the p.s.p.

The condition (3), from the point of view of macroscopical observation, allows us to consider the p.s.p. as a point object with mass m, charge e, coodinate of the center of the mass $\vec{q}$ and linear momentum $\vec{p}$:

$$m = \sum m_j, \quad e = \sum e_j, \quad \vec{q} = \sum (m_j / m)\vec{q}, \text{ and } \vec{p} = \sum \vec{p}_j. \tag{4}$$

In papers [2, 3, 4] (for details see [4]) was shown that, with the help of the small parameters l_o and $Z = \max Z_j$, $Z_j = e_j/m_j - e/m$, it is possible to obtain from (1) and (2) a closed system of equations for 15 independent variables that describe a p.s.p. as a whole, where appear 6 constants that also characterize the p.s.p.: the mass and the charge from (4), the giromagnetic factor g, the proper frequency ω_o, and other two related with the former ones, $\kappa = \sum Z_j^2 m_j = Ic^2 = 4mc^2(g-g_o)^2$ and; $g_o = e/2mc$ where the length l_o, in the case of the electron, coincides with the so called "classical radius". The 15 independent variables are: the coordinate and the momentum defined in (4), the dipole moment $\vec{d}$, the dipole speed $\vec{f} = \dot{\vec{d}}$, and the spin $\vec{S}$, defined as

$$\vec{d} = \sum e_j \vec{\xi}_j, \quad \vec{f} = \sum e_j \dot{\vec{\xi}}, \quad \vec{S} = \vec{s} - (\vec{d} \times \vec{f})/\kappa \text{ where } \vec{\xi}_j = \vec{q}_j - \vec{q}, \tag{5}$$

and $\vec{s} = \sum m_j \vec{\xi}_j \times \dot{\vec{\xi}}_j$ is the proper angular momentum. The system of equations are given by:

$$\dot{q}_\alpha = \frac{p_\alpha}{m}, \dot{p}_\alpha = mG_\alpha(\vec{q}) + eE_\alpha(\vec{q}) + \frac{e}{mc}\varepsilon_{\alpha\beta\gamma} p_\beta H_\gamma(\vec{q}) + \frac{\partial E_\alpha(\vec{q})}{\partial q_\beta} +$$

$$+ \frac{1}{2\kappa}\left(\frac{\partial^2 G_\alpha(\vec{q})}{\partial q_\beta \partial q_\gamma} + \frac{e}{m}\frac{\partial^2 E_\alpha(\vec{q})}{\partial q_\beta \partial q_\gamma}\right) d_\beta d_\gamma + \frac{1}{c}\varepsilon_{\alpha\beta\gamma}\left\{\left(\frac{p_\beta}{m} + \frac{e}{m\kappa} f_\beta\right)\frac{\partial H_\gamma(\vec{q})}{\partial q_\rho} d_\rho + \right. \tag{6}$$

$$\left. + \frac{e}{2m^2\kappa} p_\beta \frac{\partial^2 H_\gamma(\vec{q})}{\partial q_\rho \partial q_\mu} d_\rho d_\mu + f_\beta H_\gamma(\vec{q})\right\} + g\frac{\partial H_\beta(\vec{q})}{\partial q_\alpha} S_\beta + \frac{2c}{\kappa} g(g-g_o) S_\beta \frac{\partial^2 H_\beta(\vec{q})}{\partial q_\alpha \partial q_\gamma} d_\gamma$$

$$\dot{d}_\alpha = f_\alpha, \dot{f}_\alpha = -\omega_o^2 d_\alpha + \kappa E_\alpha(\vec{q},t) + \left(\frac{\partial G_\alpha(\vec{q},t)}{\partial q_\beta} + \frac{e}{m}\frac{\partial E_\alpha(\vec{q},t)}{\partial q_\beta}\right) d_\beta + \tag{7}$$

$$+ \varepsilon_{\alpha\beta\gamma}\left\{\left(\frac{\kappa}{mc} p_\beta + \frac{e}{mc} f_\beta\right) H_\gamma(\vec{q},t) + \frac{e}{m^2 c} p_\beta \frac{\partial H_\gamma(\vec{q},t)}{\partial q_\rho} d_\rho\right\} + 2c(g-g_o) S_\beta \frac{\partial H_\beta(\vec{q},t)}{\partial q_\alpha}$$

$$\dot{S}_\alpha = \varepsilon_{\alpha\beta\gamma} S_\beta \left\{ g H_\gamma(\vec{q},t) + \frac{2c}{\kappa} g(g-g_o) \frac{\partial H_\gamma(\vec{q},t)}{\partial q_\rho} d_\rho \right\} \tag{8}$$

and the energy (with accuracy of a constant terms) is:

$$H = \frac{p_\beta p_\beta}{2m} + m\phi + e\varphi + \frac{\partial\varphi}{\partial q_\beta} d_\beta + \frac{1}{2\kappa}\left(\frac{\partial^2\phi}{\partial q_\beta \partial q_\gamma} + \frac{e}{m}\frac{\partial^2\varphi}{\partial q_\beta \partial q_\gamma}\right) d_\beta d_\gamma - gS_\beta H_\beta -$$

$$- \frac{2c}{k} g(g - g_o) S_\beta \frac{\partial H_\beta}{\partial q_\gamma} d_\gamma + \frac{f_\beta f_\beta}{2\kappa} + \frac{\omega_o^2}{2\kappa} d_\beta d_\beta \qquad (9)$$

It is easy to verify that the closed system of equations (6, 7, 8) with (9) lead exactly to the conservation law of the energy. It is also possible to verify that $\vec{S}$ = const. In the absence of magnetic fields, and $\vec{S}^2$ is an integral of motion in stationary fields. If we put $\vec{d} = \vec{f} = \vec{S} = 0$, the calssical equations for a point particle (p.p.) are obtained. This means that our theory obeys the "correspondence principle". If we put $\vec{d} = \vec{f} = 0$, with $S \neq 0$, the classical equations of the quantum Pauli theory are obtained, from which it can be seen that g is the giromagnetic factor and $\vec{S}$ is the spin. It is important to remark, that considering $\vec{d} = 0$, means that W = 0, and this is not in accordance with the confinement condition (3). In other words, this model requires that $d \neq 0$ if $\vec{S} \neq 0$.

III. GENERALIZATION OF THE THEORY OF THE p.s.p. FOR NON STATIONARY FIELDS. THE SPIN-ORBIT AND SPIN-SPIN INTERACTIONS

For non stationary fields in the starting equations of motion (1) the intensities of the external fields will be time dependent. Repeating the previous procedure we obtain the closed system of equations (6, 7, 8), but with time dependent intensities of the external fields. For the variation of the energy in time is obtained the following expression:

$$\frac{d}{dt}\left\{\frac{p_\alpha p_\alpha}{2m} + \frac{f_\alpha f_\alpha}{2\kappa} - gS_\alpha H_\alpha - \frac{2c}{\kappa} g(g - g_o) S_\alpha \frac{\partial H_\alpha}{\partial q_\beta} d_\beta + \frac{\omega_o^2}{2\kappa} d_\alpha d_\alpha\right\} =$$

$$= \frac{p_\alpha}{m}\left\{mG_\alpha + eE_\alpha + \frac{\partial E_\alpha}{2q_\beta} d_\beta + \frac{1}{2\kappa}\frac{\partial^2\left(G_\alpha + \frac{e}{m}E_\alpha\right)}{\partial q_\beta \partial q_\gamma} d_\beta d_\gamma\right\} + f_\alpha\left\{E_\alpha + \frac{\partial\left(G_\alpha + \frac{e}{m}E_\alpha\right)}{\partial q_\beta}\frac{d_\beta}{\kappa}\right\} \qquad (10)$$

Is easy to verify that: if we put $\vec{d} = \vec{f} = \vec{S} = 0$ in (10) and in the equations of motion we obtain the equations for point particles in external non stationary fields. When the external fields do not depend on time, the equations of motion become the equations for the p.s.p. in external stationary fields, and from (10) is obtained the correspondent expression for the energy, with the correspondent law of conservation of the energy. For the spin are fulfilled all the considerations of the previous section.

To introduce the spin orbit interaction in the one body problem, we will consider that in the presence of an electric field with potential $\varphi(\vec{r})$, there is an additional specific interaction between the subparticles of the p.s.p., then the Lagrangian is:

$$L = \sum_j \left(\frac{m_j \dot{\vec{q}}_j \cdot \dot{\vec{q}}_j}{2} - e_j\varphi(\vec{q}_j)\right) - W - W', \text{ where : } W' = \sum_j \frac{e_j}{2c^2}\left(\dot{q}_{j\beta} - \dot{q}_\beta - \frac{ef_\beta}{m\kappa}\right)\dot{q}_\beta\varphi(\vec{q}_j) \qquad (11)$$

In this case is convenient to select as independent canonical variables:

$$\vec{q},\ \vec{P}=\sum_j \vec{P}_j,\ \vec{d},\ F_\beta=\sum_j Z_j P_{j\beta},\ \vec{S}=\sum_j \vec{q}_j\times\vec{P}_j-\vec{q}\times\vec{P}-\frac{\vec{d}\times\vec{F}}{\kappa};\quad \text{where}\quad P_{j\alpha}=\frac{\partial L}{\partial \dot{q}_{j\alpha}} \tag{12}$$

Repeating the procedure we arrive to a closed system of equations for these variables, with the Hamiltonian:

$$H=\frac{P_\beta P_\beta}{2m}+\frac{F_\alpha F_\alpha}{2\kappa}+\frac{\omega_0^2}{2\kappa}d_\beta d_\beta+e\varphi+d_\beta\frac{\partial\varphi}{\partial q_\beta}+\frac{e}{m}\frac{d_\beta d_\gamma}{2\kappa}+\frac{\partial^2\varphi}{\partial q_\beta\partial q_\gamma}+\frac{1}{2c^2}\frac{P_\alpha}{m}\left[\left(1-\frac{e^2}{m\kappa}\right)F_\alpha\varphi+\right.$$

$$\left.+cg\varepsilon_{\gamma\beta\alpha}S_\gamma\frac{\partial\varphi}{\partial q_\beta}+2c^2\left(g(g-g_0)\varepsilon_{\nu\gamma\alpha}S_\nu-g_0^2\frac{d_\gamma F_\alpha}{\kappa}\right)\frac{d_\beta}{\kappa}\frac{\partial^2\varphi}{\partial q_\beta\partial q_\gamma}\right]. \tag{13}$$

The term $H_{SO}=\frac{gc}{2mc^2}\vec{S}\cdot\left[\vec{\nabla}\varphi\times\vec{P}\right]$ for a Coulomb potential $\varphi=\frac{Z|e|}{r}$, i.e., $H_{SO}=-\frac{g|e|Z}{2mcr^3}\vec{S}\cdot\vec{L}$ is the spin orbit interaction. This result is valid not only for electrons but also for other particles and nucleons, as g is a proper characteristic of the p.s.p.

We can also check the conservation laws of the energy and the square of the spin.

In the two bodies problem we have two clusters of classical point subparticles with masses m_{1j}, m_{2k}, charges e_{1j}, e_{2k}, coordinates $\vec{q}_{1j}, \vec{q}_{2k}$, and linear momenta $\vec{p}_{1j}=m_{1j}\dot{\vec{q}}_{1j}, \vec{p}_{2k}=m_{2k}\dot{\vec{q}}_{2k}$, that are at such a distance, that their confinement potentials $W_1\left(\cdots,\vec{q}_{1j}-\vec{q}_{1j'},\cdots\right)$ and $W_2\left(\cdots,\vec{q}_{2k}-\vec{q}_{2k'},\cdots\right)$ act only over the subparticles of each p.s.p., i.e., the coordinates of the subparticles of each p.s.p. obey the relations (for any j, j´, k, k´and t)

$$\left|\vec{q}_{1j}(t)-\vec{q}_{1j'}(t)\right|\le l_{o1},\ \left|\vec{q}_{2k}(t)-\vec{q}_{2k'}(t)\right|\le l_{o2},\ \text{and}\ \left|\vec{q}_{1j}(t)-\vec{q}_{2k}(t)\right|>l_{o1}+l_{o2}.,$$

where l_{o1} and l_{o2}, are small inaccesible, for the direct observation, intervals of lenth.

The electromagnetic interaction between the subparticles of each p.s.p., considering up to terms of order $(v/c)^2$, will be considered with the help of the Darwing Lagrangian [5].

As in the previous case, we select for each p.s.p. the independent variables given by (12), and repeating the previous procedure we arrive at a closed system of 30 equations of motion that is consistent with the expression of the energy. In that expression appears the terms of the Breit interaction [6] of quantum theory:

$$U_{SO}=\frac{e_1g_2}{m_1cr^3}\left(\vec{S}_2\vec{L}_1\right)+\frac{e_2g_1}{m_2cr^3}\left(\vec{S}_1\vec{L}_2\right)\quad\text{and}\quad U_{SS}=g_1g_2\left[\frac{\left(\vec{S}_1\vec{L}_2\right)}{r^3}-\frac{3\left(\vec{r}\vec{S}_1\right)\left(\vec{r}\vec{S}_2\right)}{r^5}\right]$$

where $\vec{L}_1=\left(\vec{r}\vec{P}_1\right)$ and $\vec{L}_2=\left(\vec{r}\vec{P}_2\right)$. The terms of U_{SO} are the classical representation of the spin-orbit interactions between both p.s.p. and those of U_{SS} are the classical representations of the spin-spin interactions between the p.s.p., we have proved that the classical model of the p.s.p. in also able to give a classical explanation of the spin orbit and spin-spin interactions between two particles.

This Hamiltonian conduces exactly to the conservation laws of: the energy, the total momentum, the total angular momentum and the square of the spin for each p.s.p.

IV. ABOUT THE MOTION OF A NEUTRAL P.S.P. IN SOME STATIONARY EXTERNAL FIELDS

The trajectory $\vec{q}(t)$ and the values of other variables, $\vec{d}(t), \vec{S}(t), \cdots$, for instants $t > 0$, are uniquely determined by the motion equations (6, 7, 8) after the known values $\vec{q}_0, \vec{p}_0, \vec{d}_0, \vec{f}_0, \vec{S}_0$ of the independent variables at $t = 0$. We will consider a neutral p.s.p. ($e = g_0 = 0$). It is convenient to introduce instead of the variables $\vec{d}$ and $\vec{f}$ other physical quantities with dimensions of length and linear momentum: $\vec{\xi} = \vec{d}/\sqrt{\kappa m}$, $\vec{\eta} = m\dot{\vec{\xi}} = \sqrt{m/\kappa}\,\vec{f}$, and it is possible to show that $\vec{\xi}$ for neutral P.S.P. is proportional to the vector that joints the center of positive charge with the negative one. This means that if $\vec{\xi}$ grow up with time the condition (3) may not fulfilled and the p.s.p. desintegrates.

In the absence of external fields (free motion), the solutions of the equations (6, 7, 8) are:

$$q_\alpha = q_\alpha^0 + \dot{q}_\alpha^0 t,\ \xi_\alpha = \xi_\alpha^0 \cos\omega_0 t + \dot{\xi}_\alpha^0/\omega_0 \operatorname{sen}\omega_0 t, \text{ and } S_\alpha = S_0 \tag{14}$$

from which it can be seen that the trajectory of a free neutral p.s.p. is the same as for a free p.s., the proper frequency ω_0, is the frequency of the oscillations (rotations) of the dipole moment, and the projections of the spin are constant. The mean value in time of the dipole moment is equal to zero, using the definition:

$$\langle d_\alpha \rangle = 1/T \int_0^T \sqrt{\kappa m}\,\xi_\alpha(t)dt = 0 \quad \text{where} \quad T = 2\pi/\omega_0 \tag{15}$$

In classical or quantum theory of p.p. we have no physical characteristic analogous to the proper frequency. Only in the work of Schrödinger [1] for the "Ziterbewegung" of a free quantum relativistic spin particle does such a quantity appears with the value $\omega_0 \sim m_0c^2/\hbar$. It is important to remark that the value of ω_0 according to [1] is very large, for example for a neutron it is of the order of $10^{24}\ s^{-1}$, this means that for a reasonable period of time $\Delta t < 10^{-12}$s, the dipole moment makes more than 10^{12} oscillations (or rotations), and in a direct measurement, one measure not $\vec{d}$ but $\langle\vec{d}\rangle$ according to (15).

This means that we need to estimate the possible values of $\vec{\xi}^0$ and $\dot{\vec{\xi}}^0$. For this purpose, if we accept according to [1] that $\omega_0 = 2mc^2/3\hbar$ and that the internal energy is the relativistic rest energy mc^2, with the help of the virial theorem for the mean value of the internal kinetic and potential energies, we arrive to:

$-c < \dot{\xi}_\alpha^0 < c, \quad -3/2\chi_c < \xi_\alpha^0 < 3/2\chi_c, \chi_c = \hbar/mc$, where χ_c is the Compton wave length.

In a homogeneus gravitational field, $\vec{G}$ = const, it is possible to verify that the trajectory of the p.s.p. is the same as for a p.p., and the solutions for the dipole moment and the spin are the same as for a free p.s.p.

In a homogeneus electric field $\vec{E}$ = const, the trajectory of the p.s.p. is again the same that for a p.p. The spin $\vec{S}$ = const. But the mean value of the dipole momentum is: $\langle\vec{d}\rangle = \kappa\vec{E}/\omega_0^2$ which means that $\kappa = Ic^2$ determines the polarizability of the neutral p.s.p. This give a value of 05×10^{-3} fm^3 for the polarizability of the neutron.

In a homogeneus magnetic $\vec{H}$ = const, also takes place a polarization of the p.s.p. and its trajectory may differ from the trajectory of the p.p. only for stremly low velocities.

The motion in perpendicular homogeneous gravitational and magnetic field $\vec{G} = (G,0,0), \vec{H} = (0,0,H)$; in the solutions for the dipole momentum appears a term vGt/ω_1^2 that increases monotonically with time, where:

$\omega_1^2 = \omega_0^2 + v^2$, $v = \sqrt{\kappa/m}\,H/c$, and for long intervals of time the p.s.p. may disintegrate. For the deflection angle, when $\omega_0^2 >> v^2$, and neglecting the oscillating term, we obtain

$\theta = \text{arctg}\left\{\dfrac{\dot{q}_y(t)}{\dot{q}_x(t)}\right\} = \text{arctg}\left\{v\xi_x^0 \Big/ \left(\dot{q}_x^0 + Gt - v\xi_y^0\right)\right\}$ from where it is seen that if $G < 0$, the velocities in the denominator decrease, and the deflection angle increase.

In the case of the motion in a Coulomb field $\varphi = Z|e|/r$ we are able to calculate [7] the cross section of incidence and obtain the value $\sigma_{inc} = 2\pi[\delta/E_0]^{1/2}$, where $E_0 = m\dot{q}_0^2/2$ (the initial energy), $\delta = Z^2e^2\kappa/2\omega_0^2$. This is the same dependence of E_0 that predicts the quantum Breit-Wigner formulas.

From the examples we have studied of the motion of a neutral p.s.p. in stationary gravitational, electric and magnetic homogeneous fields and non homogeneous electric field, it can be seen that the motion of the neutral p.s.p. may differ from the motion of p.p. only for very low velocities.

REFERENCES

[1] SCHRÖDINGER, E. (1930): Sitzingsber. Preus. Acad. Wiss. **Phys-Math**. K1., 24, 418.

[2] ENTRALGO, E. and V.V. KYRYSHKIN (1990): **Il nuovo Cimento**. 103ª.(4).

[3] V.V. KYRYSHKIN and E. ENTRALGO (1990): **DAN USSR**, 312(2, 13); **Communication JINR** P2-88-878, Dubna, (1988); Topological Phases in Quantum Theory, World Scientific, Singapore, London (1989).

[4] ENTRALGO, E. (1988): **Communication JINR** P2-88-879, Dubna.

[5] DARWING (1920): **Phil Mag** 391537.

[6] BREIT, G. (1929): **Physical Review** 34(4).

[7] RODRIGUEZ; R.; A. DIAZ; E. ENTRALGO y F. GUZMAN (1996): **Rev. Méx. de Física**. 42(3).

ABOUT Λ^0 POLARIZATION PROBLEM

Julián Félix, Instituto de Física, Universidad de Guanajuato, León, México

ABSTRACT

It is very well known that Λ^0 hyperons are produced with significant polarization in *pp* reactions, where both the beam proton and the target proton are initially unpolarized. Despite the theoretical efforts to understand this phenomenon, the puzzle remains. In order to shed some light on this phenomenon, we report and discuss, from a phenomenological point of view, the present status of Λ^0 polarization problem.

1. INTRODUCTION

An ample experimental evidences on polarization of Λ^0's produced in *pp* collisions at high energies, have been collected over more than 20 years[1-15]; where both the beam proton and the target proton are initially unpolarized; these evidences show that spin particle plays a very important role in particle production mechanism; and despite some theoretical studies[16-21], the ultimate explanation of polarization remains obscure. Normally, transverse polarization is defined as: $\wp = \langle \hat{\sigma} \cdot P_{beam} \times P_{\Lambda} \rangle$; where $\hat{\sigma}$ is the spin of Λ^0; P_{beam} is the momentum of the beam and P_{Λ} is the momentum of the Λ^0. These two vectors define the Λ^0 creation plane and also the quantization axis –the normal to the creation plane-. It was useful to speak about inclusive reactions (reactions where only Λ^0 is collected and studied) and exclusive reactions (where besides Λ^0 the rest of the reaction is collected and analized); Λ^0 polarization has been studied as a function of the scaling parameter x_F and P_T (these quantities are the fraction of incident proton momentum carried by Λ^0 in the initial direction of the bean proton, in the c.m. system, and the transverse momentum with respect to the incoming beam proton direction, in that order). Another variable, that can be used to characterize Λ^0 polarization is the Λ^0 energy in the c.m. system and the complementary diffracted mass (M_X) (this means the mass of all final state particles, excepting the diffracted proton, or the mass of a subset of final state particles; for instance, in the reaction $pp \rightarrow p\Lambda^0K^+\pi^+\pi^-$, the diffracted mass, means the mass of the system $\Lambda^0K^+\pi^+\pi^-$ or the mass of the system Λ^0K^+).

2. EXPERIMENTAL FACTS ABOUT Λ^0 POLARIZATION

According to all reported data, Λ^0 polarization is consistent with zero at $P_T = 0$ and linearly decreases roughly to ~ -0.25 at $P_T = 1.2$ GeV[1-14], see Figure 1. These are evidences that the slope of the straight line fit increases as x_F increases, and for P_T above 1.2 GeV and up to 3.5 GeV, the slope of the straight line fit is zero[22]. Λ^0 polarization depends on x_F the same way it depends on P_T, see Figure 2; and we can anticipate that, if we observe the polarization as a function of x_F, the slope of the straight line would increase as P_T increases. Because P_T and x_F are correlated throught the energy and the mass Λ^0, there must be a Λ^0 polarization dependence on Λ^0 energy very similar to that on P_T or x_F. These facts have not been observed yet. From those mentioned figures, it is interesting to note that polarization is independent of the beam energy; studies of polarization with beam energy between 6 and 2000 GeV have been done[1-10]. It is a common believe that Λ^0 polarization is independent of the target nature; even though that this supposition could be plausible, there are no data enough to support this statement , because all the information about polarization is in the beam fragmentation region ($x_F > 0$) when protons do not constitute the target. Some dependence of Λ^0 polarization on the complementary diffracted mass has been observed, that dependence is alike to the dependence observed on P_T or x_F[14,15]. But none dependence has been observed of Λ^0 polarization on relative momenta of Λ^0 and other reaction particles, for example the proton, the one that does not come from Λ^0, and K^+[14,15].

3. Λ^0 POLARIZATION MODELS

Since Λ^0 polarization was discovered in 1976[1,2], some theoretical models have been proposed in order to understand this phenomenon. In spite of such efforts the underlying polarization process remains unknown. Although these models describe somewhat qualitative the trends of Λ^0 polarization data, there is no a satisfactory explanation. The old models are based on partial information about Λ^0 polarization; for instance, most of them consider that Λ^0 polarization depends only on P_T, others include a weak dependence on x_F. A strong x_F – dependence was not suspected until recently it was discovered[14,15]. In the following lines, we expose and discuss briefly the main proposed ideas to describe Λ^0 polarization data and confront their consequences with observed facts.

From a theoretical point of view, we can see that in terms of constituent quarks, when a proton fragments into a Λ^0, which has P_T different from zero, an u valence quark is replaced by a strange quark s coming from the decay of a gluon[7] –or from the quark sea[16-21]; for simplicity, the ud system is supposed to be in zero spin state, consequently the Λ^0 spin and its polarization come from the spin and polarization of s. The difference between the proposed models is the source of s polarization. For the first case, in Heller **et al** model[7], it is the s which gives Λ^0 both its transverse momentum and spin. If the gluon is polarized, so is the $s\bar{s}$ pair and this polarization is correlated with the transverse momentum direction of Λ^0. In other hand, to produce a $\bar{\Lambda}^0$, $\bar{u}$ and $\bar{d}$ quarks must be created.This model considers that $\bar{s}$ is created unpolarized when s is polarized, to fit the partial experimental evidences –apparently $\bar{\Lambda}^0$ is no polarized, on the light of partial information[7,22-24]-. For the second case, there are some models: in the fragmentation model of Anderson **et al**[18], the confined linear color field is stretched and the strange quark needed to obtain the final Λ^0 with transverse momentum different from zero is produced by an $s\bar{s}$ pair whose orbital angular momentum is balanced by the sum of $\bar{s}$ spin and s spin; a negative polarization is obtained, which value decreases as P_T increases. In the recombination model of DeGrand **et al**[16], the polarization comes from the Thomas precession effect; to minimize the associated energy with this effect –the dot product of the s spin with the Thomas precession angular frequency- the spin direction must be opposite to the Thomas precession angular frequency; one expects a negative Λ^0 polatizarion in inclusive pp production; this model predicts a zero polarization for $\bar{\Lambda}^0$ at the same x_F and P_T where Λ^0 polarization is non-zero. In the model of Troshin and Tyurin[20] Λ^0 polarization is generated during the diffractive production of Λ^0 particles, the description of such process is given on the basis of the solution of a dynamical equation written in the direct channel of the reaction and on simple ideas about the quark structure of the particles; for instance, there is a non-zero probability of hadron states containing not only valence quarks but also $q\bar{q}$; in this way diffraction dissociation is regarded as the results of the decay of an excited hadron state containing an additional $q\bar{q}$; in particular, the model considers that the system uud is not polarized and $s\bar{s}$ is polarized (the spin projection of this pair into the normal of the production plane is 1); as a result of the scattering, the spin directions of the quarks s and $\bar{s}$ can be either preserved or simultaneously reversed; for small P_T, this model gives $\wp(P_T) \cong -\frac{1}{8}\left(\frac{P_T}{2m_s}\right)^2\left(1-\frac{1}{\kappa^2}\right)$ for Λ^0 polarization distributions; where κ is the ratio of the spin-flip amplitude to non-spin-flip amplitude (a constant greater that 1), m_s is the mass of the strange quark, P_T transverse momentum of Λ^0; for $\kappa = \sqrt{3/2}$, and $x_F > 0.6$ a comparison is presented in Figure 3.

4. CONCLUSIONS

All models so far created obtain the correct sign for Λ^0 polarization and a Λ^0 polarization distribution which does not fit the data well. All models are written to obtain $\bar{\Lambda}^0$ polarization consistent with zero, this fact seems to contradict recent results. All models consider Λ^0 polarization independent, or weakly dependent, of x_F. We have not a convincing explanation for Λ^0 polarization in *pp* reactions and a honest calculation of Λ^0 polarization distribution.

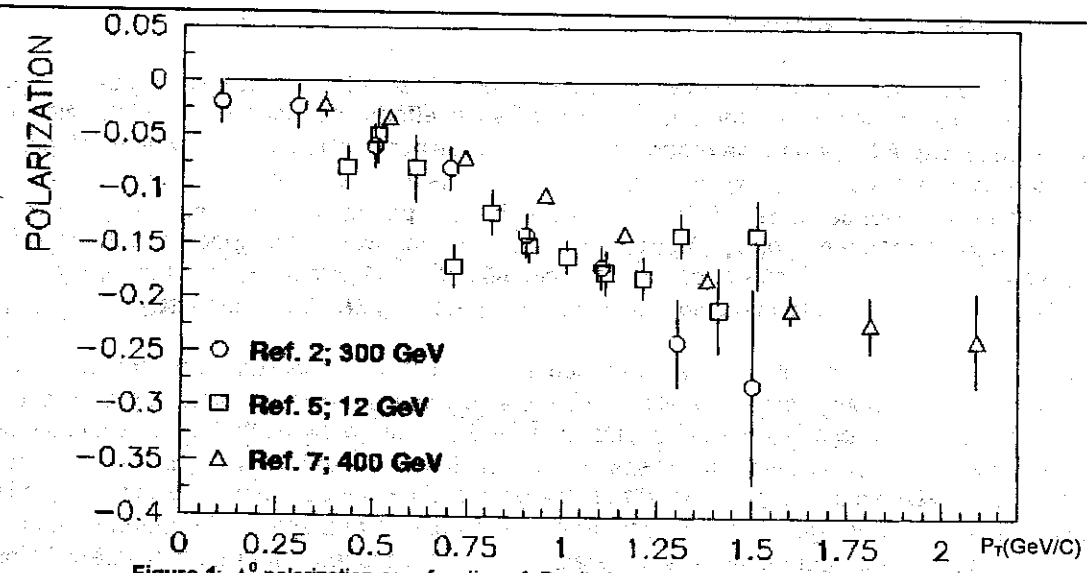

Figure 1: Λ^0 polarization as a function of P_T; its beam energy independence is shown.

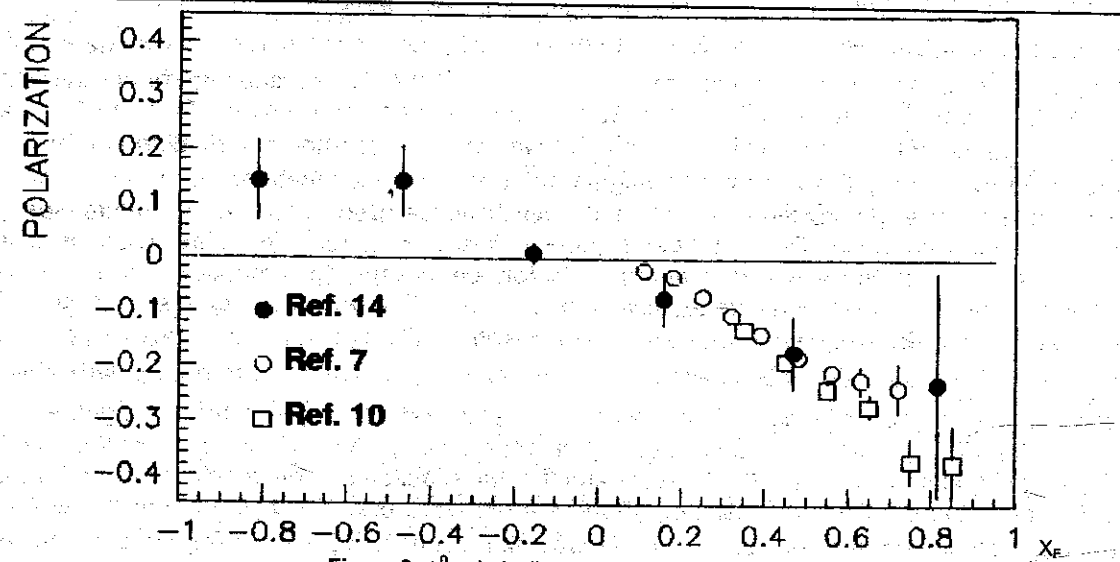

Figure 2: Λ^0 polarization as a function of X_F.

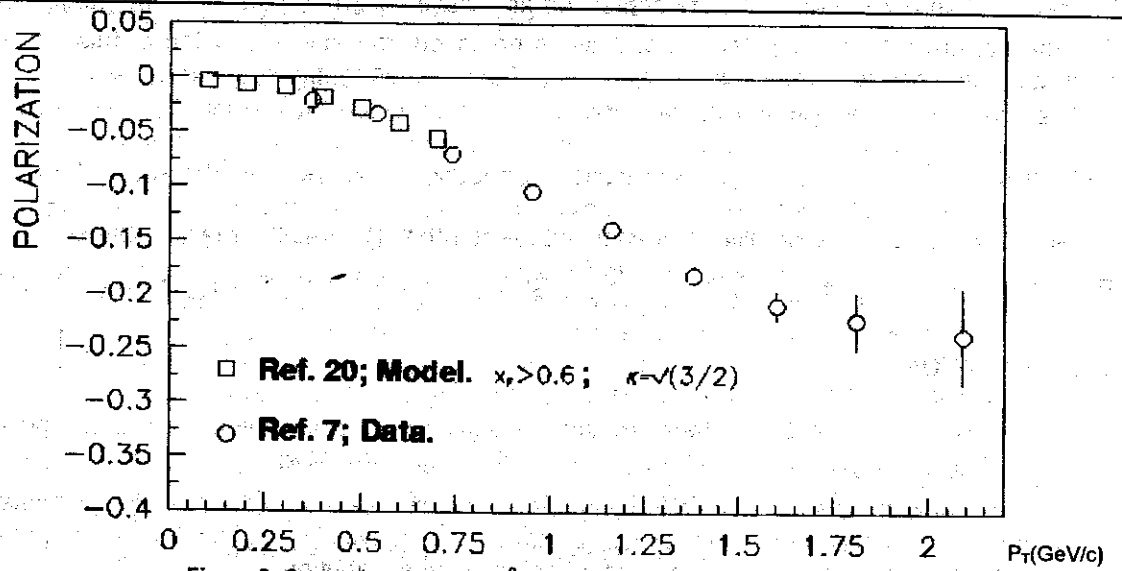

Figure 3: Comparison between Λ^0 polarization data and model is shown.

REFERENCES

1. LESNIK, A. et al (1975): Phys. Rev. Lett. 35, 770.
2. BUNCE, G. **et al** (1976): **Phys. Rev. Lett.** 36, 1113.
3. LOMANNO, F. **et al** (1979): **Phys. Rev. Lett.** 43, 1905.
4. ERHAN, S. **et al** (1979): **Phys. Lett.** 82B, 301.
5. ABE, F. **et al** (1983): **Phys. Rev. Lett.** 50, 1102.
6. RAYCHAUDHURI, K. **et al** (1980): **Phys. Lett.** 90B, 319.
7. HELLER, H. **et al** (1978): **Phys. Rev. Lett.** 41, 607.
8. ABE, F. **et al** (1983): **J. of the Phys. S. of Japan**. 52(12), 4107-4117.
9. AAHLIN; P. **et al** (1978): **Lettere al Nuovo Cimento** 21(7).
10. SMITH, A.M. **et al** (1987): **Phys. Lett** B 185, 209.
11. BLOBEL, V. **et al** (1977): **Nuclear Physics** B 122, 429.
12. HENKES, T. **et al** (1992): **Phys. Lett.** B 283, 155.
13. HELLER, K. **et al** (1977): **Phys. Rev. Lett.** 68B, 480.
14. FELIX, J. (1994): Ph. D. Thesis, U. de Guanajuato, México.
15. FELIX, J. **et al** (1996): **Phys. Rev. Lett.** 76, 22.
16. GRAND, T.A. de, and H.I. MIETTINEN (1981): **Phys. Rev.** D24, 2419.
17. MORIARTY, K.J.M. **et al** (1976): **Lett. al Nuovo Cimento** 17(366).
18. ANDERSSON, B. **et al** (1979): **Phys. Lett.** 85B, 417.
19. SZWED, J. **et al** (1981): **Phys. Lett.** 105B, 403.
20. TROSHIN, S.M. and N.E. TYURIN (1983): **Sov. J. Nucl. Phys**. 38(4). Oct.
21. SOFFER, J.and N.E. TÖRNQVIST (1992): **Phys. Rev. Lett.** 68, 907.
22. LUNDBERG, B. **et al** (1989): **Phys. Rev.** D40, 3557.
23. RAMBERG, E.J. **et al** (1994): **Phys. Lett.** B 338, 403.
24. LOMANNO, F. **et al** (1980): **Phys. Lett.** 96B, 223.

FIELD THEORY APPROACH TO K^0 - $\overline{K}^0$ AND B^0 - $\overline{B}^0$ SYSTEMS

M. Beuthe[1], G. López Castro[1,2] and J. Pestieau[1].
[1]Institut de Physique Théorique, Université Catholique de Louvain, Belgium
[2]Departamento de Física, CINVESTAV del IPN, México, D.F.

ABSTRACT
We present an approach based on field theory to describe the production and decay of unstable $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ mixed systems. Applications to describe the time evolution amplitudes of K^0 and $\overline{K}^0$ at DAPHNE and CPLEAR are presented.

1. INTRODUCTION

Neutral strange and beauty pseudoscalar mesons, $K^0\overline{K}^0$ and $B^0\overline{B}^0$, are systems of two unstable mixed states of special interest for the study of weak interactions. They are particularly suited to study the phenomena of CP violation together with the oscillations in their-time-dependent decay probabilities [1].

The traditional description of unstable neutral kaons is based on the Wigner-Weisskopf (WW) formalism [2]. In this approach, the time evolution of decaying states is governed by a Schrödinger-like equation based on a *non-hemitian* hamiltonian [3] that allows particle decays. As a result, the diagonalizing transformations, in general, are not unitary and the corresponding eigenstates are not orthogonal.

Beyond these unsatisfactory features of the WW formalism, one faces other difficulties. Projected factories of K and B mesons [4, 5] are expected to measure the CP violation and oscillation parameters to a higher accuracy than present experiments. While it is not clear whether the approximations involved in the WW formalism are valid for both the K and B systems, a consistent scheme is certainly required to compute these observables to an arbitrary degree of accuracy.

In this paper we adopt the view that the quantum mechanical behavior of a complete process involving the production and decay of unstable states can only be consistently described in the framework of quantum field theory. In QFT, the S-matrix amplitude becomes the basic object that describes the properties of a physical process among particles. This amplitude is taken between *in*- and *out*- asymptotic states which are defined as non-interacting states (stable particles) existing far away the interaction region. Therefore, as a general rule, unstable particles cannot be considered as asymptotic states.

Under these conditions, unstable particles appear only as intermediate states to which we associate Green functions (propagators) to describe the propagation amplitudes from their production to their decay spacetime locations. The form of these propagators, which are consistent with special relativity and causality, determine the time evolution of its decay probability. Since Lorentz covariance is implicit to the field theory approach, neither boost transformations nor the choice of a specific frame are required to define the time parameter in the amplitude.

In this paper we will also address some questions related to the usual treatment of CP violating parameters. As is well known, the $K^0\overline{K}^0$ (and $B^0\overline{B}^0$) system requires of two parameters to account for CP violation in the propagation (*indirect*) and decay (*direct*) of neutral kaons, usually related to ε and ε', respectively [6]. The description based on the WW formalism is not valid beyond order ε because of the aforementioned difficulty in the normalization of non-orthogonal states. Since $\varepsilon' \sim \mathcal{O}(\varepsilon^2)$ for the $K^0\overline{K}^0$ system, it becomes necessary to

establish a correct formalism [7] to account consistently for terms of order ε^2. Furthermore, this is necessary because the usual approximations for neutral kaons in the WW formalism, might fail in the case of B mesons.

On the other hand, ε and ε' can be related to the observable parameters that measure CP violation in the $K^0\overline{K}^0$ system by assuming isospin symmetry and the factorization of strong rescattering effects [6]. These approximations are rather strong assumptions in view of the smallness of direct CP violating effects [8, 9]. Without involving the isospin decomposition of the amplitudes, in this paper we shall parametrize CP violation in terms of the mixing of CP eigenstates $K_1 - K_2$ (indirect CP violation) and the parameters χ_{+-} and χ_{00} which describe the CP violating 2π decays of K_2 in our approach.

This paper is organized as follows. In section II we discuss the diagonalization of mixed propagators in momentum space for the system of unstable neutral pseudoscalar K and B mesons. In section III we focus on the space-time representations of these propagators. Section IV is devoted to the applications of our formalism to compute the time-dependent distributions of neutral kaon decays as adapted to CPLEAR and DAPHNE experiments. Our conclusions are presented in section V.

2. UNSTABLE PARTICLE PROPAGATOR IN MOMENTUM SPACE

As previously discussed, the propagator is the basic object in the S-matrix amplitude that describes the propagation of an unstable state from its production at space-time point x through its decay at point x′. In this section we study the momentum space representation of the propagator for the neutral kaon system, which will be needed to compute the S-matrix amplitudes.

Since the weak interaction couples the flavor states K^0 and $\overline{K}^0$, the renormalized propagator for these two unstable particles is a non diagonal 2×2 matrix. By imposing the CPT symmetry, we can parametrize the inverse propagator for unstable kaons of four-momentum p as follows [1]:

$$D^{-1}(p^2) = \begin{pmatrix} d & a+b \\ a-b & d \end{pmatrix} \tag{1}$$

where

$$d \equiv p^2 - m_0^2 + i m_0 \Gamma_0 , \tag{2a}$$

$$a \equiv r^2 + i s^2, \tag{2b}$$

$$b \equiv \mu^2 + i \nu^2 , \tag{2c}$$

and m_0, Γ_0, r^2, s^2, μ^2, ν^2 are real quantities.

We define the CP eigenbasis as

$$\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} \equiv S \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} \tag{3}$$

where $S = S^{-1}$. The corresponding inverse propagator is

$$\overline{D}^{-1}(p^2) \equiv S D^{-1}(p^2) S^{-1} = \begin{pmatrix} d+a & -b \\ b & d-a \end{pmatrix} . \tag{4}$$

CP conservation implies that b vanishes. For the $K^0 - \overline{K}^0$ system, b is small compared to the diagonal terms and it is predominantly imaginary [10].

Now, if we introduce the complex parameter $\hat{\varepsilon}$ as

$$\frac{\hat{\varepsilon}}{1+\hat{\varepsilon}^2} \equiv \frac{b}{2a}, \tag{5}$$

we can derive the diagonal form of the inverse propagator because Equation (4) can be rewritten as,

$$\overline{D}^{-1}(p^2) \equiv \frac{1}{1-\hat{\varepsilon}^2}\begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix}\begin{pmatrix} d+a\dfrac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2} & 0 \\ 0 & d-a\dfrac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2} \end{pmatrix}\begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix}. \tag{6}$$

Therefore, the physical basis of neutral kaons consists of two states $K_{L,S}$, of definite masses $m_{L,S}$ and decay widths $\Gamma_{L,S}$, such that

$$d_S \equiv p^2 - m_S^2 + im_S\Gamma_S = d + a\frac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2} \tag{7a}$$

$$d_L \equiv p^2 - m_L^2 + im_L\Gamma_L = d + a\frac{1-\hat{\varepsilon}^2}{1+\hat{\varepsilon}^2}, \tag{7b}$$

and the propagator $\overline{D}(p^2)$ can be written as follows

$$\overline{D}(p^2) \equiv \frac{1}{1-\hat{\varepsilon}^2}\begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix}\begin{pmatrix} d_S^{-1} & 0 \\ 0 & d_L^{-1} \end{pmatrix}\begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix}. \tag{8}$$

As already anticipated, the diagonalization of the non-hermitian matrix given in Equation (4) involves a non-unitary matrix. Furthermore, according to Equation (8), we can obtain a proper orthogonal and normalized physical basis if we define independent *ket* (in-) and *bra* (out-) states, respectively, as:

$$\begin{pmatrix} |K_S\rangle \\ |K_L\rangle \end{pmatrix} \equiv \frac{1}{\sqrt{1-\hat{\varepsilon}^2}}\begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix}\begin{pmatrix} |K_1\rangle \\ |K_2\rangle \end{pmatrix} \tag{9}$$

and

$$\begin{pmatrix} \langle K_S| \\ \langle K_L| \end{pmatrix} \equiv \frac{1}{\sqrt{1-\hat{\varepsilon}^2}}\begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix}\begin{pmatrix} \langle K_1| \\ \langle K_2| \end{pmatrix} \tag{10}$$

Notice that bra states do not correspond to simply hermitian conjugate of ket states.

The quantities $m_{S,L}$, $\Gamma_{S,L}$ and $\hat{\varepsilon}$ can be measured experimentally, while the parameters a, b, m_0 and Γ_0 can be in principle computed from the theory. The relationships between these two sets of parameters are:

$$a = \frac{1}{2}\left(\frac{1+\hat{\varepsilon}^2}{1-\hat{\varepsilon}^2}\right)\left\{m_L^2 - m_S^2 - i(m_L\Gamma_L - m_S\Gamma_S)\right\}, \tag{11a}$$

$$m_0^2 - im_0\Gamma_0 = \frac{1}{2}\left\{m_L^2 - m_S^2 - i(m_L\Gamma_L - m_S\Gamma_S)\right\}, \tag{11b}$$

$$b = \frac{\hat{\varepsilon}}{1-\hat{\varepsilon}^2}\left\{m_L^2 - m_S^2 - i(m_L\Gamma_L - m_S\Gamma_S)\right\} . \qquad (11c)$$

Since b is predominantly imaginary for the $K^0 - \overline{K}^0$ [10] and $\hat{\varepsilon} \sim \mathcal{O}(10^{-3})$, we can compute the phase of the CP violation parameter $\hat{\varepsilon}$ which is given by:

$$\phi(\hat{\varepsilon}) = \text{arctg}\left(\frac{m_L^2 - m_S^2}{m_S\Gamma_S - m_L\Gamma_L}\right)$$

$$= \text{arctg}\left(\frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}\right) + O\left(\frac{\Gamma_S}{m_S}, \frac{m_L - m_S}{m_L + m_S}\right) \qquad (12)$$

$$= (43.49 \pm 0.08)^\circ$$

where we have used $\Gamma_S/m_S \approx O(10^{-14})$ and $(m_L - m_S)/(m_L + m_S) \approx O(10^{-14})$. As is well known, this result is in excellent agreement with experimental data [11].

3. SPACE-TIME EVOLUTION OF RESONANCE PROPAGATORS

In this section we are interested in the time dependent properties of the propagation of unstable particles for the purposes of studying CP violation and the time oscillations in the kaon system. We shall therefore focus on the properties of the unstable state propagator in configuration space.

Let us first consider the propagator for a stable spin zero particle:

$$\Delta_F(x' - x) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip.(x'-x)}}{p^2 - m^2 + i\varepsilon} . \qquad (13)$$

To manifest the time dependence in the amplitude, it is necessary to put this expression into another form showing a separate time evolution for the particle and the antiparticle. A contour integration in the complex p^0 plane gives:

$$\Delta_F(x' - x) = -i\int \frac{d^3p}{(2\pi)^3} \frac{e^{-i\vec{p}.(\vec{x}'-\vec{x})}e^{-iE(t'-t)}}{2E'} \theta(t'-t)$$

$$i\int \frac{d^3p}{(2\pi)^3} \frac{e^{-i\vec{p}.(\vec{x}'-\vec{x})}e^{iE(t'-t)}}{2E} \theta(t-t') \qquad (14)$$

with $E \equiv \sqrt{\vec{p}^2 + m^2}$.

Depending of the specific process, the first (second) term in Equation (14) will survive in the time-dependent amplitude and will describe a particle (antiparticle) propagating forward in time.

Let us now consider the propagator of a spin zero resonance. The Dyson summation of self-energy graphs leads to the following renormalized propagator in momentum space representation:

$$\frac{1}{p^2 - m^2 + i\sqrt{p^2}\,\Gamma(p^2)\theta(p^2 - p_{th}^2)} , \qquad (15)$$

where p_{th}^2 is the threshold for the vanishing of the imaginary part of the self-energy in the case that we consider only one decay channel.

In order to justify the constant width approximation used in Equation (7), let us consider the 2π decay width of kaons. A direct computation of this decay width, for a kaon of squared four-momentum s, gives:

$$\Gamma(s) = \frac{m^2}{s}\left(\frac{s - s_{th}}{m^2 - s_{th}}\right)^{1/2}\Gamma \tag{16}$$

where m is the kaon mass, $s_{th} = 4m_\pi^2$ and $\Gamma \equiv \Gamma(s = m^2)$. The influence of the kaon width in the propagator is felt only for $\sqrt{s}$ values near the kaon mass, i.e., for $m - x\Gamma \le \sqrt{s} \le m + x\Gamma$, with x an arbitrary number of order 1 such that $x\Gamma/m << 1$.

Since Γ_S/m_S, $\Gamma_S/(m_S - 2m_\pi) \sim O(10^{-14})$, the form of the propagator with a constant width

$$\frac{1}{p^2 - m^2 + im\Gamma\theta\left(p^2 - p_{th}^2\right)}, \tag{17}$$

turns out to be an extremely good approximation for the renormalized propagator.

Therefore, the space-time representation of the spin zero propagator for the unstable particles can be written as:

$$\Delta_R(x' - x) = \int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-i\vec{p}.(x'-x)}}{p^2 - m^2 + im\Gamma\theta(p^2 - p_{th}^2)}\,. \tag{18}$$

Similarly as done above for the stable particle propagator, we would like to express explicitly the time dependence of $\Delta_R(x'-x)$. It becomes convenient to separate the propagator into two pieces:

$$\Delta_R(x' - x) = \Delta_R^{(1)}(x' - x)\Delta_R^{(2)}(x' - x) \tag{19}$$

with

$$\Delta_R^{(1)}(x' - x) = \int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-i\vec{p}.(x'-x)}}{p^2 - m^2 + im\Gamma}$$

$$\Delta_R^{(2)}(x' - x) = \int \frac{d^3p}{(2\pi)^3}\int_{-p_{th}^0}^{p_{th}^0}\frac{dp^0}{2\pi}\,e^{-i\vec{p}.(x'-x)}\left\{\frac{1}{p^2 - m^2} - \frac{1}{p^2 - m^2 + im\Gamma}\right\} \tag{20}$$

where $p_{th}^0 = \sqrt{\vec{p}^2 + p_{th}^2}$.

Using again the condition $\Gamma/\left(m - \sqrt{p_{th}^2}\right) << 1$, we can show that

$$\Delta_R^{(2)}(x'-x) \sim \mathcal{O}\left(\frac{\Gamma}{m - \sqrt{p_{th}^2}}\right),$$

which allows to write

$$\Delta_R(x'-x)=\int\frac{d^4p}{(2\pi)^4}\;\frac{e^{-ip.(x'-x)}}{p^2-m^2+im\Gamma}\left[1+\mathcal{O}\left(\frac{\Gamma}{m-\sqrt{p_{th}^2}}\right)\right]. \tag{21}$$

In order to made explicit the time dependence of the unstable propagator let us use the following pole decomposition

$$p^2-m^2+i\Gamma m=\left(p_0-E+\frac{i\Gamma m}{2E}\right)\left(p_0+E-\frac{i\Gamma m}{2E}\right)\left(1+\mathcal{O}\left(\frac{\Gamma^2}{m^2}\right)\right) \tag{22}$$

where $E=\sqrt{\vec{p}^2-m^2}$.

Therefore, by neglecting very small terms of order 10^{-14}, the contour integral in the complex p^0 plane with the poles located at $\pm$ (E - imΓ/2E) gives

$$\Delta_R(x'-x)=-i\int\frac{d^3p}{(2\pi)^3}\;\frac{e^{i\vec{p}.(\vec{x}'-\vec{x})}e^{-iE(t'-t)}}{2E}e^{-\frac{\Gamma}{2}\frac{m}{E}(t'-t)}\theta(t'-t)$$

$$-i\int\frac{d^3p}{(2\pi)^3}\;\frac{e^{-i\vec{p}.(\vec{x}'-\vec{x})}e^{-iE(t-t')}}{2E}e^{-\frac{\Gamma}{2}\frac{m}{E}(t-t')}\theta(t-t'). \tag{23}$$

The interpretation is similar to the one for the stable particle, except for the decay constant Γ which expresses the unstability of the particle and antiparticle. The case of $K^0\overline{K}^0$ system considered in this paper is more involved, because the propagator is a 2 × 2 matrix. This problem is circumvented by performing the diagonalization before the contour integration in the complex plane or p^0.

Notice that $\tau = t'- t$ is the time elapsed between the production and decay locations of the resonance. Note also that, contrary to non-relativistic approaches, the factor m/E naturally appears in the exponential decay factor. Therefore, no boost transformations are required to relate the *proper* time to the time parameter of a moving particle. Of course, the exponential decay gets its usual form $e^{-\Gamma\tau/2}$ [1] in the rest frame of the resonance.

4. APPLICATIONS

In this section we compute the full S-matrix amplitudes for the production and decay of neutral kaons as studied at CPLEAR and DAPHNE experiments. Then, we derive the time evolution of these transition amplitudes and introduce the CP violation parameters intrinsic to our description.

4.1. CPLEAR experiments

At the CPLEAR experiment [12], K^0 and $\overline{K}^0$ are produced at point x in the strong interaction annihilation of $p\bar{p}$, and subsequently decay at point x'to $\pi^+\pi^-$ by the effects of weak interactions. The production mechanisms of K^0 and $\overline{K}^0$ are $p\bar{p}\rightarrow K^0K^-\pi^+, \overline{K}^0K^+\pi^-$, thus neutral kaons can be tagged by identifying the accompanying charged kaon [2]. After their production, both K^0 and $\overline{K}^0$ oscillates between their two components K_L and K_S before decaying to the 2π final states. We would therefore be interested in the description of the time evolution of the full decay amplitude and its interference phenomena. It is interesting to note that despite the fact that charged kaons and pions have similar lifetimes as K_L, they can be treated as asymptotic particles in the present case.

In order to relate the different S-matrix amplitudes, let us first consider the production mechanism of $K^0\bar{K}^0$. Since strong interactions conserve strangeness, we have

$$\mathcal{M}\left(p\bar{p} \to K^-\pi^+\bar{K}^0\right) = \mathcal{M}\left(p\bar{p} \to K^+\pi^-K^0\right) = 0\,, \tag{24}$$

which, according to equation (3), implies

$$\mathcal{M}\left(p\bar{p} \to K^-\pi^+K_1\right) = \mathcal{M}\left(p\bar{p} \to K^-\pi^+K_2\right) \equiv A \tag{25a}$$

$$\mathcal{M}\left(p\bar{p} \to K^+\pi^-K_1\right) = -\mathcal{M}\left(p\bar{p} \to K^+\pi^-K_2\right) \equiv B. \tag{25b}$$

Assuming CPT invariance we obtain

$$\mathcal{M}\left(p\bar{p} \to K^-\pi^+K^0\right) = \mathcal{M}\left(p\bar{p} \to K^+\pi^-\bar{K}^0\right) \equiv C. \tag{25c}$$

Collecting all these constraints, we get

$$A = B = \frac{C}{\sqrt{2}}. \tag{26}$$

Now, let us first consider the complete process for the production of a K^0 decaying into $\pi^+\pi^-$

$$p(q) + \bar{p}(q') \to K^-(k) + \pi^+(k') + K^0(p) \to K^-(k) + \pi^+(k') + \pi^+(p_1) + \pi^-(p_2) \tag{27}$$

The full amplitude corresponding to this process can be written (the subscript + - refers to the charged of the two pions from K^0 decay):

$$T_{+-} = \int d^4x d^4x' e^{i(p_1+p_2)\cdot x'} \left(\mathcal{M}\left(K_1 \to \pi^+\pi^-\right), \mathcal{M}\left(K_2 \to \pi^+\pi^-\right)\right) \times \Delta_R^{K_1K_2}(x'-x)$$

$$\begin{pmatrix} \mathcal{M}\left(K^0 \to K_1\right) \\ \mathcal{M}\left(K^0 \to K_2\right) \end{pmatrix} \cdot \mathcal{M}\left(p\bar{p} \to K^-\pi^+K^0\right) e^{i(k+k'-q-q')\cdot x} \tag{28}$$

where $\Delta_R^{K_1K_2}(x'-x)$ is the propagator matrix for the coupled $K_1 - K_2$ system in configuration space.

With the help of equations (3), (8) and (25), this gives:

$$T_{+-} = \int d^4x d^4x' e^{i(p_1+p_2)\cdot x'} \left(\mathcal{M}\left(K_1 \to \pi^+\pi^-\right), \mathcal{M}\left(K_2 \to \pi^+\pi^-\right)\right) \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x'-x)}$$

$$\times \frac{1}{1-\hat{\varepsilon}^2} \begin{pmatrix} 1 & \hat{\varepsilon} \\ \hat{\varepsilon} & 1 \end{pmatrix} \begin{pmatrix} d_S^{-1}(p) & 0 \\ 0 & d_L^{-1}(p) \end{pmatrix} \begin{pmatrix} 1 & -\hat{\varepsilon} \\ -\hat{\varepsilon} & 1 \end{pmatrix} \begin{pmatrix} \mathcal{M}\left(K^0 \to K_1\right) \\ \mathcal{M}\left(K^0 \to K_2\right) \end{pmatrix}$$

$$\times \sqrt{2}A \cdot e^{i(k+k'-q-q')\cdot x} \tag{29}$$

$$= (2\pi)^{-4} \int d^4x d^4x' d^4p e^{i(p_1+p_2-p)\cdot x'} e^{i(k+k'-q-q'+p)\cdot x} \cdot A$$

$$\times \frac{1}{1-\hat{\varepsilon}} \left\{ \left[\mathcal{M}\left(K_1 \to \pi^+\pi^-\right) + \hat{\varepsilon}\mathcal{M}\left(K_2 \to \pi^+\pi^-\right) \right] \frac{1}{p^2 - m_S^2 + im_S\Gamma_S} \right.$$

$$\left. + \left[\hat{\varepsilon}\mathcal{M}\left(K_1 \to \pi^+\pi^-\right) + \mathcal{M}\left(K_2 \to \pi^+\pi^-\right) \right] \frac{1}{p^2 - m_L^2 + im_L\Gamma_L} \right\} \tag{30}$$

$$= (2\pi)^{-4}\delta^{(4)}(q+q'-k-k'-p_1-p_2)\frac{1}{1+\hat{\varepsilon}} \cdot A\mathcal{M}\left(K_1 \to \pi^+\pi^-\right)$$

$$\times \left\{ (1+\chi_{+-}\hat{\varepsilon}) \frac{1}{(p_1+p_2)^2 - m_S^2 + im_S\Gamma_S} \right.$$

$$\left. + (\hat{\varepsilon}+\chi_{+-}) \frac{1}{(p_1+p_2)^2 - m_L^2 + im_L\Gamma_L} \right\} \tag{31}$$

where

$$\chi_{+-} \equiv \frac{\mathcal{M}(K_2 \to \pi^+\pi^-)}{\mathcal{M}(K_1 \to \pi^+\pi^-)} \,, \tag{32}$$

is the parameter describing direct Cp violation in our approach.

To obtain the time dependence of the full amplitude where an originally pure K^0 state decays to $\pi^+\pi^-$, we must insert Equation (23) into Equation (28) and we get:

$$T_{+-} = -i(2\pi)^4\delta^{(4)}(q+q'-k-k'-p_1-p_2)\frac{1}{1+\hat{\varepsilon}} A\mathcal{M}\left(K_1 \to \pi^+\pi^-\right)$$

$$\times \left\{ (1+\chi_{+-}\hat{\varepsilon}) \int dt \frac{1}{2E_S} \left[e^{-i(E_S-E)t} e^{-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}\theta(t) + e^{i(E_S-E)t} e^{\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}\theta(-t) \right] \right.$$

$$\left. + (\hat{\varepsilon}+\chi_{+-}) \int dt \frac{1}{2E_L} \left[e^{-i(E_L-E)t} e^{-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}\theta(t) + e^{i(E_L-E)t} e^{\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}\theta(-t) \right] \right\} \tag{33}$$

where $E = p_1^0 + p_2^0$ is the total energy of the $\pi^+\pi^-$ system,

$$E_S = \sqrt{(\vec{p}_1+\vec{p}_2)^2 + m_S^2} \,,$$

$$E_L = \sqrt{(\vec{p}_1+\vec{p}_2)^2 + m_L^2} \,,$$

and we have defined the time t in Equation (33) as the time elapsed from the production to the decay locations of K^0. Thus, the transition amplitude T(t) describing the time evolution of the system for $t > 0$ is given by the integrand proportional to $\theta(t)$ in Equation (33), namely

$$T_{+-}(t) = -i(2\pi)^4\delta^{(4)}(q+q'-k-k'-p_1-p_2)\frac{1}{1+\hat{\varepsilon}} A\mathcal{M}\left(K_1 \to \pi^+\pi^-\right)e^{iEt}$$

$$\times\left\{\frac{1}{2E_S}e^{-iE_S\hat{t}}e^{-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}(1+\chi_{+-}\hat{\varepsilon})+\frac{1}{2E_L}e^{-iE_Lt}e^{-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}(\hat{\varepsilon}+\chi_{+-})\right\}. \quad (34)$$

Let us now consider the analogous process where a pure $\bar{K}^0$ state is initially produced and the decay to $\pi^+\pi^-$, i.e. $p\bar{p}\to K^+\pi^-\bar{K}^0\to K^+\pi^-\pi^+\pi^-$ [12]. Following the same procedure as in the case of K^0 production and decay, we can get the following expression for the time evolution of $\bar{K}^0$ decays:

$$T'_{+-}(t)=(-i)(2\pi)^4\delta^{(4)}(q+q'-k-k'-p_1-p_2)\frac{1}{1-\hat{\varepsilon}}A_{\bar{M}}(K_1\to\pi^+\pi^-)e^{iEt}$$

$$\left\{\frac{1}{2E_S}e^{-iE_S\hat{t}}e^{-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}(1+\chi_{+-}\hat{\varepsilon})-\frac{1}{2E_L}e^{-iE_Lt}e^{-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}(\hat{\varepsilon}+\chi_{+-})\right\}. \quad (35)$$

Let us notice that if we were interested in the $\pi^0\pi^0$ decay mode of neutral kaons, we would have to replace in Equations (34) and (35) $\mathcal{M}(K_1\to\pi^+\pi^-)$ by $\mathcal{M}(K_1\to\pi^0\pi^0)$ and χ_{+-} by χ_{00} where

$$\chi_{00}\equiv\frac{\mathcal{M}(K_2\to\pi^0\pi^0)}{\mathcal{M}(K_1\to\pi^0\pi^0)} \quad (36)$$

Using Equations (9(and (10), we can express the ratio of CP-violating to CP-conserving decay amplitudes of K_L, K_S states in terms of the CP-violating parameters in our approach:

$$\eta^{+-}\equiv\frac{\mathcal{M}(K_L\to\pi^+\pi^-)}{\mathcal{M}(K_S\to\pi^+\pi^-)}=\frac{\hat{\varepsilon}+\chi_{+-}}{1+\chi_{+-}\hat{\varepsilon}}, \quad (37)$$

and

$$\eta^{00}\equiv\frac{\mathcal{M}(K_L\to\pi^0\pi^0)}{\mathcal{M}(K_S\to\pi^0\pi^0)}=\frac{\hat{\varepsilon}+\chi_{00}}{1+\chi_{00}\hat{\varepsilon}}. \quad (38)$$

As is well known, the parameters η^{+-} and η^{00} are commonly used to express the violation of CP in the two pion decays of K_L (see for example pages 422-425 in [11]). Note that the above relations between measurable quantities and the parameters that quantify direct and indirect violation of CP, are derived without relying on assumptions based on isospin symmetry, contrary to the relations obtained for the η parameters in terms of the usual parameters ε and ε'. Since the parameters $\chi_{+-,00}$ are expected to be very small, we can neglect terms of $O(\chi_i\chi_j\varepsilon)$ in the above equations and use isospin symmetry to show that in that limit,

$\chi_{+-}=\epsilon'$

$\chi_{00}=-2\varepsilon'$.

Finally, let us mention that Equations (34) and (35) reduce to the two well known expressions for the time evolution used in the analysis of the CPLEAR collaboration [12], when we choose the center of mass frame of the two pion produced in $K^0-\bar{K}^0$ decays.

4.2. Neutral kaon production ad DAPHNE

In this section we consider the oscillations of the pair of neutral kaons produced in e^+e^- annihilations at DAPHNE [4]. The results obtained in the present formalism for the $K^0\bar{K}^0$ system can be straightforward generalized to describe the same phenomena in pair production of neutral B mesons in the $\Upsilon(4s)$ region [5].

Neutral and charged kaons will be copiously produced ($\sim 10^9$ pairs $K^0\bar{K}^0$/year) in e^+e^- collisions operating at a center of mass energy around the mass of the $\phi(1020)$ meson [4]. The ϕ mesons produced in e^+e^- annihilations decay at point x into $K^0\bar{K}^0$ pairs, and subsequently each neutral kaon oscillates between its K_L–K_S components before decaying to final states $f_1(p)$ and $f_2(p')$ at spacetime points y and z:

$$\phi(q) \to K^0\bar{K}^0 \to f_1(p)f_2(p') \tag{39}$$

where q, p and p´are the corresponding four-momenta.

Since each final state can be produced by either K^0 or $\bar{K}^0$, we must add coherently the two amplitudes arising from the exchange of K^0 or $\bar{K}^0$ as intermediate states. Conservation of angular momenta forces the system of neutral kaons to be in a p-wave. Taking into account the charge conjugation properties of the electromagnetic current, the pair of neutral kaons are found to be in a total antisymmetric wavefunction [13]. Thus, the relative sign of the two contributions to $\phi \to f_1f_2$ decays must be negative. The S-matrix amplitude for the process indicated in Equation (39) is:

$$\begin{aligned}
T_{f_1f_2} &= \int d^4x d^4y d^4z e^{ip.y+ip'.z}\mathcal{M}(\phi \to K^0\bar{K}^0)e^{-iq.x} \\
&\times\left\{\left(\mathcal{M}(K_1 \to f_1)\,\mathcal{M}(K_2 \to f_1)\right)\Delta_R^{K_1K_2}(y-x)\begin{pmatrix}\mathcal{M}(K^0 \to K_1)\\ \mathcal{M}(K^0 \to K_2)\end{pmatrix}\right\} \\
&\times\left\{\left(\mathcal{M}(K_1 \to f_2)\,\mathcal{M}(K_2 \to f_2)\right)\Delta_R^{K_1K_2}(z-x)\begin{pmatrix}\mathcal{M}(\bar{K}^0 \to K_1)\\ \mathcal{M}(\bar{K}^0 \to K_2)\end{pmatrix}\right\}
\end{aligned} \tag{40}$$

Let us define $\mathcal{M}_{ij} \equiv \mathcal{M}(K_i \to f_j)$. With the help of Equations (3) and (8), we can reexpress the previous amplitude as:

$$\begin{aligned}
T_{f_1f_2} &= \int d^4x d^4y d^4z e^{ip.y+ip'.z-iq.x}\int\frac{d^4k}{(2\pi)^4}\frac{d^4k'}{(2\pi)^4}e^{-ik.(y-x)-ik'.(z-x)}\frac{1}{2(1-\hat{\varepsilon}^2)} \\
&\left\{\left[(\mathcal{M}_{11}+\hat{\varepsilon}\mathcal{M}_{21})d_S^{-1}(k)+(\hat{\varepsilon}\mathcal{M}_{11}+\mathcal{M}_{21})d_L^{-1}(k)\right]\right. \\
&\times\left[(\mathcal{M}_{12}+\hat{\varepsilon}\mathcal{M}_{22})d_S^{-1}(k')-(\hat{\varepsilon}\mathcal{M}_{12}+\mathcal{M}_{22})d_L^{-1}(k')\right] \\
&-\left[(\mathcal{M}_{11}+\hat{\varepsilon}\mathcal{M}_{21})d_S^{-1}(k)-(\hat{\varepsilon}\mathcal{M}_{11}+\mathcal{M}_{21})d_L^{-1}(k)\right] \\
&\left.\times\left[(\mathcal{M}_{12}+\hat{\varepsilon}\mathcal{M}_{22})d_S^{-1}(k')-(\hat{\varepsilon}\mathcal{M}_{12}+\mathcal{M}_{22})d_L^{-1}(k')\right]\right\} \\
&\cdot\mathcal{M}\left(\phi \to K^0\bar{K}^0\right) \\
&=(2\pi)^4\delta^{(4)}(q-p-p')\frac{1}{1-\hat{\varepsilon}^2}\times\mathcal{M}\left(\phi \to K^0\bar{K}^0\right) \\
&\left\{-(\mathcal{M}_{11}+\hat{\varepsilon}\mathcal{M}_{21})(\hat{\varepsilon}\mathcal{M}_{12}+\mathcal{M}_{22})\frac{1}{p^2-m_S^2+im_S\Gamma_S}\cdot\frac{1}{p'^2-m_L^2+im_L\Gamma_L}\right.
\end{aligned} \tag{41}$$

$$+(\hat{\varepsilon}\mathcal{M}_{11}+\mathcal{M}_{21})(\mathcal{M}_{12}+\hat{\varepsilon}\mathcal{M}_{2})\frac{1}{p^2-m_L^2+im_L\Gamma_L}$$

$$\cdot\frac{1}{p'^2-m_S^2+im_S\Gamma_S}\Bigg\}. \qquad (42)$$

As anticipated, the relative sign of the two contributions is negative.

As in the previous subsection, in order to obtain a time evolution of the amplitude, we can insert the explicit time-dependent propagator, Equation (23), into the amplitude (41). The result is

$$T_{f_1f_2}=\int dtdt'(T(t,t')\theta(t)\theta(t')+\text{other terms in }\theta(\pm t),\theta(\pm t'))$$

where t and t′are the times taken by unstable kaons to propagate from the common production point (x) up to their disintegration into f_1 at point y and f_2 at point z, respectively.

Thus, the explicit time evolution of the decaying amplitude is given by

$$T(t,t')=\frac{1}{1-\varepsilon^2}(2\pi)^4\delta^{(4)}(q-p-p')e^{ip^0t+ip'^0t'}\frac{1}{4E_SE_L}\cdot\mathcal{M}\left(\phi\to K^0\bar{K}^0\right)$$

$$\Bigg\{-[\mathcal{M}_{11}+\hat{\varepsilon}\mathcal{M}_{21}][\hat{\varepsilon}\mathcal{M}_{12}+\mathcal{M}_{22}]\times e^{-iE_S(p)t-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t}\cdot e^{-iE_L(p')t'-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t'}$$

$$+[\hat{\varepsilon}\mathcal{M}_{11}+\mathcal{M}_{21}][\mathcal{M}_{12}+\mathcal{M}_{22}]\times e^{-iE_S(p')t'-\frac{1}{2}\Gamma_S\frac{m_S}{E_S}t'}\cdot e^{-iE_L(p)t-\frac{1}{2}\Gamma_L\frac{m_L}{E_L}t}\Bigg\}$$

where $E_{S;L}(p)\equiv\sqrt{\vec{p}^2+m_{S,L}^2}$.

As we have already pointed out in the case of the CPLEAR experiment, no boost transformations are required to adequate the time evolution of the decay amplitude to a givem reference frame. Observe that, due to the initial antisymmetrisation of the $K^0\bar{K}^0$ system, $T(t,t)=0$ if $f_1=f_2$ and $p=p'$as noted in Reference [13].

5. DISCUSSIONS AND CONCLUSIONS

We have already discussed in the introduction the problems intrinsic to the Wigner-Weisskopf approximation. Other papers have appeared recently criticizing the old approach [14, 15, 16, 17] but they all present shortcomings which we will discuss in detail in an extended version of this paper. Their introduction of a proper time parameter for each particle forces them to use boosts to express the answer in a common frame. As we have shown in this article, relativistic quantum field theory yields results for the time evolution valid in any frame. An interference term showing time oscillation has then to be converted to a space evolution by the classical formula $t=\frac{E}{|\vec{p}|}x$. We remark here that this formula applies to particles observed in the detector and not to unstable K_S and K_L states.

A similar discussion based on field theory was considered sometime ago in Reference [3]. The field theory formalism, considering unstable particles as intermediate states between in and out stable states avoids asking questions without answers about these intermediate particles. Moreover the result is relativistically

correct and valid in any frame. Finally, we have introduced the CP violation parameters $\hat{\varepsilon}$ and χ_{+-}, χ_{00} (cf equations (5), (32) and (36)) without relying on the Wigner-Weisskopf effective hamiltonian or approximations based on isospin symmetry.

REFERENCES

[1] See for example: P.K. KABIR, "The CP Puzzle" (Academic Press, New York, 1968).

[2] WIGNER, E. and V.F. WEISSKOPF (1930): **Z. Phys.** 63, 54 ibid, 65 (1930) 18.

[3] SACHS, R.G. (1963): **Ann. of Phys** 22, 239.

[4] **The DAΦNE Physics Handbook** I, Eds. L. MAIANI, G. PANCHERI and N. PAVER, (Frascati, 1992).

[5] B factories at SLAC and KEK, see description in Reference 11, 130.

[6] See for example: L. L. CHAU, **Phys. Rep.** 95 (1983) 1.

[7] WANG, Q. and A.I. SANDA (1997): **Phys. Rev.** D55, 3131.

[8] E731 collaboration, L.K. GIBBONS *et al:* **Phys. Rev. Lett.** 70, (1993) 1203; **Phys. Rev.** D55 (1997) 6625.

[9] NA31 collaboration, G.D. BARR *et al:* (1993): **Phys. Lett.** B317, 233.

[10] See for example: L. OKUN, **Quarks and Leptons**, (North Holland, Amsterdam, 1982), 97.

[11] The Particle Data Group R. M. BARNETT *et al.*: **Phys. Rev.** D54 Part I (1996).

[12] CPLEAR collaboration, R. ADLER *et al.*, **Phys. Lett.** B363 (1995) 243; ibid. B363 (1995) 237.

[13] LIPKIN, H.J. (1968): **Phys. Rev.** 176, 1715.

[14] LIPKIN, H.J. (1995): **Phys. Lett.** B348, 604.

[15] KAYSER, B. Proc. of the 28th International Conference on High Energy Physics, (World Scientific, Warsaw, (1996) 1135; B. KAYSER and L. STODOLSKY, **Phys. Lett.** 359 (1995) 343.

[16] LOWE, J. *et al.* (1996): **Phys. Lett.** B384, 288.

[17] SRIVASTAVA, S.; A., WIDOM and E. SASSAROLI (1995): **Z. Phys.** C66, 601.